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The Role of Aggregate Preferences for Labor Supply – Evidence from Low-Paid Employment

Luke Haywood and Michael Neumann

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DIW Berlin
German Institute for Economic Research
Mohrenstr. 58
10117 Berlin

Tel. +49 (30) 897 89-0
Fax +49 (30) 897 89-200
http://www.diw.de

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The role of aggregate preferences for labor supply - evidence from low-paid employment

Luke Haywood & Michael Neumann*

8th March 2017

Abstract

Labor supply in the market for low-paid jobs in Germany is strongly influenced by tax exemptions - even for individuals to whom these exemptions do not apply. We present compelling evidence that an individual's choice set depends on other workers' preferences because firms cater their job offers to aggregate preferences in the market. We estimate an equilibrium job search model which rationalizes the strong earnings bunching at the tax exemption threshold using German administrative data. We then simulate modifications to the tax schedule that remove the discontinuity and thus the bunching at the threshold. Results highlight the indirect costs of (discontinuous) tax policies which are shown to be reinforced by firm responses: Workers who would work anyway are hurt by subsidies benefiting groups who enter the market as a result of tax incentives.

Keywords: Tax exemptions, Welfare-to-work, Labor Demand, Job search, Firm responses, Bunching.

JEL Codes: J64 ; J31 ; J22 ; J23

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1 Introduction

It is largely recognized that workers face important constraints on the labor market in choosing labor supply (e.g. Stewart and Swaffield (1997)). Employed workers cannot easily change hours, and unemployed workers might not be able to find jobs with their preferred hours. This has important implications for tax policy design as constraints limit workers' ability to make optimal choices (Chetty et al. (2011)). Many researchers have, therefore, introduced hours constraints into labor supply models (e.g. Van Soest et al. (1990), Bloemen (2008), Beffy et al. (2015)). Job search models provide a way of explaining some of these constraints without resorting to ad-hoc restrictions on the set of labor supply choices available to individuals. Constraints in search models result from informational deficiencies - individuals are only aware of specific job offers and firms' vacancies do not reach all potential workers.

In equilibrium, these frictions can result in additional hours constraints imposed by firms (Chetty et al., 2011): If firms cannot easily change the hours of a posted job offer, they have an incentive to package their hours-wage bundles according to average preferences of job-seeking workers. The restricted choice set of any particular individual worker will depend on other workers' preferences. We term this the *population labor supply effect*. Population labor supply effects represent a channel through which firms mediate individual responses to tax policies. We show how individual responses to tax incentives are reinforced. Tax incentives for one group of workers have unintended effects for other groups of workers (Chetty et al., 2011).

We present direct evidence of a population labor supply effect induced by non-linear tax reductions in Germany: We show a large mass point in the earnings distribution of workers who are subject to a continuous tax schedule - bunching at points where *other* workers face important tax incentives. We use this specific institutional setting to estimate an equilibrium job search model which accounts for population labor supply effects and allows for a discontinuous tax schedule. We estimate our model using a combination of German administrative and household survey data. We find that the tax and SSC exemption for low earnings provoke substantial distortions. With our estimated structural model we then test different counter-factual policy simulations. For example, we show that the distortions can be removed by smoothing the tax schedule. These simulations highlight the indirect costs of (discontinuous) tax policies.

Wage earnings below the minijob threshold are exempted from social security contributions (SSC) and income taxes. We focus on minijobs, a tax policy in the tradition of other *Making Work Pay* (MWP) policies like the Earned Income Tax Credit in the US or the Working Family Tax Credit in the UK (Bargain and Orsini, 2006). Earnings exceeding the minijob threshold are subject to full SSC as well as income taxes, generating huge bunching in the earnings distribution at the threshold. This suggests negative earnings responses for workers who may be willing to work more - a typical feature of MWP policies (Bargain et al., 2010). Recent research underlines firms' role in responses to tax reforms in general (Kopczuk and Slemrod, 2006) and bunching in particular (Chetty et al., 2011; Best, 2014; Gudgeon and Trenkle, 2016). The population labor supply effect is one channel: firms post discontinuously many job offers at the threshold in order to attract more potential workers. Individuals seeking to earn more than the minijob threshold are faced with reduced availability of higher-paying jobs. This study complements theoretical work by Tazhitdinova (2015) who discusses the strong bunching in the earnings distribution due to firms' reactions to individuals' tax treatment at the German minijob threshold. The evidence provided here serves as compelling evidence in favor of an *additional* population labor supply effect.

We make two main contributions. First, we include discontinuous taxes into a standard equilibrium job search model (Shephard (2017) includes continuous taxes). In line with the bunching literature (Saez, 2010), we believe important additional welfare costs are caused by the discontinuous nature of taxes. We show that these are further increased by firm responses. In contrast to previous equilibrium job search models, we are able to rationalize discontinuous bunching at tax thresholds in the interior of the earnings distribution. We are thus able to analyze counter-factual policy reforms that aim at removing the welfare costs induced by tax non-linearities. Second, our unique institutional setting allows us to empirically study population labor supply effects. Tax subsidies offered to certain workers with low earnings end up affecting many other workers in the labor market. We estimate our equilibrium job search model and quantify these distorting effects. We thus contribute to the body of evidence on *Making Work Pay* (MWP) policies (e.g. Blundell (2000) for the US, Bargain and Orsini (2006) for Europe). We show that firms can reinforce the negative earnings responses of already employed workers which are found when evaluating MWP policies (Bargain et al., 2010). We complement work discussing unintended equilibrium consequences of MWP policies (Rothstein, 2010; Andrew, 2010;

Kolm and Tonin, 2011; Tazhitdinova, 2015) with population labor supply as an additional channel.

In section 2 we briefly discuss the market for low-paid employment in Germany. Section 3 presents our model and simulation results. In section 4 we present our data and some descriptive statistics supporting our basic modeling choices. Section 5 discusses identification and estimation. Section 6 presents estimation results and an application of the model. Section 7 concludes.

2 Minijobs in Germany

This study focuses on regular low-paid employment[1], a sector subject to special tax and social security treatment. We focus on the institutions as they existed from 1999-2003. In 2003, another reform took place, which we discuss in section 6.3. The distortion we focus on continues to exist, but identification is not as clean after the most recent reform, since all small jobs are now covered, regardless of other employments.

Employment contracts qualify for special tax and SSC treatment if monthly earnings do not exceed 325 € and jobs have fewer than 15 hours per week.[2] These employment relations are designated as minijobs and are not subject to employee SSC[3]. First, crossing the threshold results in a decrease of net earnings because earnings above the threshold are entirely subject to SSC (around 21% of gross earnings), i.e. the budget set features a notch[4]. Whilst minijobs do not create an entitlement to social security (and jobs with earnings above do), we do not believe that entitlement effects for neither health nor pension insurance are important in the market for low-paid employment: First, most individuals with low labor income are however either health-insured through their partners, the state or have access to specific student health care provision. Second, pension contributions

[1] We use *low-paid* employment to describe jobs with low earnings. This covers all employment relations that yield regular earnings below 800 € per month. We exclude short-term employment consisting of jobs that last for less than a certain number of days per year as they are subject to different rules.

[2] The monthly threshold may be exceeded for two months within a year as long as monthly earnings are on average below the threshold in that year.

[3] Employers' SSC are 10 (health) and 12% (pension) of minijob earnings and thus approximately correspond to employers' SSC for other employment contracts.

[4] In March 2002, a policy called *Mainzer Modell* aimed at improving incentives by balancing the additional SSC due to crossing the threshold by a subsidy which was gradually withdrawn for higher household earnings. The notch with respect to income taxation remained. We nevertheless limit the empirical analysis to the period before March 2002, see section 4.

from low-paid employment would not suffice to increase pension benefits above the social assistance level which is available independent of contributions. We thus treat SSC in this market as a pure tax.

Second, minijob earnings are not liable to income tax. Since earnings above this level were subject to income tax, the size of the additional tax notch depends on the other factors influencing a person's income tax above the minijob notch (in particular, spousal earnings). Since the general income tax allowance was above the minijob threshold, there is no additional income tax notch for singles with no other income. It could become very substantial for married individuals, though.

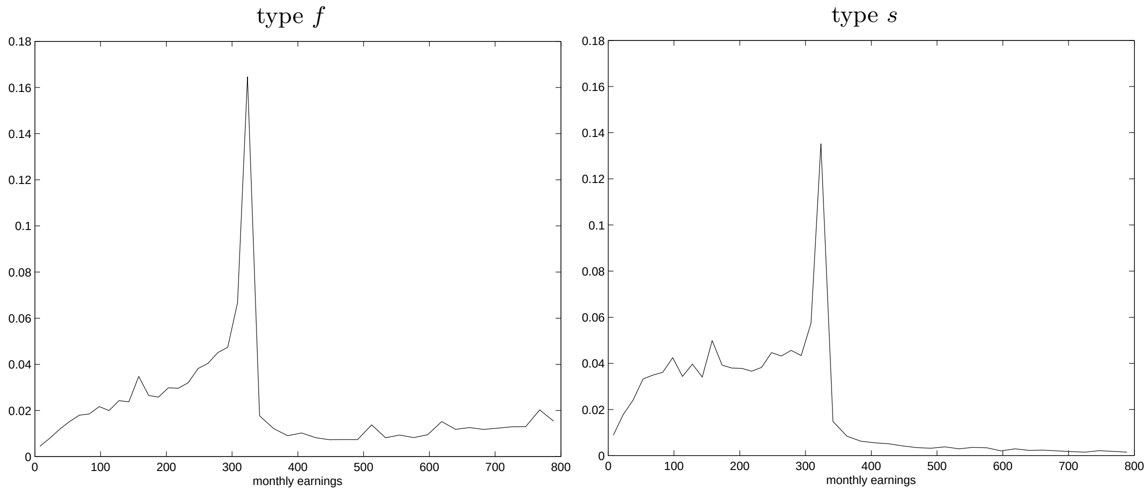
The size of the drop in net earnings is thus composed of SSC and income taxation and varies across individuals. There are thus two sources generating the notch at the minijob earnings threshold. Importantly for this study, if the worker has another job subject to SSC on top of the minijob, a low-paid job cannot qualify as a minijob[5] and is therefore subject to full SSC and income taxation. There is thus no incentive to bunch at the minijob threshold. Nevertheless, we see strong bunching of earnings for workers with main jobs (we call these workers "second jobbers", since they seek a second job in the labor market we consider), as can be seen in figure 1. To explain this phenomenon, we set up an equilibrium model including firms' wage-setting and workers' job search.

3 The Model

Population labor supply effects are constraints on the labor supply choice set imposed by firms in equilibrium. They arise when firms cannot easily adjust their job offers to individual job seekers with different preferences or tax treatment. Firms then have an incentive to tailor their job offers to the expected preferences of potential workers. To model population labor supply effects, an equilibrium search model with wage posting is the most appropriate: First, job search models provide a convenient way of modeling the limited labor supply options most people face: job offers need to be sought and, once found, typically come with fixed hours. Second, by contrast to simple search models, equilibrium models endogenize the job offer distribution. If earnings could be adjusted costlessly (in a simple bargaining framework), we should expect no bunching at thresholds that do not apply to individuals personally. The data suggests otherwise, leading us to

[5] Unless joint income of two minijobs did not exceed the threshold, a rare case in our data.

Figure 1: Distribution of observed earnings by types of workers (Note: type s have no incentive to bunch)



Notes: The graph includes all spells classified as type s or f as outlined in section 4. Note large bunching at the minijob threshold.
Source: SIAB.

assume that firms make take-it-or-leave-it offers along the lines of Burdett and Mortensen (1998) and Bontemps et al. (1999)[6]. The endogenous job offer distribution then reflects the preference distribution of the whole population of job seekers.

We are mainly interested in the sector for low-paid employment and assume that these jobs form a separate labor market. Section 4.3 shows that the industry sector distributions of minijobs and full-time employments are very different and that the former are mainly concentrated in cleaning, security and other firm services, as well as retail. The focus on the low-paid market implies that first jobs of second job holders are not modeled but treated as constant and exogenous.

3.1 Model Basics

The labor market is composed of a continuum of workers and firms. Some workers have another job but are nonetheless active in the market for low-paid jobs. We refer to these as *type s* workers (since they are seeking a second job), of which there are n^s. These workers are not qualified for the tax exemption, which is available for workers seeking a first job in the low-paid market, i.e. *type f* workers. Thus the budget set of type f workers

[6] An alternative modeling choice would be to allow for negotiations for a fraction of workers along the lines of Chetty et al. (2011). In our market for low-paid employment, our assumption of take-it-or-leave-it offers appears particularly credible, see (Shephard, 2017).

exhibits a notch at gross earnings $z = z^*$ while it is smooth for type s workers. Omitting the individual index for simplicity, net earnings c for workers of type j are then given by

$$c = \begin{cases} z & \forall z \leq z^* \ \& \ j = f \\ z(1 - t^{ssc})(1 - t^{inc}) & \forall z > z^* \ \& \ j = f \\ z(1 - t^{ssc})(1 - t^{inc}) & \forall j = s, \end{cases} \quad (1)$$

where t^{ssc} denotes employees' social security contribution rate and t^{inc} the income tax rate. We assume utility v depends on consumption and leisure $\bar{h} - h$ ($\bar{h}$ is total available hours which we set to 12 hours a day). α gives the elasticity of v with respect to net earnings and leisure, such that we have $v = c^{\alpha}(\bar{h} - h)^{(1-\alpha)}$.

Workers search when unemployed and when employed with uniform search intensity by drawing from a known job offer distribution $F(.)$. The exogenous job offer arrival rate (λ) is allowed to differ between type f and type s workers, but is independent of employment status. Workers lose their job at an exogenous rate δ which also differs between worker types. Workers seek to maximize expected steady-state future utility v. Unemployed workers accept jobs according to their individual tax treatment and if the job exceeds the reservation utility v^r. Firms may freely enter the market and seek to maximize profits $\pi = [p - w]hl(v)$ where p is turnover per working hour, $w = \frac{z}{h}$ the gross hourly wage rate and $l(v)$ the size of a firm's labor force which offers jobs with utility v (each firm only offers one type of job).

The remainder of this section first presents a stylized model with no variation in hours worked. In this scenario, we can analytically rationalize bunching in the interior of the earnings distribution (section 3.2). We then resort to numerical solutions to allow for heterogeneous hours and to relax assumptions on taxes (section 3.3). This more realistic version of the model is brought to the data in section 5.

3.2 A Stylized Model of Population Labor Supply Effects

The objective of this section is to show how a notch in the budget set of one group of workers affects the earnings distribution of all workers. We can thereby rationalize bunching in the interior of the earnings distribution. Two simplifying assumptions enable us to solve the model analytically. Both are relaxed in the next section. First, job offers contain no variation in hours. This implies that utility increases monotonously in net earnings, which thus completely describe a job offer.

Second, type f workers only accept job offers with earnings up to the threshold. This is a simplification in order to account for the different incentives facing our two sub-populations: On the one hand, type-s have no incentive to locate above or below the threshold, since no tax deductions apply. The population of type-s thus consists of a selected sample of individuals willing to work despite being taxed - we address this issue when we consider the effect of reducing taxes for this population, see section 6.3. On the other hand, the observed sample of type-f workers is very different from type-s workers: A significant fraction (and for in this section we assume, the universe) of these workers are never observed to accept earnings above the threshold. We model the difference in tax incentives by assuming punitive tax rates for type-f workers above the thresholds and zero taxes for type-s workers - both assumptions are relaxed in the following section.

Importantly, the prominence of type f workers generates a strong incentive for firms to over-proportionally offer such jobs. We now use information about worker mobility to establish the firm size distribution $l(z)$, critical in determining firms' optimal job offers in equilibrium.

Worker mobility

In equilibrium, the flows of workers of each type j moving between unemployment and employment must balance.

$$\delta^j(n^j - u^j) = \begin{cases} \lambda^j u^j & \text{for } j = s \\ \lambda^f u^f F(z^*) & \text{for } j = f \end{cases} \tag{2}$$

where u^j denotes the number of unemployed type j workers and $\kappa^j = \frac{\lambda^j}{\delta^j}$. The flows differ across types since type f workers do not accept jobs with earnings $z > z^*$. The measures of unemployed individuals are thus:

$$u^j = \begin{cases} \frac{n^j}{1+\kappa^j} & \text{for } j = s \\ \frac{n^j}{1+\kappa^j F(z^*)} & \text{for } j = f \end{cases} \tag{3}$$

Similarly, in the steady-state the flow of workers of each type from unemployment to jobs with earnings no greater than z must equal the flow of workers of the same type leaving that group. The latter comprises employees moving from a job with value no greater than z to unemployment or to a better job (the left-hand side of equation 4 with $G^j(.)$ denoting the cumulative density function of realized earnings for type j workers).

$$\lambda^j F(z) u^j = \begin{cases} [\delta^j + \lambda^j (1 - F(z))] G^j(z)(n^j - u^j) & \text{for } j = s \\ [\delta^j + \lambda^j (F(z^*) - F(z))] G^j(z)(n^j - u^j) & \text{for } j = f \ \& z \leq \ z^* \end{cases} \tag{4}$$

Firm size

In the steady-state the number of workers of type j which is employed at a firm offering jobs with earnings z can be expressed by equation 5, which follows Burdett and Mortensen (1998).

$$l^j(z) = \lim_{\epsilon \to 0} \frac{(G^j(z) - G^j(z - \epsilon))(n^j - u^j)}{F(z) - F(z - \epsilon)} \quad \text{for } j \in (s, f) \tag{5}$$

Using expressions 2, 4 and 5, appendix A shows that firm size is increasing in z both below and above z^*, but with a discontinuity at z^*, since type f workers do not accept jobs above this threshold value:

$$\begin{aligned} l(z) &= l^s(z) + l^f(z) \\ &= \begin{cases} \frac{n^s \kappa^s}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))} + \frac{n^f \kappa^f}{(1+\kappa^f(F(z^*)-F(z)))(1+\kappa^f(F(z^*)-F(z-\epsilon)))} & \forall z \leq z^* \\ \frac{n^s \kappa^s}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))} & \forall z > z^* \end{cases} \end{aligned} \tag{6}$$

Equilibrium job offer distribution

If different job offers z by ex ante identical firms are viable in equilibrium, free entry implies that their expected profits must be equal. Firms offering jobs with low earnings achieve higher profits per worker but attract fewer workers than firms offering jobs with higher earnings, as in Burdett and Mortensen (1998). However, when earnings exceed z^*, n^f individuals drop out creating a discontinuity. The endogenous offer distribution might, therefore, include a mass point because firms do not have an incentive to offer slightly higher earnings.

Proposition (I) *If we observe offers above z^*, there must be a mass point of job offers at z^*. The earnings offer distribution above z^* is continuous up to the highest earnings offers, $\bar{z}$.*

Proof: See Appendix B.

The intuition of proposition (I) is that equal profits at and marginally above the threshold can only hold when the loss of type f workers is balanced by a discontinuously large number of type s workers that can be attracted by exceeding the threshold. This requires a mass point at z^*. An earnings offer slightly above the threshold is able to attract all type s workers currently earning z^* (i.e. located at the mass point). As we observe positive mass above the threshold in the data (section 4, proposition (I) implies that there is a mass point at $z = z^*$ (i.e. that $f(z^*) > 0$).

Proposition (II) *If there is a mass point at z^*, there will be a gap in the offer distribution just below the threshold.*

Proof: See Appendix B

To understand this proposition, note that a mass point in our setting implies that any job offer with earnings just below the mass point ($z^* - \varepsilon$) will earn less profits, since margins per worker are only slightly higher, but firm size will be discontinuously lower since there is a mass of firms (offering z^*) that can poach a worker employed at earnings $z^* - \varepsilon$.

Proposition (III) *There may or may not exist earnings offers below the threshold z^* in equilibrium. If there are earnings below z^*, the earnings offer distribution will then be continuous between the left limit of the gap, z'', and reservation earnings, z^r.*

Proof: See Appendix B

The intuition for proposition (III) follows from (I) and (II): Given the lack of offers just below the mass point, i.e. a gap in the offer distribution, the question arises whether any earnings lower than z^* are offered in equilibrium. This is the case when the increase in profit per worker is able to balance the lower firm size vis-à-vis locating at the mass point.

Simulation

The top-right panel of figure 2 shows the features of the earnings distribution simulated using our stylized model[7]: We observe that earnings above the minijob earnings threshold

[7] We simulated the model with the following parameter values: $p = 800$ €; $z^* = 325$ €; $\lambda^s = 0.2$;

(325 €) increase smoothly on the top-left panel up to a maximum of $\bar{z} = 689$ € in our simulation. At the minijob threshold z^*=325 € there is a large mass point, following proposition (I). There is a gap below the threshold in line with proposition (II). Firms do not offer any earnings within that interval. The additional margin of reducing offered earnings compensates for the discontinuously lower firm size below the mass point not until a firm reduces its earnings to z'', which in our simulation is at 127 €. The resulting equilibrium cumulative offer distribution increases smoothly up to $z = z'' = 127$ and is constant in the interval $z \in [127, 325)$ (see the top-left panel of figure 2).

The corresponding earnings distributions of the two types of workers are clearly influenced by the job offer distribution (figure 2): Although type s workers have no tax incentive to bunch at z^*, the earnings distributions of both types exhibit a mass point here (for type f workers the mass point is more prominent and - since these workers accept no job offers above the minijob level - there is no mass above). The upper right panel of figure 2 plots the resulting joint earnings distribution.

3.3 Heterogeneous hours, taxes and worker heterogeneity

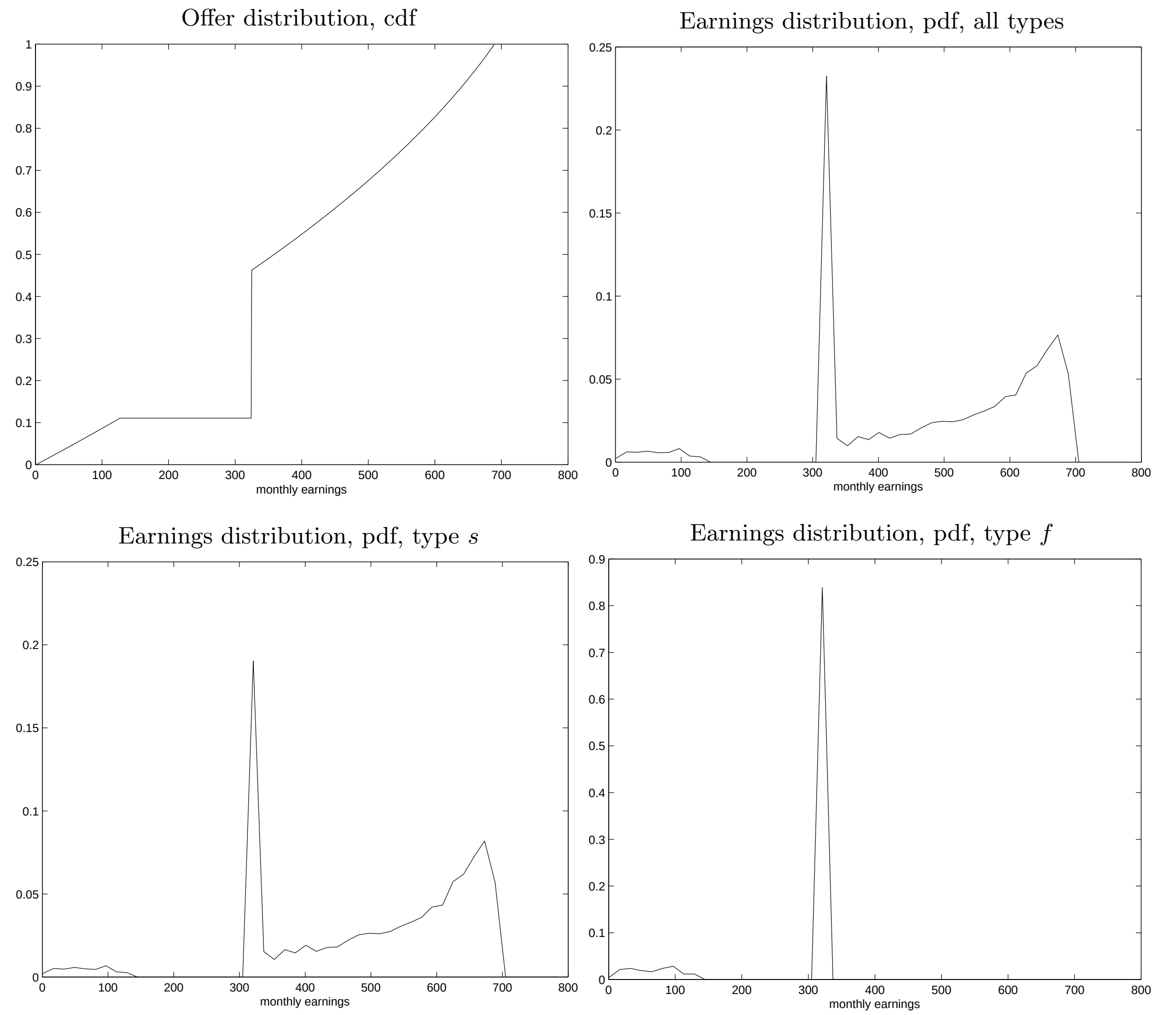
We now enrich the model with two important aspects which prevent an analytical solution: First, working hours may differ. This is particularly relevant as the minijob threshold is based on monthly earnings, not hourly wages. Although there are clear spikes at round numbers, survey data shows variation in hours of low-paid workers (figure 3). This section thus builds an equilibrium model taking into account variation in hours. Including variation in working hours in our model allows both for the existence of earnings offers above the threshold and a lack of a gap below the threshold. Second, although type f workers bunch very strongly at the threshold, we observe a positive mass of first jobs above the threshold, contrary to our assumption so far. This section thus allows for individual variation in tax and SSC rates for t^{inc} and t^{ssc} in equation 1. This generates variation in incentives for type-f workers to accept jobs above the threshold. Additionally, we include further variation in workers' willingness to accept only a minijob compared to a job with higher earnings. We now discuss these new aspects in turn.

Variance in working hours

We assume that hours vary across firms and workers care about hours and earnings.

$\lambda^f = 0.2$; $\delta = 0.1$; $n^s = 1$; $n^f = 0.1$; $z^r = 0$.

Figure 2: Offer and earnings distribution by types of workers



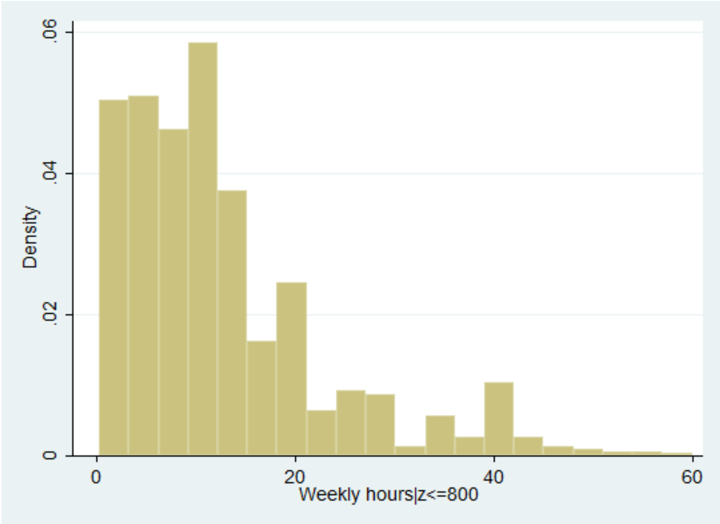
Notes: The graph of type-f workers uses a different scale.

Source: own calculations.

Following Chetty et al. (2011) we assume, however, that hours of work is not a choice variable of firms but predetermined by, for example, technology. The attractiveness of a job offer (for workers and thus also for firms) depends crucially on how many job offers are viewed as superior in terms of utility and could poach a worker away from another firm. Consider a scenario where every firm has a different requirement of weekly hours and where the required hours of work vary continuously across firms[8]. Equal wage offers

[8] Allowing for discrete variation in hours is a non-trivial complication since this generates several thresholds for different wage rates, corresponding to the mini-job earnings threshold with different working hours. The same strategic arguments as in the case of homogeneous hours will prevent firms from locating just below a mass point in the utility offer distribution. We demonstrate this in a version of the model with two hours sectors in Appendix C and note that the model becomes intractable for many discrete hours levels.

Figure 3: Hours distribution - less than 800 €/month



Source: GSOEP wave 2001

of different firms will correspond to different utility levels[9]. This will also mean that every firm will face a different wage threshold w^* corresponding to the wage level that generates monthly earnings of z^*. As the number of weekly hours is not a choice variable, the offered utility at z^* will similarly differ between firms. Given continuous variation in hours, a mass point in the earnings distribution (for example at the threshold) does not result in a mass point in the utility distribution. There is thus no reason for a gap in the offer distribution below the threshold (as in the model without variation in hours).

SSC and Taxes

The SSC rate, t^{ssc}, is constant for all earnings above the threshold and for workers whose employment constitutes a second job (see equation 1). The SSC rate is set to the average legal value over the analysis period (20.69 %). The income tax rate, t^{inc}, depends among others on household characteristics and thus varies across individuals. These characteristics are not observed in our administrative data, so for our simulation as well as the empirical part, we impute tax rates based on an auxiliary survey data set and a micro-simulation model which accounts for the most important aspects of the German tax-transfer system (Appendix D). Tax rates of type s workers vary between roughly 8 % and just below 20 %. Approximately half of type f workers do not pay any taxes

[9]This is true unless workers' marginal utility of leisure is zero, such that the utility function is independent of hours. We do not consider this trivial case.

even when they exceed the minijob threshold, since they do not exceed the general tax allowance. Tax rates of the remaining type f workers amount to up to 20 % (Appendix D).

Worker heterogeneity among type-f workers (those with no other job)

The aim of the tax exemption we study is to increase employment. We have argued that taking into account individual tax rates implies that some individuals without another job (type-f) may have much higher taxes than others. On top of small tax notches, there may be other reasons why workers accept jobs above the threshold. For example, they may have very bad outside options, meaning that they accept job offers even if a job's earnings imply very low net earnings. To take these differences into account, we add a simple form of worker heterogeneity within the population of workers with no other job (type f): A fraction θ of type f workers only seek minijobs and do not accept any offers with earnings above the threshold. We designate these exclusive minijobbers fm. In the current equilibrium, their labor supply is conditional on obtaining the tax exemption. The remaining type f workers accept all offered jobs out of unemployment - and will engage in on-the-job search moving to any job with higher utility in the market for low-paid employment. We designate this group by fa. Their budget set exhibits a notch at the minijob threshold which varies across individuals. Both groups of type f workers thus generate an incentive for firms to offer earnings below or at the threshold.

Worker mobility and firm size

As in the case without variation in hours, workers either accept all jobs (type s and fa) or only minijobs with earnings up to the threshold (type fm). We thus adapt equation 2 accordingly. As before, in equilibrium the flow of workers into and out of jobs with utility no greater than v must coincide:

$$\lambda^j F(v) u^j = \begin{cases} [\delta^j + \lambda^j(1 - F(v))] G^j(v)(n^j - u^j) & \text{for } j \in (s, fa) \\ [\delta + \lambda^f(1 - F(v|z \leq z^*))F(z^*)] G^{fm}(v)(n^{fm} - u^{fm}) = \lambda^f F(v) u^{fm} & \text{for } j = fm \ \& \ z \leq z^* \end{cases} \tag{7}$$

where $(1-F(v|z \leq z^*))F(z^*)$ describes the probability of another firm's offer exceeding v with earnings at or below the threshold. For an individual firm with hours h, the strategic choice is then whether to offer utility $v(z, h)$ or utility $v(z', h)$, where strictly speaking the choice variable is the wage rate, but given fixed hours, different wage rates monotonously translate to different earnings and utilities. Profits are then given by

$$\pi(w) = \begin{cases} (p-w)h(\frac{n^s\kappa^s}{(1+\kappa^s(1-F(v(w))))^2} + \frac{(1-\theta)n^f\kappa^f}{(1+\kappa^f(1-F(v(w))))^2} + \frac{\theta n^f\kappa^f}{(1+\kappa^f(1-F(v(w)|z\leq z^*)))^2}) & \forall z \leq z^* \\ (p-w)h(\frac{n^s\kappa^s}{(1+\kappa^s(1-F(v(w))))^2} + \frac{(1-\theta)n^f\kappa^f}{(1+\kappa^f(1-F(v(w))))^2}) & \forall z > z^* \end{cases}. \quad (8)$$

Characterization of Equilibrium

The model does not have a closed-form solution, but we can sketch some important characteristics of the equilibrium solution.

First, variation in hours implies that the *utility offer distribution* will be smooth. That is, the utility distribution will not exhibit a mass point or gap. If there was a mass point in the utility distribution, a firm could increase profits by slightly increasing the offered wage (and therefore utility). As hours of work are assumed to be distributed continuously, utility $v(z^*)$ varies across firms. We have a wage threshold function $w^*(h)$, where $w^*(h)\ h \equiv z^*\ \forall\ h$. The attractiveness of firms' offers will depend importantly on whether or not a given wage offer corresponds to an earnings level that lies above or below the earnings threshold z^*: This is because for some workers (the any jobbers, fa) utility decreases discontinuously at z^* and a positive number of workers (the exclusive minijobbers, fm) do not accept any offers above. Given a smooth utility offer distribution this implies that there will be a gap in the distribution of earnings above z^* because firm size drops discontinuously when the earnings offer exceeds the threshold. By contrast, there will be no gap below the threshold. The key difference to the case with homogeneous hours is that a mass point at z^* does not imply that firms offering $z^* - \varepsilon$ will attract discontinuously less workers than those offering z^* - attractiveness depends on the ranking of job offers, in terms of utility, not earnings.

Second, in equilibrium, earnings offers will be set such that firms with different hours requirements maximize their profits given other firms' earnings offers and hours requirements. The decision between offering earnings below, at or above the threshold will be assessed by firms according to their hours requirements: Firms with sufficiently small hours ($h \leq \bar{h} = \frac{z^*}{p}$) will offer earnings below the threshold. By contrast, firms with sufficiently high hours ($h > \bar{\bar{h}} = v^{-1}(w^*, \underline{v})$) will offer earnings above the threshold[10]. For

[10]This holds as long as the reservation utility level is strictly positive. As $v(.)$ is monotonously increasing in w in all points except at the threshold $v(w^*, h) < \underline{v}$, this implies that a firm with h or more will not attract any workers by offering $w \leq w^*$.

firms with hours requirements between these two thresholds ($\bar{\bar{h}} > h > \bar{h}$), the common Burdett and Mortensen (1998) trade-off holds: higher marginal profits per worker (the first term in equation 9 representing the first order condition of profit-maximization for a firm with hours requirement h) may be offset by a smaller firm size (the second term in equation 9). Identifying the best offer for these firms is not simple in our setting as firm size of a given firm is not monotonously increasing in the offered wage rate. Exceeding the threshold might even lead to a decrease in firm size as type f workers either suffer a utility drop at the threshold (any jobbers, type fa) or do not accept any jobs with $z > z^*$ (exclusive minijobbers, type fm).

$$\frac{\partial \pi(w)}{\partial w} = -h\left(\frac{n^s \kappa^s}{(1+\kappa^s(1-F(v(w))))^2} + \frac{(1-\theta)n^f \kappa^f}{(1+\kappa^f(1-F(v(w))))^2}\right) \tag{9}$$

$$+ (p-w)h\left(\frac{2n^s {\kappa^s}^2 \frac{\partial F(v(w))}{\partial v(w)} \frac{\partial v(w)}{\partial w}}{(1+\kappa^s(1-F(v(w))))^3} + \frac{2(1-\theta)n^f {\kappa^f}^2 \frac{\partial F(v(w))}{\partial v(w)} \frac{\partial v(w)}{\partial w}}{(1+\kappa^f(1-F(v(w))))^3}\right) \tag{10}$$

The change in firm size by increasing the wage rate crucially depends on $\frac{\partial v(w)}{\partial w}$ which varies between firms with different hours. Assuming a Cobb Douglas utility function, $v(w)$ is a monotonously increasing concave function in all points except the threshold where $v(w)$ drops discontinuously.

Further, $v(w)$ varies between firms. There is an optimal number of hours h^* in the sense that for any given wage w, utility is largest for the firm with $h = h^*$. The popularity of a firm depends on the preference parameter α. At a given w, $\frac{\partial v(w)}{\partial w}$ is larger for 'better' firms. Profit-maximizing earnings of 'better' firms are thus more likely to be high and above the threshold. Firms with low hours requirements need to offer a high wage rate to reach threshold earnings. Since $v(w)$ is concave, a high w implies a small value of $\frac{\partial v(w)}{\partial w}$. Profit-maximizing earnings are likely to be small and below the threshold. In the end, whether a firm offers earnings above the threshold depends on the equilibrium offer distribution $F(v)$. $\frac{\partial F(v(w))}{\partial v(w)}$ needs to be sufficiently high for $w > w^*$ such that enough additional individuals who already hold a job (type s) as well as individuals without a job who accept any job (type fa) can be attracted to balance the loss in marginal profit *as well as* the loss of workers who only accept minijobs (type-fm workers).

Simulation

Since we have no analytic solution for the equilibrium earnings distribution, we solve for it as the solution to a fixed-point problem (in $F(z)$), see appendix F.2. Figure 2 presents results based on simulating the model using realistic parameter values (estimation is relegated to section 5). The simulated cumulative earnings offer distribution exhibits a continuously decreasing slope up to the threshold. The jump at z^* implies a mass point in the density. It is flat above the threshold where the density exhibits a gap. From roughly 500 € it increases again up to approximately 850 € which is the highest earnings level in the simulated market.

This translates into a realized earnings distribution which slowly increases up to a discontinuous mass point at z^* where it sharply drops. This is consistent with the observed distribution (figure 1 above). The increase in the predicted distribution is driven by type f workers. The density for type s workers up to the threshold is slightly decreasing. This is not the case in our data although the observed density is indeed much flatter for type s workers (figure 1).

4 Data

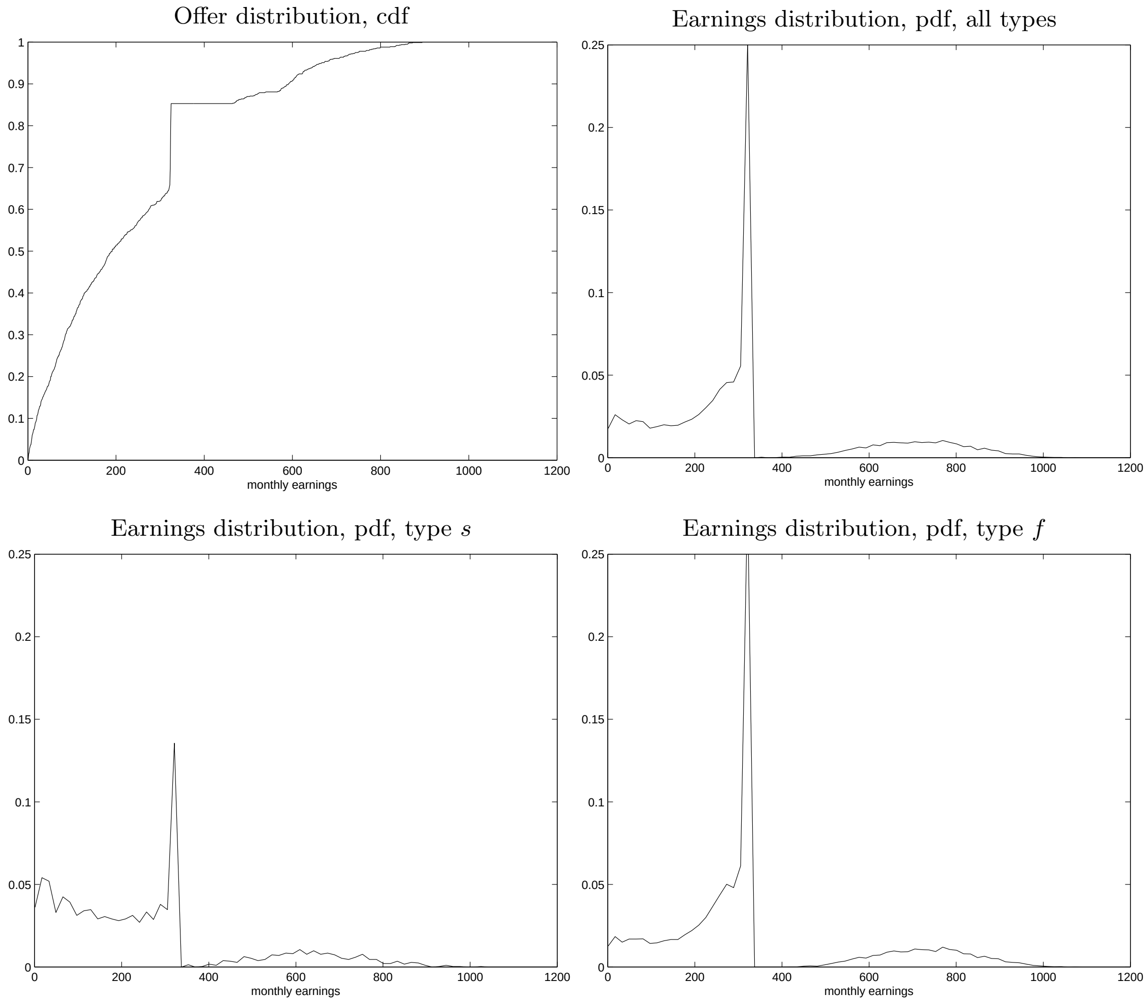
We use administrative data collected by the German social security system. The Sample of Integrated Labor Market Biographies (SIAB) is a representative two percent sample of all individuals for whom an employer's report to the social security system exists[11]. For the present analysis, its specific characteristics makes the SIAB the most suited data set. First, accurate total gross earnings for a period of an employment spell are observed[12]. Second, with approximately 1.6 million sampled employees, the sample size is large enough to include a substantial number of individuals holding second jobs. It does not include civil servants and the self-employed. Third, the SIAB consists of complete employment biographies of the sampled individuals. We can, therefore, differentiate between first and second jobs and observe, for example, the exact day a second employment spell starts. It also includes unemployment spells. Fourth, we observe important individual and firm characteristics: age, sex, occupation and education, and for firms, industry sector, size and wage structure.

Our data have two limitations. First, the number of hours worked is not precisely measured. Hours information only exists in three broad categories. Second, we do not

[11] *Stichprobe der Integrierten Arbeitsmarktbiografien.* We use the weakly anonymized version of the data via on-site use at the research data center of the IAB in Berlin.

[12] A period lasts at most until the end of the calender year. Gross earnings are capped at the earnings cap of pension insurance which is not problematic here as we focus on low-paid jobs.

Figure 4: Offer and earnings distribution by types of workers



Notes: Underlying parameter values as estimated in section 6.1: $\lambda^s = 0.0329$, $\lambda^f = 0.2506$, $\delta^s = 0.0570$, $\delta^f = 0.0379$, $\theta = 0.3715$, $\alpha = 0.8648$. Optimization error is assumed as discussed in section 5.2 ($\sigma = 71.21$). Optimization error is the reason for realized earnings spanning the support of the offer distribution.

Source: own calculations.

observe information about earnings of spouses or other income sources. This information is however needed to calculate precise individual income tax rates. We thus use household data to complement our administrative data and impute individual tax rates. The German Socio Economic Panel (GSOEP)[13] is a yearly representative household survey which includes information on labor market status, earnings, hours and all tax-relevant individual and household information.

[13] See Wagner et al. (2007) for a detailed description.

4.1 Sample definition

We restrict the sample to employees[14] aged 17 to 65 years. For individuals with two employment spells, a second job is the one with lower earnings (very few individuals have more than two jobs, these are dropped). We do not classify very short overlaps between jobs (less than 5 days) as parallel employment spells. Interruptions of less than one month of otherwise similar spells are ignored unless the interruption is filled by another spell[15]. We exclude employees who additionally receive benefits from unemployment insurance or social assistance. Due to high benefit withdrawal rates, low-paid employment is not attractive for these individuals. We include spells which ended after April 1999 or started before March 2002 in the analysis. Every spell is classified as type s or type f, with the same individuals sometimes seeking a main job, and sometimes a second job.

- Type f workers seek or have a main job in the market for low-paid jobs. Unemployed individuals who are observed to be job-seeking are classified as type f if they held a job paying up to 800 € at least once in the sample period. The reason for this restriction is to ensure that workers are actually active in the market for low-paid jobs. Employed type f workers currently hold a single job with earnings of less than 800 €.

- Type s workers already have a main job, but are interested in or have a second job. Two conditions have to be fulfilled for a spell to be classified as type s. First, an individual needs to have a job in the market for better-paid jobs[16]. Second, the individual must be potentially interested in a second job. In absence of information on job-seeking for the whole sample, we restrict our sample to individuals who had at least one second job with earnings up to 800 € in the analysis period.

The market of low-paid jobs is dominated by main jobs (type f). Second job spells (type s) account for around 10 % of all employment spells in our sample (table 1)[17]. While 84 % of type f workers are employed, but only 38 % of type s workers. While a high fraction of unemployment spells are right-censored (69.32 % for type s and 58.87 %

[14]We exclude trainees, interns, employees in the military, individuals in old-age retirement, freelance home workers, people with special needs, short-term employees, seamen and artists.

[15]If an individual has two parallel full-time employments the employment with lower earnings is excluded.

[16]We use all employment spells with earnings above 1000 €. We choose this figure to be sensibly above the maximum value for the market of low-paid jobs, to avoid individuals switching between markets when earnings vary.

[17]This is approximately also true for the number of employed individuals. In June 2001, for example, 96,789 individuals in our sample hold a main job, 9,441 a second job.

for type f, table 1), by far the most frequent reason for censoring is the end of the analysis period. The probability of being right-censored is considerably lower for employment spells (33.31 % for type s and 39.35 % for type f). As minijobs are only included in the data as of April 1999, all spells which started before April 1999 are treated as left-censored. This is the case for roughly 20 % of the sample. Job separation (60.90 % for type s and 51.91 % for type f) is much more frequent than job-to-job transitions (5.8 % for type s and 8.73 % for type f).

Table 1: Total number of spells by type and employment

Status \ Type	s	f
Unemployed	49,984	55,592
unemp. to job	16,290	19,281
right-censored	33,694	36,311
left-censored	11,176	6,485
Employed	30,121	287,255
job to job	1,690	23,803
job to unemp.	17,394	138,861
right-censored	11,037	124,591
left-censored	10,508	80,814

Notes: The by far most frequent reason for right-censoring is the end of the analysis period. For unemployment spells another reason is the start of a new employment with earnings higher than 800 €. Employment spells are treated as right-censored when their defining conditions lose validity before a transition occurs. This includes for example the increase of earnings beyond 800 € or for type s spells the end of the main job.

Source: SIAB; own calculations

4.2 Hours and taxes

While SSC rates are homogeneous in Germany, income tax rates depend on household characteristics. This implies that the size of the notch at the threshold varies between workers. In particular, marital status and spousal income are crucial. Since we have no information on these characteristics in our administrative data, we impute tax rates using detailed information on all tax-relevant characteristics provided by the GSOEP (Junge, forthcoming). For the sake of calculation speed we allocate individuals to three tax groups instead of allowing for continuous variation. Type s workers face larger tax rates than type f workers (table 2), and this is mainly driven by first job earnings. Appendix D presents the procedure for imputing tax rates in more detail.

The model outlined in the previous section assumes continuous variation in hours of work. Since we do not have information on hours of work in the data, we use the GSOEP to impute hours of work based on variables included in the SIAB: sex, gross earnings, sector and education. As the hours distribution resembles a log-normal distribution (see figure 3), a generalized linear model with log link is used for imputation. We then predict individual hours of work in our sample and let firms draw from the smoothed predicted empirical hours distribution. Mean predicted hours per month are approximately 40 for type s workers translating into 10 weekly working hours (table 2). Low-paid main jobs come with on average 55 hours per month. Within-type variation in hours is substantial. The average estimated hourly wage rate is approximately 5.50 € for both worker types[18]. We relegate details on imputation of hours to appendix E.

Table 2: Imputed hours, wage and tax rates

	type s		type f	
	mean	std	mean	std
weekly hours	8.95	4.61	12.84	7.66
wage rate	5.56	1.48	5.76	1.37
tax rate	32.61	3.27	28.03	6.02

Notes: Variation in tax rates is between groups. Tax rates include SSC (20.69 pp.).

Source: SIAB, GSOEP wave 2001, own calculations.

4.3 Descriptives

The earnings response to the tax exemption is dramatic. The observed earnings distribution of low-paid jobs as first jobs features a huge peak at the threshold for minijobs (figure 1 in section 2, left panel). This group of workers (type f) is subject to the minijob threshold implying a strong incentive to locate below the 325 € threshold. This is not the case for second job earnings as the minijob regulations do not apply. Nevertheless, the earnings distribution of type s workers features a clear peak at the minijob threshold and hardly any mass beyond (right panel of figure 1). We interpret this as compelling evi-

[18] This is quite low, in particular relative to the minimum wage of 8.50 € introduced a decade later. However, the median wage rate in the GSOEP sample used for the hours imputation is 7.2 € and the model fit is fairly good (see appendix E).

dence for the existence of a population labor supply effect[19]. Firms seem to offer contracts based on employees looking for minijobs as main jobs, consistent with our assumption of both types drawing from the same take-it-or-leave-it offer distribution. Furthermore, the earnings densities of both types increase gradually up to the mass point which is of similar relative size. The fraction of employees below the threshold is higher for type s ($\approx$ 90 %) than for type f workers ($\approx$ 75 %), though. We therefore allow the offer arrival rate to vary between types in our model. This is consistent with observed differences in unemployment spells, which are shorter for type f workers (see table 3). By contrast, the average duration in a job is similar for both types.

Table 3: Spell durations

Status \ Type	s	f
Unemployment spell		
uncensored	5.76	4.14
only right-censored	9.40	4.62
only left-censored	12.74	9.42
left+right-censored	10.87	6.2
Job spell		
uncensored	5.56	5.45
only right-censored	10.75	10.85
only left-censored	10.32	12.19
left+right-censored	28.94	28.09
Employment spell		
uncensored	6.10	8.49
only right-censored	11.76	14.77
only left-censored	11.94	17.72
left+right-censored	29.37	29.04

Source: SIAB, own calculations

The distribution of sectors of main and second jobs in the low-paid market are fairly similar (first two columns in table 4): The dominant sectors are “cleaning, security and other firm services” as well as wholesale/retail, with the former more frequent as a second job and retail more common as a first job. This also supports the assumption that first and second jobs in the low-paid market are drawn from the same job offer distribution. By contrast, manufacturing is the major sector where second job holders work in their first job (third column in table 4). This suggests that low-paid employment is indeed a separate market.

[19] As we can observe whether a job benefits from the tax exemption, there is no measurement error with respect to misclassification in the data. Interestingly, the small spike slightly below 200 € is also an indication of a population labor supply effect. Workers receiving unemployment benefits have an incentive to earn at most 165 €. Although our sample excludes these workers, we see a small spike at 165 €.

Table 4: Distribution of sectors, 2000-2002

Sector	Low-paid job Type f	Type s	1st job of Type s
D Manufacturing	11.67	10.27	30.02
G Wholesale & retail	22.22	13.74	13.53
H Hotels & restaurants	9.08	8.62	2.21
I Transport	4.77	8.70	5.87
K Cleaning, security & other firm services	18.98	28.70	10.52
L Public admin	2.18	2.97	7.62
N Health	10.87	8.08	12.72
O Other services	7.74	9.18	4.21

Source: SIAB

In our sample most workers holding a low-paid job as main job are relatively young women (table 5). East Germans are slightly over-represented and over 90% have at most vocational training. Unemployed type f workers are relatively more likely male, East German and not highly educated. Survey data shows that individuals with minijobs as main job can be categorized as housewives/-men, students, pensioners and as registered job-seekers. Housewives constitute the largest and registered job-seekers the smallest group (Körner et al., 2013). Only roughly one quarter say that they do not work more because they cannot find a suitable position, in line with our differentiation between workers willing to gain higher earnings and workers exclusively seeking minijobs (fa and fm). Furthermore, minijobs have not been found to serve as stepping stones to better-paid jobs (Freier and Steiner, 2007), such that we can treat this market as independent. The gender ratio of workers holding or seeking side jobs is fairly balanced (table 5). The share of East German workers is smaller and type s workers are slightly better educated than type f workers. The age distribution is roughly similar across types. The main jobs of type s workers are mainly full-time positions with on average slightly more than 2000 € earnings per month. This is similar for employed and unemployed type s workers.

5 Identification and Estimation

We seek to estimate seven parameters[20]: One offer arrival rate for each worker type (λ_s and λ_f), one job destruction rates per worker type (δ_s and δ_f), the relative weight of consumption and leisure in the utility function (α), the fraction of type f workers who do not accept jobs with earnings exceeding the threshold (θ) and the level of productivity per hour (p). In this section we first discuss identification of these parameters (section 5.1) before presenting our two-step maximum likelihood (ML) procedure which is used for

[20]Reservation utility is set to zero as we observe jobs with very small utility.

Table 5: Socio-demographic characteristics by type, 2001

	type s		type f	
	unemployed	employed	unemployed	employed
Female	48.20	45.15	52.67	76.26
Age	36.51	39.73	40.06	38.77
East German	9.07	7.76	29.82	13.22
Education				
Intermediate	12.99	12.97	28.13	24.24
Voc. training	74.81	74.71	65.16	67.16
Grammar	6.72	6.15	3.71	4.84
University	5.48	6.17	3.00	3.76
First job				
Monthly earnings	2025	2131	.	.
Part-time	17.4 %	18.3 %	.	.

Source: SIAB; own calculations

estimation (section 5.2).

5.1 Identification

We present arguments for identification of our key parameters, and whilst these could mostly also be used to construct estimators, we use a different estimation procedure, outlined in the following section. The frictional parameters are identified by observed durations in (un-)employment. Type s workers are assumed to accept all jobs. Their job offer arrival rate λ^s is thus identified by the observed duration in unemployment for type s workers. An unobserved fraction of type f workers also accepts all offers out of unemployment - we denote these by fa. Type f workers who accept jobs above the threshold are necessarily of type fa, who accept all jobs. The duration in unemployment of this observed group identifies the job offer arrival rate of type f workers, λ^f. Duration in employment (for different types, and including job-to-job mobility) identifies δ_s and δ_f. The upper support of the wage distribution identifies the underlying productivity of jobs. Alternately, the job offer distribution is non-parametrically identified from the distribution of accepted earnings by unemployed type s workers who accept all jobs.

The distribution of realized earnings (in particular, the proportion of earnings below, at or above the threshold) is informative about both the preference parameter α and θ, the proportion of individuals who exclusively accept minijobs. A high θ implies a high fraction of workers who do not accept offers with earnings $z > z^*$, increasing the incentive for firms to offer earnings at or below the threshold. The same is true for a low α implying a low relative weight of earnings (vis-à-vis leisure) in the utility function. Differences in unemployment durations across worker types are also informative of θ. For unemployed workers who only accept minijobs (type fm workers) the probability of a match in any one period is $\lambda_f F(z^*)$. For other workers this probability is simply λ_f or λ_s, respectively. Conditional on λ_f and the offer distribution, the difference in unemployment duration between type s and f is thus informative about θ. The same is true for the difference in unemployment duration between type f workers who accept earnings offers below and above the minijob threshold.

5.2 Estimation procedure

We use a two-step ML procedure. In the **first step**, the frictional parameters are estimated based on duration data. The preference parameters are then estimated in the **second step** based on data on labor market spells and earnings. Although less efficient, two-step estimation procedures are more robust. In our case, joint estimation is numerically extremely costly as well.

For the **first step**, we use workers' relevant employment spells. The duration in employment is used to estimate the job separation rates following equation 11, which consists of the probability that an employment spell is drawn and the exponentially distributed duration of employment, t_{emp}, with transition rate δ,

$$L^j_{e1}(t_{emp}) = \left(1 - \frac{n^j \delta_j}{\delta_j - \lambda_j}\right) \delta_j^{1-d} exp[-\delta_j(t_{emp})] \quad \text{for } j \in (s, f), \tag{11}$$

where d is a dummy for censored observations (left- or right-censored). We estimate job offer arrival rates using the duration in unemployment (equation 12 where t is spell length). Exclusive minijobbers (type fm workers), of course, do not accept jobs with $z > z^*$. Their duration in unemployment depends on the offer distribution. We therefore only use type f workers who accept earnings above the threshold.

$$L^j_{u1}(t) = \frac{n^j \delta_j}{\delta_j - \lambda_j} \lambda_j^{1-d} exp[-\lambda_j(t)] \quad \text{for } j \in (s, fa) \tag{12}$$

The **second step** uses information on durations as well as earnings in order to estimate θ and α. While the endogenous job offer distribution $F(.)$ depends on structural parameters, we do not have an analytical expression. In our setting the restrictions on the earnings distribution help identify α, the parameter of the utility function and θ, the unobserved fraction of type f workers who do not accept any jobs with $z > z^*$ (type fm). In contrast to the non-parametric two-step procedure proposed by Bontemps et al.

(1999)[21] we therefore calculate $f(.)$ and $F(.)$ by numerically solving the firms' problem of choosing the profit maximizing earnings level in every iteration (see Appendix F for more details).

The likelihood contribution of an unemployed worker is composed of the probability of being unemployed, the spell duration and the realised wage of the following employment spell (if observed, i.e. $d_r = 0$).

$$L^j_{u2}(t,z) = \frac{n^j \hat{\delta}_j}{\hat{\delta}_j - \hat{\lambda}_j} \left(\hat{D^j_u}\right)^{1-d} exp[-\hat{D^j_u} t] * \left(\int_{-\infty}^{\infty} \hat{f}_z(z+\eta) dh(\eta) \right)^{(1-d_r)} \quad \text{for } j \in (s, fa, fm) \tag{13}$$

where $\hat{D^j_u}$ denotes the estimated arrival rate of acceptable offers. For $j \in (s, fa)$ it holds that $\hat{D^j_u} = \hat{\lambda}_j$. For $j = fm$, $\hat{D^j_u} = \hat{\lambda}_j \hat{F}_z(z^*)$. The third part of equation 13 is the probability that the accepted job has true earnings $\tilde{z_u} = z_u + \eta$ where z denotes observed earnings and $\eta \sim N(0, \sigma^2)$ represents random optimization error. Firms might not always offer exactly desired earnings because, for example, weekly hours or the wage rate usually is a round number. Frictions are a standard assumption in the bunching literature where bunching is usually found as a hump around the threshold rather than an exact spike (Saez, 2010)[22].

For an employed individual the likelihood contribution is then

[21] While Bontemps et al. (1999) similarly have no analytical expression for the job offer distribution, they can apply the following two-step procedure: First non-parametrically estimate the cdf and pdf of the realized wage distribution (G and g). Using these estimates and their relation to the offer distribution (equation 7), the likelihood contributions can be expressed in terms of the realized wage distribution and the model parameters. Once the transition parameters are estimated from workers' mobility patterns, these are used to transform the observed distribution $G(.)$ to the offer distribution $F(.)$ which is the object required in the likelihood. This strategy only exploits equations with respect to *workers'* behavior, the offer distribution is basically treated as exogenous. Firms' behavior leading to the endogenous offer distribution is not exploited, but is key to our identification argument.

[22] We do not allow for the case that true earnings are below the threshold while observed earnings are above or vice versa: When $z > z^* > \tilde{z}$ or $z < z^* < \tilde{z}$ we set $z = \tilde{z}$. The reason is that the observed distribution of earnings drops very sharply at exactly the threshold while it decreases much more gradual to the left (figure 1). We argue that the cost of crossing the threshold is so high that it exceeds potential adjustment costs or costs for exact declaration.

$$L_{e2}^{j}(t,h,w) = \left(1 - \frac{n^j \hat{\delta}_j}{\hat{\delta}_j}\right) - \hat{\lambda}_j(\hat{D_e^j}(h,w))^{1-d} exp[-\hat{D_e^j}(h,w)t] *$$
$$(\delta^{1-j2j}\hat{\lambda}_j(\hat{D_e^j}(h,w))^{j2j})^{(1-d_r)} *$$
$$\left(\int_{-\infty}^{\infty} \hat{g}_z(z_e+\eta)dh(\eta)\right)^{(1-d_r)} \quad \text{for } j \in (s, fa, fm), \tag{14}$$

where $j2j$ indicates whether an uncensored employment spell ends with a job-to-job transition ($j2j = 1$) or a transition to unemployment ($j2j = 0$). It holds that $\hat{D_e^j}(v) = \hat{\delta}_j + \hat{\lambda}_j(1 - \hat{F}(v))$ for $j \in (s, fa)$. For $j = fm$, $\hat{D_e^j}(v) = \hat{\delta}_j + \hat{\lambda}_j(1 - \hat{F}(v|z \leq z^*))\hat{F}(z^*)$. The third part is the probability of a transition to unemployment (δ) or to a better job ($\hat{D_e^j}(h_e, w_e)$), if observed (i.e. if $d_r = 0$). The last part is the probability that the current job has true earnings $\tilde{z_e} = z_e + \eta$. The density of realized earnings, g_z, is derived from equation 7 and the estimation of the offer distribution. While we observe whether an individual is of type s or f, we cannot distinguish whether or not a worker would be willing to accept an offer above the threshold (whether she is type fa or fm) if observed wages are below the threshold or not observed. For $k \in (u, e)$ the likelihood for type f workers is therefore

$$L_k^f(t,z) = \begin{cases} \theta L_{k2}^{fm}(t,z) + (1-\theta)L_{k2}^{fa}(t,z) & \text{for } z \leq z^* \\ L_{k2}^{fa}(t,z) & \text{for } z > z^* \end{cases} \tag{15}$$

We then numerically estimate α, θ and σ by maximizing the likelihood contributions (see appendix F for further details).

6 Results

In the following we discuss the estimation results, the model fit and simulate counterfactual policy reforms which smooth the discontinuity generated by the minijob tax exemption.

6.1 Estimation results

The job separation rate is larger for second jobs (type s) than for main jobs (type f, table 6). A second job is terminated every 18 months, a low-paid main job every 26 months. The job arrival rate differs even more strongly. Type s workers receive an offer on average every 30 months while type f workers receive on average three job offers per year. Second jobs are, thus, less stable and workers need more time to find one (this makes sense as workers seeking second jobs may have less time to search for a job).

We estimate that 37 % of type f workers in the market do not accept offers with earnings exceeding the threshold. This group of workers represents roughly two million individuals who are only in the market due to the tax exemption. At first sight the minijob policy thus seems to achieve its main objective to facilitate new employment relationships. Conversely, this also implies that more than 60 % of workers whose main job is low-paid employment would also work if there were no subsidy (type fa workers). From the government's point of view the increase in employment thus comes at the expense of lower taxes and SSC revenues from this group. Intensive margin effects of type fa workers are discussed in the following section when we simulate removing the tax exemption.

The elasticity of consumption (leisure) with respect to utility is 0.86 (0.14) and not statistically different from unity. The standard deviation of the optimization error is estimated to be 71 €/month. This implies, for example, that about 15 % of jobs with observed earnings of less than 250 € actually have true earnings at the threshold. The transition parameters are estimated very precisely. Standard errors are larger for the remaining parameters (table 6). The reason is that standard errors for the latter include the uncertainty rooted in the random elements of the estimation procedure (section 5)[23].

Table 6: Parameter estimates

Parameter	p.e.	s.e.
δ_s	0.0570	0.0005
δ_f	0.0379	0.0002
λ_s	0.0329	0.0002
λ_f	0.2506	0.0028
θ	0.3715	0.0775
α	0.8648	0.1121
σ	71.212	6.963

Notes: p.e. – mean point estimate of 50 bootstrap repetitions, s.e. – standard deviation of 50 bootstrap repetitions; length of period is one month
Source: SIAB; own calculations.

[23]Precision can be increased by basing these random procedures on more repetitions. The calculation of the offer distribution is, for example, repeated ten times per parameter set. Increasing this number increases precision but is costly in terms of computing time.

6.2 Model Fit

We simulate the low-paid market based on the estimated parameter estimates. The predicted realized earnings distribution reflects the main characteristics of its observed counterpart fairly well (figure 5). For type f workers it increases gradually up to a discontinuous mass point where it drops sharply. The mass beyond the threshold is relatively small. This is perfectly consistent with the observed distribution. The size of the mass point is over-estimated, though. For type s workers, by contrast, the size of the mass point in the predicted and observed distribution almost coincide. Our model also correctly predicts that the side job earnings distribution is fairly flat to the left of the threshold.

Two more deviations between predicted and observed earnings distributions concern both worker types. First, our model does not predict any mass slightly above the threshold although some workers are observed to be located there. As utility and profits could be increased by decreasing earnings to the threshold, our model cannot rationalize this[24]. Second, the endogenous upper earnings limit in the simulated low-paid market is over 1000 € month (represented by the jumps at the upper limit in figure 5) while we cut the observed earnings data at 800. Driving the upper support of the earnings distribution is our assumption on productivity. We use the 95th quantile of the observed wage distribution as an estimator for the homogeneous productivity p which may be high for some firms [25].

The employment probability is fit very well for both worker types (table 7). The model also correctly predicts that hours, wages and earnings are higher for type f than for type s workers. The fit of the earnings level is relatively good albeit on average slightly high.

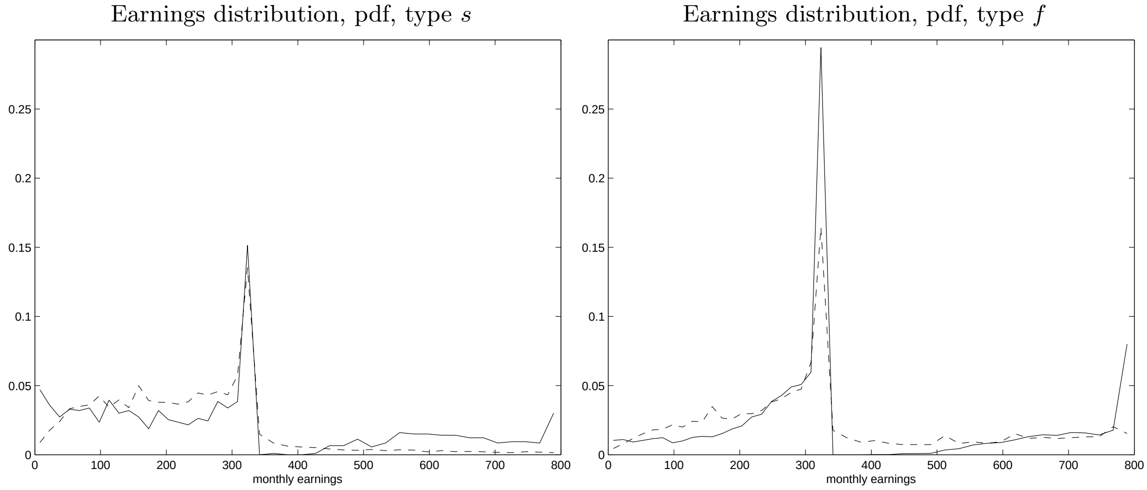
6.3 Smoothing the discontinuity

The main objective of MWP policies is to facilitate employment relationships which would not be formed without the minijob tax exemption. This often comes at the cost of an incentive for already employed individuals to reduce hours of work (Bargain et al., 2010): The withdrawal of the subsidy usually implies high implicit marginal tax rates. Crossing the minijob threshold in earnings implies a substantial increase in the average tax rate, in Germany, i.e. there is a notch. This generates not only large bunching, but also incentives

[24]Other studies interpret observed mass in a dominated region as evidence for "optimization frictions" (Kleven and Waseem, 2013). Although we indeed include an optimization error in the model, we do not allow earnings to cross the threshold as a result, since this would smooth the discontinuity at the threshold (section 5.2).

[25]One could attempt to estimate p jointly with the other parameters, this would increase the state space by a further parameter, however, which is costly in terms of computing time (see section F).

Figure 5: Offer and earnings distribution by types of workers



Notes: The solid line represents the predicted distribution, the dashed line the observed one.
Source: own calculations.

Table 7: Fit of moments

	Type s Mean		Type f Mean	
Measure	obs	pred	obs	pred
$P(e)$	37.60	37.36	83.79	86.11
z	223.62	279.91	320.16	365.97
w	5.56	4.25	5.76	4.54
h	8.95	13.94	12.84	18.41

Notes: $\bar{w}$ =*mean wage*, $\bar{h}$ =*mean weekly hours*, $\bar{z}$ =*mean earnings*, $P(e)$ =*employment probability*, *unrest.* =all earnings, *unrest.* = $z \leq 800$
Source: SIAB; own calculations.

for many firms to offer minijobs, creating externalities for workers who are not directly affected by the tax exemption. We now evaluate these aspects using three policy reform scenarios which remove the discontinuity induced by the minijob tax exemption. First, we simulate completely removing the tax exemption. Second, we replace the minijob regulation by a smoother subsidy schedule which retains the objective to generate employment. It emphasizes the particular distortion induced by discontinuous tax policies like the German minijob regulation. Third, we predict the effects of a reform implemented in 2003 aimed at reducing the discontinuity at the minijob threshold.

For these policy simulations we need to make assumptions about labor demand. When tax exemptions are reduced, all exclusive minijobbers (type fm workers) leave the market. How many of the jobs vacated by these jobs are offered to other workers? I.e. how much substitution is there across workers? When the jobs are lost because the productivity of the activity is too small to allow for an acceptable wage rate, substitutability is arguably low. By contrast, low-productivity workers or workers with specific preferences might be substituted more easily. To test the sensitivity of our analysis to labor demand we make two very different sets of assumptions.

In the first scenario, we assume that our frictional parameters are exogenous and constant. The number of vacancies in the economy falls in response to the increase in labor costs and the reduction in labor supply - resulting in a constant job-finding rate. In a second scenario (relegated to appendix G), we assume that the offer arrival rate increases for all remaining workers such that the total number of offers per month stays constant[26].

Removing the minijob tax exemption

When the tax exemption is removed, all exclusive minijobbers (type fm) leave the market. This amounts to roughly one and a half million employment relationships (table 8). Naturally, these workers unambiguously lose utility when the tax exemption is removed. We now discuss the effects of the removal of the tax exemption on workers who are not directly affected because they have another job (type s) and on workers who also accept earnings above the threshold (type fa). The reason they are affected lies in firms' responses[27].

As expected, the earnings offer distribution becomes smooth when the tax exemption is removed (left panel of figure 6). Since jobs with earnings above the threshold become relatively more attractive for workers, firms offer more of these jobs. This reveals that firms are responsible for negative effects of the minijob regulation on earnings of already employed - average gross earnings of workers employed in the low-paid market increase (table 8) as lower-paid exclusive minijobbers (fm workers) leave the market. However, the likelihood of offers *below* the threshold increases as well. The mass point at the threshold in the status-quo scenario is thus distributed over almost the entire support when the minijob tax exemption is removed. Why are there more low-earnings-offers in the market now? Our results show that workers who hold a main job (type s workers) receive fewer

[26]Alternative scenarios are possible, e.g. assuming fixed and exogenous hours for given firms. The relative size of firms with different hours might then respond to a change in financial incentives. When tax rates increase, for example, leisure becomes relatively more attractive and so firms with fewer hours expand.

[27]Note that in this section we assume constant frictional parameters. Appendix G simulates an alternative labor demand scenario.

offers per time period than type fm workers (see table 6). Since the latter have now left the market, the new equilibrium is less competitive. This means that the likelihood that another firm outbids a job offer decreases. Firms therefore offer less valuable jobs.

The population labor supply effect thus goes beyond the visible bunching reaction: Whilst the population of exclusive minijobbers (fm workers) created a particular incentive for firms to offer minijobs, the increased competition of workers that seek higher wages (up to the threshold) also creates an incentive for firms to create good offers up to the threshold. As a combined result of these two forces, the resulting realized earnings distribution of workers with a main job (type s workers) monotonously decreases as an increase in hours is overcompensated by a decrease in the wage rate. The average utility of type s workers also decreases. This implies that type s workers benefit more from the positive externality of the higher offer arrival rate than they lose due to the discontinuous tax schedule.

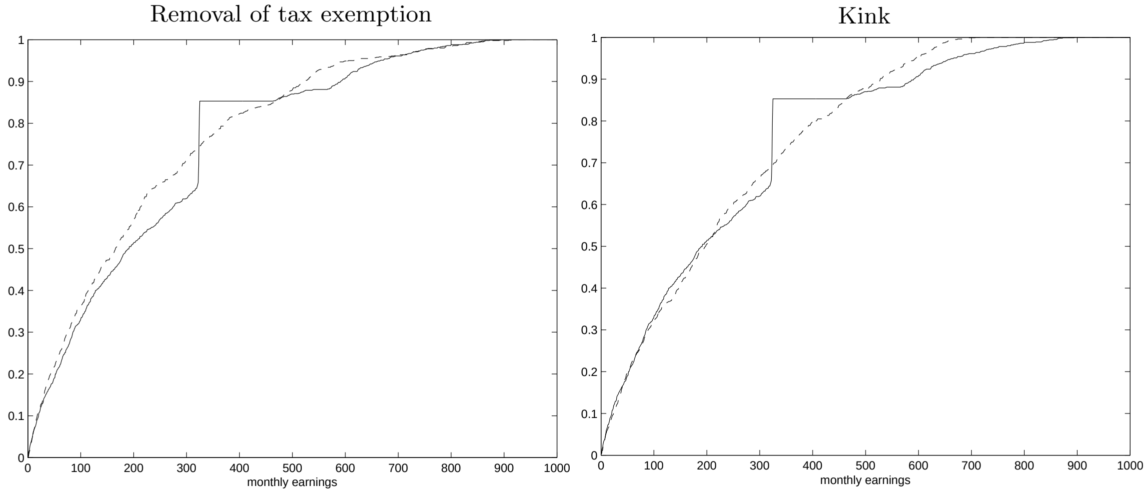
The earnings distribution of workers with no main job (type f workers), by contrast, increases up to approximately 500 € and decreases beyond. This difference mirrors the much higher offer arrival rate of type f workers (and potentially also the on average lower income tax rate), driven by both higher wage rates and an increased number of hours of work. These workers are hampered by the minijob threshold in the sense that gross earnings are lower than they would be. But note that they still profit substantially from the tax exemption - and the special tax treatment in the minijob-scenario outweighs the distortion of earnings.

Why are workers with and without main jobs (type s and f workers) affected so differently? Firms offer considerably more jobs with earnings below the threshold when the tax exemption is removed resulting in lower average starting earnings (and wages). The opportunities of advancement also increase, though, as firms offer more jobs with earnings above the threshold as well. The latter is mainly beneficial for workers with no main job who accept offers above the threshold (type fa workers), as they receive considerably more offers.

Total hours and gross earnings decrease by roughly 23 % and 16 %, respectively (table 8). Tax revenues increase by 110 million Euro. This opens scope for compensation. When the additional tax revenue is equally distributed to all individuals by tax-free lump-sum transfers, net earnings and utility of all workers remaining in the market (types s and fa)

would increase strongly (table 8). Both worker types thus clearly benefit from removing the tax exemption. The reform is not Pareto improving in terms of utility, though, as the exclusive minijobbers (type fm workers) lose albeit being compensated. This highlights that a tax exemption has redistributive effects: workers who would not otherwise work (with rich spouses, students and pensioners) gain at the expense of workers seeking low-paid employment independent of any tax exemption.

Figure 6: Cumulative earnings offer distribution - status quo vs. counter-factual



Notes: The solid line represents the status quo, the dashed line the counter-factual.
Source: SIAB, own calculations.

Table 8: Effects of smoothing the discontinuity

	Removal of tax exemption				Kink			
Change in	Total	type s	type fa	type fm	Total	type s	type fa	type fm
$\bar{w}$(*euro*)	0.23	-0.20	0.34	.	0.08	0.08	0.16	-0.04
$\bar{h}$	11.92	0.96	2.13	.	7.57	1.02	0.91	21.49
$\sum h$ (%)	-22.58	0.44	1.67	-100	10.04	1.69	0.98	38.87
$\bar{z}$ (euro)	91.70	-4.56	52.81	.	37.97	5.30	8.99	99.97
$\sum z$ (%)	-15.62	-2.74	11.65	-100	10.94	1.89	2.10	40.03
$\bar{c}$	18.79	-2.66	-7.55	.	41.54	3.50	28.43	77.23
$\bar{v}$	6.34	-3.72	-6.47	-199.34	28.45	3.58	20.67	36.08
jobs	-1644	0	0	-1644	13.5	0	0	13.5
taxes	110.56	-2.39	199.17	-86.21	23.13	1.54	-50.08	71.67
After lump-sum transfers								
$\bar{c}$	35.51	14.06	9.16	.	45.04	7.00	31.92	80.73
$\bar{v}$	53.00	40.02	41.48	-174.02	71	36.56	64.64	78.32

Notes: $\bar{w}$, $\bar{z}$ and $\bar{c}$ in €; $\sum h$ and $\sum z$ in %; jobs in 1000s; taxes in million €; jobs and taxes extrapolated to population; $\bar{w}$, $\bar{h}$, $\bar{z}$, and $\bar{c}$ conditional on employment, $\bar{v}$ based on all individuals
Source: SIAB; own calculations.

Smoothing the tax schedule

In the following we simulate a reform which replaces the minijob regulations by a smooth subsidy schedule. Average tax rates increase gradually in the interval between 325 € and 800 € up to their full values. The complete tax exemption below 325 € remains. The notch is thus replaced by a kink. The objective is to prevent additional distortions induced by a discontinuous tax schedule while positive employment effects are retained. Note that the withdrawal of the subsidy implies high implicit marginal tax rates in the interval between 325 and 800 €.

This hypothetical reform makes the tax schedule more generous, raising the question of whether additional workers (akin to type fm) participate in the market. We make two assumptions regarding labor supply. We assume, first, that no additional individuals enter the labor market due to tax subsidies beyond 325 €. Second, we postulate that labor supply of exclusive minijobbers (type fm workers) varies with the average tax rate. In the status quo, type fm workers do not accept any jobs with earnings above the threshold. Now, the average tax rate increases gradually with earnings beyond the threshold. Type fm workers will thus also accept some jobs above the threshold. We assume that $\theta(1 - H(t(z)))n_f$ workers accept a job offer with earnings z, where $H(.)$ is the cumulative uniform distribution (equation 16). Intuitively, this implies that type fm workers gradually leave the labor market when earnings – and therefore average taxes – increase.

$$H(t(z)) = \begin{cases} 0 \quad \text{for } z \in (0, 325] \\ \frac{t(z)-325}{800-325} \quad \text{for } z \in (325, 800] \\ 1 \quad \text{for } z > 800 \end{cases} \tag{16}$$

The resulting earnings offer distribution is smooth (right panel of figure 6). Most firms which offered jobs at the threshold in the status quo now offer jobs with higher earnings. The profile of the offer distribution is now steeper above the threshold with the highest earnings offer being considerably lower than in the status quo. The reason is twofold. First, the high implicit marginal tax rate induced by the withdrawal of the subsidy creates an incentive for workers to reduce earnings. Second, a marginal increase in offered earnings now results in a marginal decrease of exclusive minijobbers (type fm workers) above the threshold also.

Relative to the status quo all groups of workers increase their average gross earnings (table 8, right panel). This is driven by both wage rate and hours effects. Gross earnings of type fm workers increase the most, the average increase (9 €) for type fa workers is fairly modest. This is particularly true when compared to the increase resulting from the removal of the tax exemption (52.81 €). This again highlights the negative impact of the high implicit marginal tax rate introduced by the reform which has often been found in the previous literature on MWP policies (Bargain et al., 2010).

In terms of utility, all workers without main jobs (type f workers), both those who only work due to tax subsidies and those who would work anyway - benefit substantially from the reform. Besides the modest increase in gross earnings, workers who accept jobs above the threshold (type fa workers) mainly benefit from the additional tax subsidies beyond 325 € leading to substantially higher net earnings. Exclusive minijobbers (type fm workers) benefit from a slight increase in employment because the likelihood of an acceptable offer increases for this worker group. As workers with a main job (type s workers) are not eligible, their increase in utility is much smaller but still positive. The smoothing of the discontinuity is thus also beneficial for workers not directly affected. Overall hours and gross earnings increase by approximately ten percent. Total tax revenue increases by 23 million €. From the government's point of view, the positive behavioral responses more than compensate for the increase in generosity of the tax schedule. A budget neutral version of the reform redistributes the additional tax revenues evenly to all individuals as lump-sum transfers. This increases the positive effects on net earnings and utility for all groups of worker (table 8). This shows how inefficient the discontinuity of the minijob policy is.

The 2003 Reform

The reform of the minijob regulations in 2003 aimed at smoothing the tax schedule. Under the new regulations the SSC burden increases gradually for jobs with earnings exceeding the minijob threshold until it reaches its full amount at 800 Euro/month. While the size of the notch is significantly reduced, average tax rates still increase discontinuously at the threshold for most workers. First, the SSC burden for jobs with earnings exceeding the minijob threshold is at least 4%. Second, the income tax schedule is not changed in the same way. Workers with other income sources (often, spousal income) continue to have a large notch at the threshold. Third, the 2003 reform changed the rules to allow earnings from at most one second job to benefit from the minijob tax exemption. The notch for second job earnings is usually particularly big as first job earnings exceed the general income tax allowance and second job earnings are subject to full SSC immediately when exceeding the minijob threshold. That is, while the incentive for type f workers to locate at or below the threshold decreases, the reform generates an additional incentive for type s workers to do so. Can the reform still reduce the distorting effects of the minijob non-linearity?

In the following we use our model to predict the effects of the reform on the earnings distribution. Now that tax exemptions are extended to second jobs, we assume that $\theta_s = \theta_f$. This assumption implies that the fraction of workers who only enters the market due to the minijob regulations is the same for the populations seeking first or second jobs[28]. As before we further assume θ_s and θ_f are constant, implying that the increase of the minijob threshold from 325 to 400 Euro/month as well as the SSC reductions for jobs with earnings below 800 € do not increase the population of job seekers.

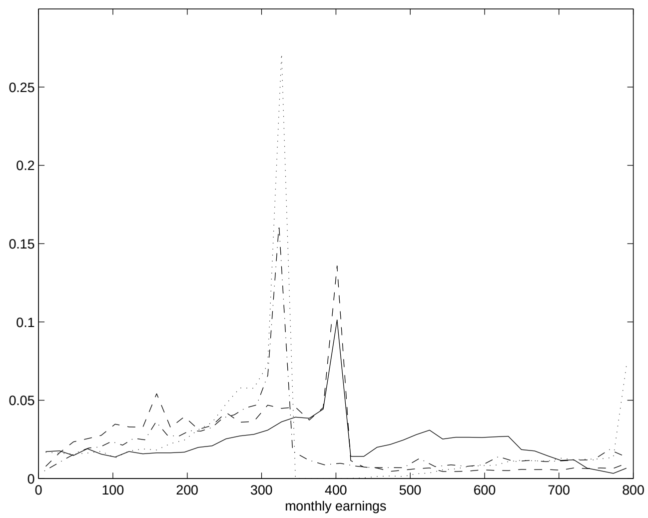
We now compare our simulated to the observed data from 2005. The most obvious effect of the reform is that the spike moves from 325 Euro to 400 Euro (figure 7). In the observed distribution, the spike at 325 Euro almost entirely vanishes by 2005, suggesting that adjustment frictions are rather low. In line with the aim of the policy, the size of the observed spike decreases from 2001 to 2005 as our model predicts. The predicted decrease is stronger than the observed decrease. The extension of the tax exemption to second jobs cannot entirely explain why so many workers continue to bunch at the threshold level. The increase in the level of the threshold (from 325 € to 400 €) as well as a reduction of red tape for creating a minijob may also have contributed to labor demand effects not included here. The observed continued bunching shows that the negative externalities imposed on other workers with population labor supply effects continue. They are not as easily visible, however, as both type s and type f workers are now eligible for the tax exemption.

7 Conclusion

We present a simple equilibrium job search model in which firms tailor their offers to the aggregate preferences of job seekers. As a result, a worker's labor supply choice set depends on other workers' preferences. This *population labor supply effect* represents a

[28]Future work could attempt to exploit the reform in 2003 to estimate the extensive margin as a function of the average tax rate. In 2003 the minijob threshold was increased to 400 €/month. Gradually decreasing SSC subsidies beyond the threshold smoothed the discontinuity in the SSC schedule. Side jobs became eligible for the tax exemption as well (see Appendix 6.3 for more details). The former two measures change the average tax rate at different levels of earnings. This can then be linked to the number of new low-paid main jobs.

Figure 7: Predicted and observed earnings distributions in 2001 and 2005, both worker types



Notes: The dotted and the solid line represents the earnings distribution in 2001 and 2005 as predicted by our model. The dashed-dotted and the dashed line represent observed earnings distributions for 2001 and 2005.
Source: own calculations, SIAB

channel through which firms mediate responses to tax policies - even when labor costs are not directly affected. We apply our model to a unique setting where we observe strong reactions to a *making work pay* policy for workers who are not directly affected. In Germany earnings below a threshold are exempted from income tax and employees' SSC, generating a substantial discontinuity in the budget set. These so-called minijobs aim at increasing employment. In the period of analysis this special tax treatment did not apply for second jobs. Yet, second job earnings feature strong bunching at the threshold. We interpret this as compelling evidence for the existence of a population labor supply effect.

We set-up an equilibrium job search model which allows for a discontinuous tax schedule. We are able to rationalize the discontinuous bunching in the earnings distributions of main and second jobs as firm responses to workers' aggregate preferences. We structurally estimate our model exploiting this specific institutional setting. We estimate the extensive margin effect of the tax exemption: 1.5 million people only take on a low-paid job due to the tax exemption.

We then test counter-factual policy reforms which remove the discontinuity induced by the tax exemption: Apart from creating jobs, the tax exemption has substantial negative effects on earnings of individuals who would also work without a subsidy. A first scenario completely removes the tax exemption. This increases earnings of the latter by on average over 50 €/month (ca. 11% of earnings). In a second reform scenario, we replace the minijob tax exemption by a smooth subsidy schedule. This prevents the distortions generated by a discontinuous tax schedule while retaining the positive employment effect. This reform prevents earnings bunching at the threshold, increasing total earnings in the low-paid market by over ten percent and - although the new tax schedule is more generous - increasing total tax revenues as well. While removing the tax exemption generates a group of workers who unambiguously lose utility, our second reform scenario improves welfare for all groups of workers.

Negative effects on earnings of already employed individuals is a usual side effect of MWP policies like the Earned Income Tax Credit in the US or the Working Family Tax Credit in the UK (Blundell, 2000; Bargain et al., 2010). We show that firms contribute crucially to these earnings effects. In both scenarios that reduce the tax exemption, firms offer more jobs above the threshold than in the status quo. The abundance of job offers at the threshold benefits some while hurting others. In our case individuals with rich spouses, pensioners and students benefit at the cost of unemployed workers seeking any kind of employment. More generally, less dominant groups in the labor market will find it difficult to get acceptable offers if their preferred hours differ from average preferences. Population labor supply effects might, for example, play a role in reinforcing low levels of labor supply of women when a substantial part of women desire working part-time. However, our model also reveals less obvious reactions: The increase in competitiveness of the labor market incites firms to post high-utility offers below the threshold. When employee tax subsidies are withdrawn, firms respond by *reducing* their earnings offers. Firms' responses represent reactions to changes in labor supply preferences or, put differently, population labor supply effects. We thus complement other work which discusses how unintended consequences of making work pay policies unfold in equilibrium (Rothstein, 2010; Andrew, 2010; Kolm and Tonin, 2011; Tazhitdinova, 2015). While these studies focus on channels like incidence and displacement effects, we add population labor supply as a potential channel.

The tax exemption studied here was reformed in 2003. Unfortunately, the negative welfare effects highlighted here remain very relevant: The distortion has not been removed. On the one hand, the reform attenuated the distorting effect of the minijob non-linearity by smoothing the drop in net earnings at the threshold. However, on the other hand, the tax exemption was generalized to include second jobs as well as first jobs. (This makes identification of the population labor supply effect less clear, which is why we focus on the period prior to 2003 in our estimation.) The fraction of workers in the market eligible for the tax exemption thus increased, raising the incentive for firms to offer jobs below or at the threshold. Thus the problematic redistributive aspects highlighted here remain.

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A Derivation of firm-size

We now derive the number of type s and f workers a firm can attract by offering earnings z, based on equations 2, 4 and 5. We start with type s workers:

$$\begin{aligned}
l^s(z) &= \frac{\frac{F(z)}{1+\kappa^s(1-F(z))} - \frac{F(z-\epsilon)}{1+\kappa^s(1-F(z-\epsilon))}}{F(z)-F(z-\epsilon)}(n^s-u^s) \\
&= \frac{\frac{F(z)(1+\kappa^s(1-F(z-\epsilon)))-F(z-\epsilon)(1+\kappa^s(1-F(z)))}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^s-u^s) \\
&= \frac{\frac{F(z)+F(z)\kappa^s-F(z)\kappa^sF(z-\epsilon)-(F(z-\epsilon)+F(z-\epsilon)\kappa^s-F(z-\epsilon)\kappa^sF(z))}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^s-u^s) \\
&= \frac{\frac{F(z)+F(z)\kappa^s-(F(z-\epsilon)+F(z-\epsilon)\kappa^s)}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^s-u^s) \\
&= \frac{\frac{(F(z)-F(z-\epsilon))(1+\kappa^s)}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^s-u^s) \\
&= \frac{(1+\kappa^s)}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}(n^s-u^s) \\
&= \frac{(1+\kappa^s)}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}\frac{n^s\kappa}{1+\kappa^s} \\
&= \frac{n^s\kappa}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}
\end{aligned}$$

Type f workers do not accept jobs with $z > z^*$. For $z \leq z^*$ we thus get

$$
\begin{aligned}
l^f(z) &= \frac{\frac{F(z)}{(1+\kappa^f(F(z^*)-F(z)))F(z^*)} - \frac{F(z-\epsilon)}{(1+\kappa^f(F(z^*)-F(z-\epsilon)))F(z^*)}}{F(z)-F(z-\epsilon)}(n^f-u^f) \\
&= \frac{\frac{F(z)((1+\kappa^f(F(z^*)-F(z-\epsilon)))F(z^*))-F(z-\epsilon)((1+\kappa^f(F(z^*)-F(z)))F(z^*))}{((1+\kappa^f(F(z^*)-F(z)))F(z^*))((1+\kappa^f(F(z^*)-F(z-\epsilon)))F(z^*))}}{F(z)-F(z-\epsilon)}(n^f-u^f) \\
&= \frac{\frac{(F(z)+F(z)\kappa^f F(z^*)-F(z)\kappa^f F(z-\epsilon)-(F(z-\epsilon)+F(z-\epsilon)\kappa^f F(z^*)-F(z-\epsilon)\kappa^f F(z)))F(z^*)}{F(z^*)^2(1+\kappa^f(F(z^*)-F(z)))(1+\kappa^f(F(z^*)-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^f-u^f) \\
&= \frac{\frac{(F(z)+F(z)\kappa^f F(z^*)-(F(z-\epsilon)+F(z-\epsilon)\kappa^f F(z^*)))}{F(z^*)(1+\kappa^f(F(z^*)-F(z)))(1+\kappa^f(F(z^*)-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^f-u^f) \\
&= \frac{\frac{(F(z)-F(z-\epsilon))(1+\kappa^f F(z^*))}{F(z^*)(1+\kappa^f(F(z^*)-F(z)))(1+\kappa^f(F(z^*)-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^f-u^f) \\
&= \frac{(1+\kappa^f F(z^*))}{F(z^*)(1+\kappa^f(F(z^*)-F(z)))(1+\kappa^f(F(z^*)-F(z-\epsilon)))}(n^f-u^f) \\
&= \frac{(1+\kappa^f F(z^*))}{F(z^*)(1+\kappa^f(F(z^*)-F(z)))(1+\kappa^f(F(z^*)-F(z-\epsilon)))}\frac{n^f\kappa^f F(z^*)}{1+\kappa^f F(z^*)} \\
&= \frac{n^f\kappa^f}{(1+\kappa^f(F(z^*)-F(z)))(1+\kappa^f(F(z^*)-F(z-\epsilon)))}
\end{aligned}
$$

The total firm size is then the sum of the number of workers of each type, which gives expression 6.

B Sketch of Proof of Propositions (I) - (III)

Proposition (I) *If we observe offers above z^*, there must be a mass point of job offers at z^*. The wage offer distribution above z^* is continuous up to some $\bar{z}$.*

Assume there exists no mass point (i.e. $f(z^*) = 0$), the offer distribution for $z < z^*$ is then continuous and profits at the threshold are

$$\pi(z^*) = (p - z^*)\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)))^2} + n^f\kappa^f. \tag{17}$$

Profits associated with offering earnings slightly above the threshold (for $\epsilon \to 0$) are:

$$
\begin{aligned}
\pi(z^*+\epsilon) &= (p-(z^*+\epsilon))\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*+\epsilon)))(1+\kappa^s(1-F(z^*-\epsilon+\epsilon)))} \\
&= (p-z^*)\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)))^2}
\end{aligned} \tag{18}
$$

Assuming $f(z^*) = 0$ it holds that $\pi(z^*+\epsilon) < \pi(z^*)$ implying that there is a gap to the right of the threshold.

Is there an earnings level $z' > z^* + \epsilon$ where the equal profit condition holds again? Equation 19 makes use of $F(z') = F(z^*)$ which holds if there is a gap in the interval $z \in (z^*, z')$.

$$\begin{aligned}\pi(z') &= (p - z')\frac{n^s \kappa^s}{(1+\kappa^s(1-F(z')))(1+\kappa^s(1-F(z'-\epsilon)))} \\ &= (p-(z'))\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)))^2}\end{aligned} \tag{19}$$

As $(p - z') < (p - z^*)$ it holds that $\pi(z') < \pi(z^*)$ implying that no job with earnings $z > z^*$ will be offered if there is no mass point at z^*. Allowing for a mass point at z^*, $\frac{\partial \pi(z^*)}{\partial f(z^*)} < 0$ and $\frac{\partial \pi(z^*+\epsilon)}{\partial f(z^*)} = 0$ imply that there might be a value for $f(z^*)$ for which the equal profit condition between z^* and $z^*+\epsilon$ holds ($\pi(z^*+\epsilon) = \pi(z^*)$). For earnings $z' \in [z^*+\epsilon, \overline{z}]$, the usual trade-off between profit per workers and firm size ensures that the equilibrium offer distribution is continuous in that interval and determined by $\pi(z') = \pi(z^* + \epsilon)$.

Proposition (II) *If there is a mass point at z^*, there will be a gap in the offer distribution just below the threshold.*

We need to show that a mass point in the wage offer distribution is only consistent with equal profits if there is a gap in the wage offering distribution. We compare profits $\pi(z^*)$ with profits $\pi(z^* - \epsilon)$. Profits of a job offer with earnings at the threshold are given by

$$\begin{aligned}\pi(z^*) &= (p - z^*)l(z^*) \\ &= (p - z^*)\left(\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)))(1+\kappa^s(1-F(z^*-\epsilon)))} + \frac{n^f\kappa^f}{(1+\kappa^f(F(z^*)-F(z^*)))(1+\kappa^f(F(z^*)-F(z^*-\epsilon)))}\right) \\ &= (p-z^*)\left(\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)))(1+\kappa^s(1-F(z^*)+f(z^*)))} + \frac{n^f\kappa^f}{(1+\kappa^f f(z^*))}\right)\end{aligned} \tag{20}$$

Profits slightly below the threshold are given by (for $\epsilon \to 0$):

$$\begin{aligned}\pi(z^*-\epsilon) = (p-(z^*-\epsilon))(&\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*-\epsilon)))(1+\kappa^s(1-F(z^*-2\epsilon)))} + \\ &\frac{n^f\kappa^f}{(1+\kappa^f(F(z^*)-F(z^*-\epsilon)))(1+\kappa^f(F(z^*)-F(z^*-2\epsilon)))}) \\ = (p-z^*)(&\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*-\epsilon)))^2} + \frac{n^f\kappa^f}{(1+\kappa^f(f(z^*)))^2}) \\ = (p-z^*)(&\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)+f(z^*)))^2} + \frac{n^f\kappa^f}{(1+\kappa^f(f(z^*)))^2})\end{aligned}$$

Given proposition (I), the data implies that there is a mass point at $z = z^*$ (i.e. that $f(z^*) > 0$). When $f(z^*) > 0$, it holds that $(1+\kappa^j(1-F(z^*)+f(z^*))) > (1+\kappa^j(1-F(z^*)))$ and $(1+\kappa^f(f(z^*))) > 1$. Therefore, $\pi(z^*) > \pi(z^* - \epsilon)$ and there will be a gap to the left of the threshold.

Proposition (III) *There may or may not exist wage offers below the threshold* z^* *in equilibrium. The wage offer distribution will then be continuous between* $z \in [\underline{z}, z'']$ *for* $z'' < z^*$.

Define the highest wage offer below the threshold as z'', such that $F(z'') = F(z^* - \epsilon)$. Note that since there is a gap left of the threshold, if in equilibrium a z''-offer exists, it may be significantly below z^*. In equilibrium any z''-offer must make the same amount of profits as the threshold wage offer z^*.

$$\begin{aligned}\pi(z'') &= (p - z'')\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z'')))(1+\kappa^s(1-F(z''-\epsilon)))} + \\ &\quad \frac{n^f\kappa^f}{(1+\kappa^f(F(z^*)-F(z'')))(1+\kappa^f(F(z^*)-F(z''-\epsilon)))} \\ &= (p - z'')\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)+f(z^*)))^2} + \frac{n^f\kappa^f}{(1+\kappa^f(f(z^*)))^2} \end{aligned} \tag{21}$$

Comparing equations 20 and 21 illustrates that $\pi(z'') = \pi(z^*)$ can hold as $\pi(z'')$ increases with decreasing z''. That is, there might be a certain size of the gap where $\pi(z'') = \pi(z^*)$ holds.

Using that $F(\underline{z}) = 0$, we now determine the lowest wage offer $\underline{z}$ that will be made (if there are wage offers below z^*).

$$\begin{aligned}\pi(\underline{z}) &= (p - \underline{z})\frac{n^s\kappa^s}{(1+\kappa^s(1-F(\underline{z})))(1+\kappa^s(1-F(\underline{z}-\epsilon)))} + \\ &\quad \frac{n^f\kappa^f}{(1+\kappa^f(F(z^*)-F(\underline{z})))(1+\kappa^f(F(z^*)-F(\underline{z}-\epsilon)))} \\ &= (p - \underline{z})\frac{n^s\kappa^s}{(1+\kappa^s)^2} + \frac{n^f\kappa^f}{(1+\kappa^f(F(z^*)))^2} \end{aligned} \tag{22}$$

If $z'' < z^*$ exists and $\underline{z} < z''$ there will then be a continuity of wage offers between these two values, generating equal profits with the standard trade-off between margins and firm-size.

C Discrete variation in working hours

In the main text, we assume a continuous variation in hours worked. We here now discuss the model equilibrium with discrete variation in hours. For simplification, we allow for two different numbers of hours h_k where $k \in [0, 1]$. In the market for low-paid jobs, this may correspond to 10 and 20 hours of working. We later consider how this model generalizes to three or more hours sectors.

Firms set wage rates, w, workers care about wage rates as well as hours. In order to simplify notation, we follow Shephard (2017) and define $q_1(w) = w$ and $U(q_0(w), h_0) = U(q_1(w), h_1) = U(w, h_1)$, so $q_0(w)$ is a function that denotes the wage rate that makes individuals indifferent between working with few ($k = 0$) hours at $q_0(w)$ or working more ($k = 1$) hours at w. Depending on preferences, individuals may require a low-hours premium or accept a low-hours wage penalty.

C.1 Worker mobility

The flow from and into unemployment is equivalent to before (see equation 2). The flow of workers of type $j \in (s, fa, fm)$ from and into jobs with hours h_k and wage rate w is

$$D^j(w) g_k^j(q_k(w)) e_k^j = \lambda^j f_k(q_k(w))(u^j + G_0^j(q_0(w) - \epsilon)e_0^j + G_1^j(w - \epsilon)e_1^j) \tag{23}$$

with $D^j(w) = [\delta + \lambda^j((1 - F_1(w)) + (1 - F_0(q_0(w))))]$ for $j \in (s, fa)$. Equation 24 states the corresponding definition for workers of type fm who do not accept jobs with wage rates larger than w_k^*.

$$D^{fm}(w) = \begin{cases} [\delta + \lambda^j((F_1(w_1^*) - F_1(w)) + (F_0(w_0^*) - F_0(q_0(w))))] & \forall w \leq w_1^* \text{ and } q_0(w) \leq w_0^* \\ [\delta + \lambda^j(F_1(w_1^*) - F_1(w))] & \forall w \leq w_1^* \text{ and } q_0(w) > w_0^* \end{cases} \tag{24}$$

The LHS of equation 23 pertains to workers who leave a job in a sector k with wage $q_k(w)$. For $k = 1$ this group consists of workers who move from sector 1 to sector 0 ($\lambda^j(1 - F_0(q_0(w)))g_1^j(w)e_1^j$ for $j \in (s, fa)$), who move to a better paying job within sector 1 ($\lambda^j(1 - F_1(w))g_1^j(w)e_1^j$ for $j \in (s, fa)$) and who become unemployed ($\delta^j g_1^j(w)e_1^j$). The RHS pertains to workers who start a job in sector k with wage rate $q_k(w)$. For $k = 1$ this consists of workers who move from sector 0 to sector 1 ($\lambda^j f_1(w) G_0^j(q_0(w) - \epsilon)e_0^j$), who changes jobs within sector 1 ($\lambda^j f_1(w) G_1^j(w - \epsilon)e_1^j$) and who come from unemployment ($\lambda^j f_1(w)$). The overall flow (i.e. both sectors) between jobs with wage rate of no greater than w and unemployment is:

$$\begin{aligned} &(G_0^j(q_0(w))e_0^j + G_1^j(w)e_1^j)D^j(w) = \lambda^j u^j F_0(q_0(w)) + \lambda^j u^j F_1(w) \\ &= \lambda^j u^j + \lambda^j u^j - \lambda^j u^j (1 - F_0(q_0(w))) - \lambda^j u^j (1 - F_1(w)) \end{aligned} \tag{25}$$

Using equation 2 gives

$$G_0^j(q_0(w))e_0^j + G_1^j(w)e_1^j = \frac{\delta n^j - u^j D^j(w)}{D^j(w)}. \tag{26}$$

By combining equations 23 and 26 we obtain

$$g_k^j(q_k(w))e_k^j = \frac{\lambda^j f_k(q_k(w))(u^j + \frac{\delta n^j - u^j(D^j(w-\epsilon))}{D^j(w-\epsilon)}}{D^j(w)} \tag{27}$$

C.2 Firm size

The number of workers of type j in steady-state employed at a firm in sector k which offers wage rate $q_k(w)$ is

$$
\begin{aligned}
l_k^j(q_k(w)) &= \frac{g_k^j(q_k(w))e_k^j}{f_k(q_k(w))} \\
&= \frac{\lambda^j \delta n^j}{D^j(w)D^j(w-\epsilon)}.
\end{aligned} \tag{28}
$$

The steady state firm size is then

$$
\begin{aligned}
l_k(q_k(w)) &= l_k^s(q_k(w)) + l_k^{fa}(q_k(w)) + l_k^{fm}(q_k(w)) \\
&= \begin{cases} \frac{\lambda^s \delta n^s}{D^s(w)D^s(w-\epsilon)} + \frac{\lambda^f \delta n^{fa}}{D^{fa}(w)D^{fa}(w-\epsilon)} + \frac{\lambda^f \delta n^{fm}}{D^{fm}(w)D^{fm}(w-\epsilon)} & \forall w \leq w^* \\ \frac{\lambda^s \delta n^s}{D^s(w)D^s(w-\epsilon)} + \frac{\lambda^f \delta n^{fa}}{D^{fa}(w)D^{fa}(w-\epsilon)} & \forall w > w^* \end{cases}.
\end{aligned} \tag{29}
$$

Following the standard arguments of profit equalization, we find the following (the reasoning is parallel to the case without hours variation):

Proposition (A1) *There can be (at most one) mass point in the wage offer distribution at the threshold in each sector, i.e. at wages* $w_k^* \equiv \frac{z^*}{h_k}$.

Sketch of Proof: The following argument closely mirrors the argument in the case of homogeneous hours. We compare profits at the threshold value with profits above. We find that if there exist offers above, there must be a mass point at the threshold.

The profit of a sector k firm offering wage rate $q_k(w)$ can be expressed as $\pi_k(q_k(w)) = (ph_k - q_k(w)h_k)l_k(q_k(w))$. We first state the profits of a type-1-firm, assuming that $q_0(w_1^*) \leq w_0^*$.

$$
\begin{aligned}
\pi_k(w_1^*) &= \frac{\lambda^s \delta n^s}{D^s(w_k^*)D^s(w_k^*-\epsilon)} + \frac{\lambda^f \delta n^{fa}}{D^{fa}(w_k^*)D^{fa}(w_k^*-\epsilon)} + \frac{\lambda^f \delta n^{fm}}{D^{fm}(w_k^*)D^{fm}(w_k^*-\epsilon)} \\
&= \frac{\lambda^s \delta n^s}{[\delta + \lambda^s((1-F_1(w_1^*)) + (1-F_0(q_0(w_1^*))))][\delta + \lambda^s((1-F_1(w_1^*-\epsilon)) + (1-F_0(q_0(w_1^*)-\epsilon)))]} + \\
&+ \frac{\lambda^f \delta n^{fa}}{[\delta + \lambda^f((1-F_1(w_1^*)) + (1-F_0(q_0(w_1^*))))][\delta + \lambda^f((1-F_1(w_1^*-\epsilon)) + (1-F_0(q_0(w_1^*)-\epsilon)))]} + \\
&+ \frac{\lambda^f \delta n^{fm}}{[\delta + \lambda^j(F_0(w_0^*) - F_0(q_0(w_1^*)))][\delta + \lambda^j(f_1(w_1^*) + (F_0(w_0^*) - F_0(q_0(w_1^*)-\epsilon)))]}
\end{aligned} \tag{30}
$$

Evaluated marginally above the threshold, profits are

$$\pi_k(w_1^*+\epsilon) = \frac{\lambda^s \delta n^s}{D^s(w_k^*+\epsilon)D^s(w_k^*)} + \frac{\lambda^f \delta n^{fa}}{D^{fa}(w_k^*+\epsilon)D^{fa}(w_k^*)}$$
$$= \frac{\lambda^s \delta n^s}{[\delta+\lambda^s((1-F_1(w_1^*+\epsilon))+(1-F_0(q_0(w_1^*+\epsilon))))][\delta+\lambda^s((1-F_1(w_1^*))+(1-F_0(q_0(w_1^*))))]} +$$
$$+ \frac{\lambda^f \delta n^{fa}}{[\delta+\lambda^f((1-F_1(w_1^*+\epsilon))+(1-F_0(q_0(w_1^*+\epsilon))))][\delta+\lambda^f((1-F_1(w_1^*))+(1-F_0(q_0(w_1^*))))]} \tag{31}$$

Equations 30 and 31 show that the equal profit condition can only hold if there is a mass point in the offer distribution of sector 1 at w_1^*. By symmetry, note that the same argument can be made with respect to a type-0 firm. However, if the utility of a threshold offer lies in the "gap area" due to a threshold in another sector, it may be the case that there is no mass point in that sector. This explains the restriction "at most one" in Proposition (IV) and completes our discussion.

We now consider the influence of thresholds in other hours sectors on the wage distribution. Consider a firm of type-1, i.e. seeking a worker to work for h_1 hours. The impact of a potential mass point in the offer distribution of sector 0 at w_0^* depends on the relation between w_1^*, $q_0(w_1^*)$ and w_0^*.

Proposition (A2) *There will be no wage offers at wage levels (and in a certain interval below this level) that offer the same utility as is available at threshold wages $w_{j\neq k}^*$ in other sectors.*

The intuition for Proposition (A2) is the following: It is a dominated strategy to offer a wage rate that is equal in utility to an offer made by several other firms. A slightly higher offer will attract all workers from these firms at only marginal cost. By Proposition (IV), wage offers at earnings thresholds generate mass points in the wage offer distributions. Thus for example a type-1 firm will offer a wage rate slightly larger than $\tilde{w}_1$ (where $U(\tilde{w}_1, h_1) = U(w_0^*, h_0)$.) in order to additionally attracts workers from this positive mass of sector 0 firms. This implies that there must be a gap in the wage offer distribution at $\tilde{w}_1$. How much below this utility value an offer can be sustained in equilibrium will depend on the parameters of the model in an analogous way to the potential existence of offers below the threshold offer in the homogeneous case.

Sketch of proof: Let $\tilde{w}_1$ denote the wage rate which satisfies $U(\tilde{w}_1, h_1) = U(w_0^*, h_0)$. If $\tilde{w}_1 > w_1^*$ the profits of a sector 1 firm offering wage rate $\tilde{w}_1$ and slightly above are:

$$
\begin{aligned}
\pi_k(\tilde{w}_1) &= \frac{\lambda^s \delta n^s}{D^s(\tilde{w}_k)D^s(\tilde{w}_k-\epsilon)} + \frac{\lambda^f \delta n^{fa}}{D^{fa}(\tilde{w}_k)D^{fa}(\tilde{w}_k-\epsilon)} + \frac{\lambda^f \delta n^{fm}}{D^{fm}(\tilde{w}_k)D^{fm}(\tilde{w}_k-\epsilon)} \\
&= \frac{\lambda^s \delta n^s}{[\delta+\lambda^s((1-F_1(\tilde{w}_1))+(1-F_0(w_0^*)))][\delta+\lambda^s((1-F_1(\tilde{w}_1-\epsilon))+(1-F_0(w_0^*-\epsilon)))]}+ \\
&+ \frac{\lambda^f \delta n^{fa}}{[\delta+\lambda^f((1-F_1(\tilde{w}_1))+(1-F_0(w_0^*)))][\delta+\lambda^f((1-F_1(\tilde{w}_1-\epsilon))+(1-F_0(w_0^*-\epsilon)))]}+ \\
&+ \frac{\lambda^f \delta n^{fm}}{[\delta+\lambda^j((F_1(w_1^*)-F_1(\tilde{w}_1))+(F_0(w_0^*)-F_1(w_0^*))))][\delta+\lambda^j((F_1(w_1^*)-F_1(\tilde{w}_1-\epsilon))+(F_0(w_0^*)-F_1(w_0^*-\epsilon)))]} \\
&= \frac{\lambda^s \delta n^s}{[\delta+\lambda^s((1-F_1(\tilde{w}_1))+(1-F_0(w_0^*)))][\delta+\lambda^s((1-F_1(\tilde{w}_1))+(1-F_0(w_0^*)+f_0(w_0^*))))]}+ \\
&+ \frac{\lambda^f \delta n^{fa}}{[\delta+\lambda^f((1-F_1(\tilde{w}_1))+(1-F_0(w_0^*)))][\delta+\lambda^f((1-F_1(\tilde{w}_1))+(1-F_0(w_0^*)+f_0(w_0^*)))]}+ \\
&+ \frac{\lambda^f \delta n^{fm}}{[\delta+\lambda^j((F_1(w_1^*)-F_1(\tilde{w}_1)))][\delta+\lambda^j((F_1(w_1^*)-F_1(\tilde{w}_1))+f_0(w_0^*))]}
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
\pi_k(\tilde{w}_1-\epsilon) &= \frac{\lambda^s \delta n^s}{D^s(\tilde{w}_k-\epsilon)D^s(\tilde{w}_k-2\epsilon)} + \frac{\lambda^f \delta n^{fa}}{D^{fa}(\tilde{w}_k-\epsilon)D^{fa}(\tilde{w}_k-2\epsilon)} + \frac{\lambda^f \delta n^{fm}}{D^{fm}(\tilde{w}_k-\epsilon)D^{fm}(\tilde{w}_k-2\epsilon)} \\
&= \frac{\lambda^s \delta n^s}{[\delta+\lambda^s((1-F_1(\tilde{w}_1-\epsilon))+(1-F_0(w_0^*-\epsilon)))][\delta+\lambda^s((1-F_1(\tilde{w}_1-2\epsilon))+(1-F_0(w_0^*-2\epsilon)))]}+ \\
&+ \frac{\lambda^f \delta n^{fa}}{[\delta+\lambda^f((1-F_1(\tilde{w}_1-\epsilon))+(1-F_0(w_0^*-\epsilon)))][\delta+\lambda^f((1-F_1(\tilde{w}_1-2\epsilon))+(1-F_0(w_0^*-2\epsilon)))]}+ \\
&+ \frac{\lambda^f \delta n^{fm}}{[\delta+\lambda^j((F_1(w_1^*)-F_1(\tilde{w}_1-\epsilon))+(F_0(w_0^*)-F_1(w_0^*-\epsilon)))][\delta+\lambda^j((F_1(w_1^*)-F_1(\tilde{w}_1-2\epsilon))+(F_0(w_0^*)-F_1(w_0^*-2\epsilon)))]} \\
&= \frac{\lambda^s \delta n^s}{[\delta+\lambda^s((1-F_1(\tilde{w}_1))+(1-F_0(w_0^*)+f_0(w_0^*)))]^2}+ \\
&+ \frac{\lambda^f \delta n^{fa}}{[\delta+\lambda^f((1-F_1(\tilde{w}_1))+(1-F_0(w_0^*)+f_0(w_0^*)))]^2}+ \\
&+ \frac{\lambda^f \delta n^{fm}}{[\delta+\lambda^j((F_1(w_1^*)-F_1(\tilde{w}_1))+f_0(w_0^*))]^2}
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
\pi_k(\tilde{w}_1+\epsilon) &= \frac{\lambda^s \delta n^s}{D^s(\tilde{w}_k+\epsilon)D^s(\tilde{w}_k)} + \frac{\lambda^f \delta n^{fa}}{D^{fa}(\tilde{w}_k+\epsilon)D^{fa}(\tilde{w}_k)} + \frac{\lambda^f \delta n^{fm}}{D^{fm}(\tilde{w}_k+\epsilon)D^{fm}(\tilde{w}_k)} \\
&= \frac{\lambda^s \delta n^s}{[\delta+\lambda^s((1-F_1(\tilde{w}_1+\epsilon))+(1-F_0(w_0^*+\epsilon)))][\delta+\lambda^s((1-F_1(\tilde{w}_1))+(1-F_0(w_0^*)))]}+ \\
&+ \frac{\lambda^f \delta n^{fa}}{[\delta+\lambda^f((1-F_1(\tilde{w}_1+\epsilon))+(1-F_0(w_0^*+\epsilon)))][\delta+\lambda^f((1-F_1(\tilde{w}_1))+(1-F_0(w_0^*)))]}+ \\
&+ \frac{\lambda^f \delta n^{fm}}{[\delta+\lambda^j((F_1(w_1^*)-F_1(\tilde{w}_1+\epsilon)))][\delta+\lambda^j((F_1(w_1^*)-F_1(\tilde{w}_1)))]} \\
&= \frac{\lambda^s \delta n^s}{[\delta+\lambda^s((1-F_1(\tilde{w}_1))+(1-F_0(w_0^*)))]^2}+ \\
&+ \frac{\lambda^f \delta n^{fa}}{[\delta+\lambda^f((1-F_1(\tilde{w}_1))+(1-F_0(w_0^*)))]^2}+ \\
&+ \frac{\lambda^f \delta n^{fm}}{[\delta+\lambda^j((F_1(w_1^*)-F_1(\tilde{w}_1)))]^2}
\end{aligned} \tag{34}
$$

As $f_0(w_0^*) > 0$ and $\epsilon \to 0$, it holds that $\pi_k(\tilde{w}_1-\epsilon) < \pi_k(\tilde{w}_1) < \pi_k(\tilde{w}_1+\epsilon)$. This implies that there will be no wage offers of value $\tilde{w}_1$. As $(ph-wh)$ increases with decreasing w, there might be a wage rate w' where it holds that $\pi_k(w') = \pi_k(\tilde{w}_1+\epsilon)$. This implies that $f_1(.)$ exhibits a gap in the interval (w',$\tilde{w}_1$]. If $\tilde{w}_1 < w_1^*$, the terms in equations 32 and 34 referring to workers of type fm drop out. Although this might reduce the extent of the gap, $\pi_k(\tilde{w}_1) < \pi_k(\tilde{w}_1 + \epsilon)$ still holds. If $\tilde{w}_1 = w_1^*$ the necessary size of the mass point at w_1^* to balance the loss of type fm workers decreases (in comparison to $\tilde{w}_1 \neq w_1^*$). How large the gap is, i.e. whether any offers will be made below $\tilde{w}_1$ will depend on the economic environment captured by the parameters of the model.

D Predicting income tax rates

In Germany, income tax rates depend on household characteristics, but the SIAB data set used in this analysis does not include these. We thus impute tax rates based on another data set, the GSOEP. The tax simulation accounts for the most important aspects of the German tax-transfer system (Junge, forthcoming). For type f workers, explaining tax rates using variables included in both data sets does not yield satisfying predictions. As we aim at predicting the marginal tax rate directly above the threshold, gross earnings, the most valuable predictor vary only marginally. Other variables hardly add explanatory power (Junge, forthcoming). We therefore replicate the SIAB sample in the GSOEP data and differentiate three tax groups based on the distribution of simulated tax rates. Observations in the estimation sample are then allocated randomly the mean tax rate of one of these groups. Since earnings at the threshold are less than the general tax allowance, almost 50% of type f workers have an income tax rate of zero (table 9). The imputed tax rate for the next 40 % is about eight per cent, for the highest decile it is approximately 20 %.

Tax rates for type s workers are explained by a Tobit model with first job earnings and sex as explaining variables (table 10) [29]. Based on the same information in the SIAB data we then predict the individual tax rates. Type s workers are also allocated to three groups (table 9). Tax rates are higher for type s workers as they already have first job earnings. As our sample is restricted to observations with first job earnings of more than 1000 € (section 4.1), all observations exceed the general tax allowance and thus have a strictly positive marginal tax rate.

E Predicting hours of work

We do not have information on hours of work offered in the market. We, therefore, seek to impute actual working hours by means of survey data (GSOEP). To do that we model hours based on variables included in both data sets: sex, gross earnings, sector and

[29] To increase precision we use all second jobs for the estimation of tax rates. We do not restrict the sample with respect to first job earnings as done for the main estimation.

Table 9: Income tax groups

	Type s			Type f		
Group	%	t	std	%	t	std
1	25	8.32	1.75	48.27	0	0
2	65	12.99	4.17	41.73	7.92	4.64
3	10	27.6	8.52	10	19.47	4.50

Notes: t= mean average income tax rate at 326 €; std=standard deviation; type *s*: aggregated predicted tax rates based on model in table 10; type *f*: aggregated observed tax rates
Source: Own calculations based on GSOEP wave 1999-2002

Table 10: Estimation results: tax rates type *s* workers

	coeff.	s.e.
Male	-0.0422	0.003
Yearly first job earnings	4.27e-06	7.50e-08
Constant	0.0354	0.002

Notes: coeff.=regression coefficient, s.e.=standard errors.
Source: Own calculations based on GSOEP wave 1999-2002

education. As the hours distribution resembles a log-normal distribution (see figure 3), a generalized linear model with log link is used for imputation. As in the main analysis the estimation sample comprises two groups of employees. These are, first, employees with only one job and earnings of less than 800 €/month. We use their actual hours of work per week. The sample includes, second, employees holding a main job with earning above 1000 €/month and a side job with earnings below 800 €/month. Weekly hours of work in the side job are used as dependent variable. The model is estimated jointly for both groups (table 11). The resulting hours distribution increases sharply up to a global peak at approximately 50 hours per month (figure 8). The right tail covers hours up to 200. The mean is fit very well. The variation is however smaller in the predicted data (table 12).

F Numerical approaches

This section describes two numerical approaches applied in the estimation procedure. First, the numerical calculation of the job offer distribution $F(.)$ and, second, the numerical optimization of the likelihood.

F.1 Job offer distribution

In the case with homogeneous hours the offer distribution can be characterized by a system of equal profit conditions. Firms with different hours, however, may make different profits in equilibrium. The offer distribution in the case of heterogeneous hours is, thus, calculated by solving the maximization problem successively for all firms in the market. Firms take

Table 11: Estimation results: hours

	coeff.	s.e.
Gross Earnings	0.00	0.00
Sex	0.03	0.05
Sector		
Manufacturing	-0.38	0.11
Energy, Water	-0.28	0.20
Construction	-0.21	0.14
Wholesale and retail	-0.26	0.10
Hotels and restaurants	-0.23	0.15
Transport	-0.40	0.14
Finance	-0.43	0.17
Real Estate	-0.44	0.11
Public Admin	-0.51	0.14
Education	-0.48	0.13
Health	-0.57	0.11
Other Services	-0.28	0.12
Households	-0.35	0.14
Other Sector	-0.33	0.11
Education		
Other Education	0.56	0.13
Basic	0.26	0.09
Middle Voc.	0.28	0.09
Higher Voc.	0.18	0.10
Constant	1.83	0.14

Notes: coeff.=regression coefficient, s.e.=standard errors., dependent variable: weekly hours
Source: Own calculations based on GSOEP wave 2001

Table 12: Fit of hours distribution

	Observed	Predicted
mean	57.25	57.14
$p(25)$	23.11	34.45
$p(50)$	43.33	52.17
$p(75)$	86.67	76.91

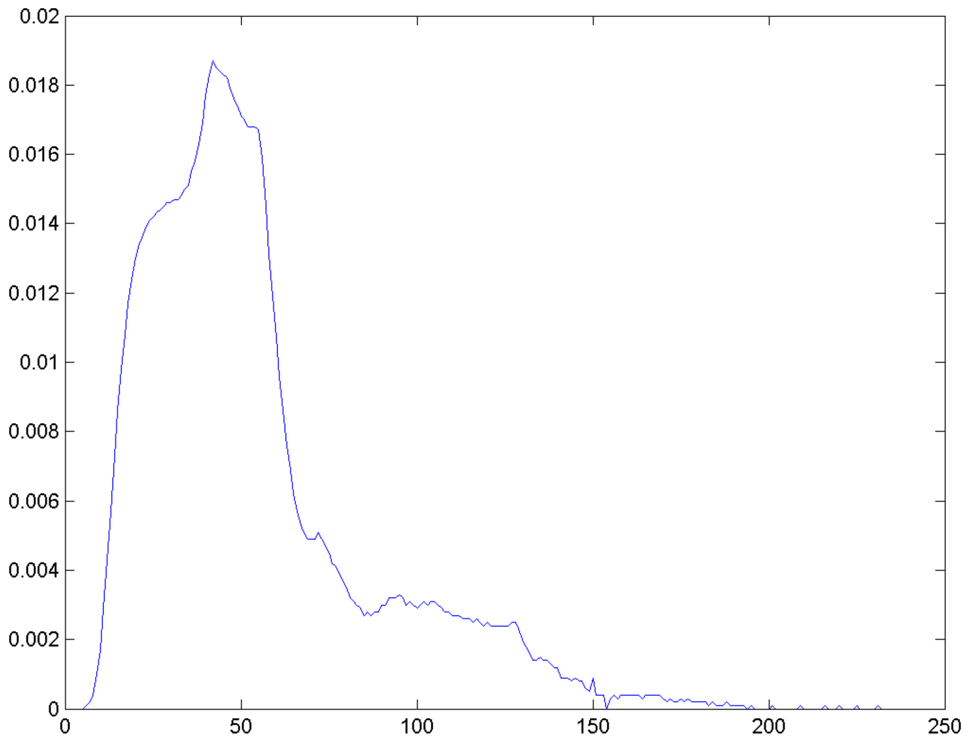
Notes: $p(x)$: xth quantile
Source: SIAB; own calculations.

the current offer distribution as given which is then updated once a firm has decided on a wage offer. The order of firms deciding on an offer is determined by random sampling with replacement. We run the algorithm based on 100 firms which first draw once from the predicted hours distribution (see above). Firms can chose between 100 different wages. The algorithm is stopped after 1000 draws. After this, the distribution is fairly stable: The estimated fraction of offers with earnings below or at the threshold, for example, has a standard deviation of less than 0.02 when the algorithm is repeated ten times. To increase precision, we take the average offer distribution of ten repetitions.

F.2 Optimization of likelihood

The likelihood is optimised with respect to θ, α and σ by a two-step grid search procedure (Hansen, 2016). Gradient-based approaches are inappropriate because the costly calcula-

Figure 8: Probability density function of monthly hours of work



Source: SOEP, SIAB, own calculations.

tion of the job offer distribution has to be repeated every iteration. It further contains random elements rendering the likelihood function non-smooth.

For θ and α we evaluate the likelihood in the first step at 11 equally spaced grid points in the interval $[0, 1]$. In the second step the grid is narrowed down to steps of 0.02. For σ we use the 50, 100 and 150 as grid points in the first step. The grid is narrowed down to steps of 20 in the second step.

G Robustness test: Extensive margin reactions

Extensive margin reactions in our model are exclusively driven by type fm workers who leave the market when the tax exemption is disposed. Due to the assumption of exogenous and constant frictional parameters, the overall number of vacancies posted by firms decreases. We now perform a robustness test in this respect.

In this alternative scenario, type fm workers are substituted by workers of other types. We model this by assuming that the number of vacancies per period posted by firms stay constant. Offer arrival rates for type s and fa workers are thus increased. Keeping the ratio between both constant, this implies $\lambda_s = 0.0513$ and $\lambda_f = 0.3908$. This is comparable to introducing a matching function, accounting for on-the-job search and differing arrival rates across worker groups.

Both type s and fa workers can substantially increase their net earnings and utility - albeit the latter forgoes the tax exemption. Total hours still decrease by about 12%. This is overcompensated by an increase in average wage rate, though, such that total gross earnings increase. Tax revenues increase accordingly. The results of such a simulation can be interpreted as an upper bound. When economic incidence of the tax exemption, for example, is not entirely on employees, firms profit from lower wage rates for minijobs relative to jobs subject to full SSC. Removing the tax exemption might then lead to a decrease in vacancies. The substantial differences between both scenarios imply that labor demand responses are crucial for comprehensively evaluating the minijob regulation. Our model indeed includes firms catering to the aggregate preferences in the market. It, however, abstracts from other important margins of firm responses.

Table 13: Removing the tax and SSC exemption - upper bound scenario

Change in	Total	type s	type f	type fa
$\bar{w}$	0.78	0.09	0.92	.
$\bar{h}$	18.05	4.98	9.92	.
$\sum h$	-10.01	41.27	15.15	-100
$\bar{z}$	179.98	36.67	156.08	.
$\sum z$	9.72	47.45	41.38	-100
$\bar{c}$	83.96	24.98	70.31	.
$\bar{v}$	69.11	20.25	55.81	-199.34
jobs	-1360.5	163	120.5	-1644
taxes	341.04	39.64	387.61	-86.21

Notes: $\bar{w}$, $\bar{z}$ and $\bar{c}$ in €; $\sum h$ and $\sum z$ in %; jobs in 1000s; taxes in million €; jobs and taxes extrapolated to population; $\bar{w}$, $\bar{h}$, $\bar{z}$, and $\bar{c}$ conditional on employment, $\bar{v}$ based on all individuals
Source: SIAB; own calculations.

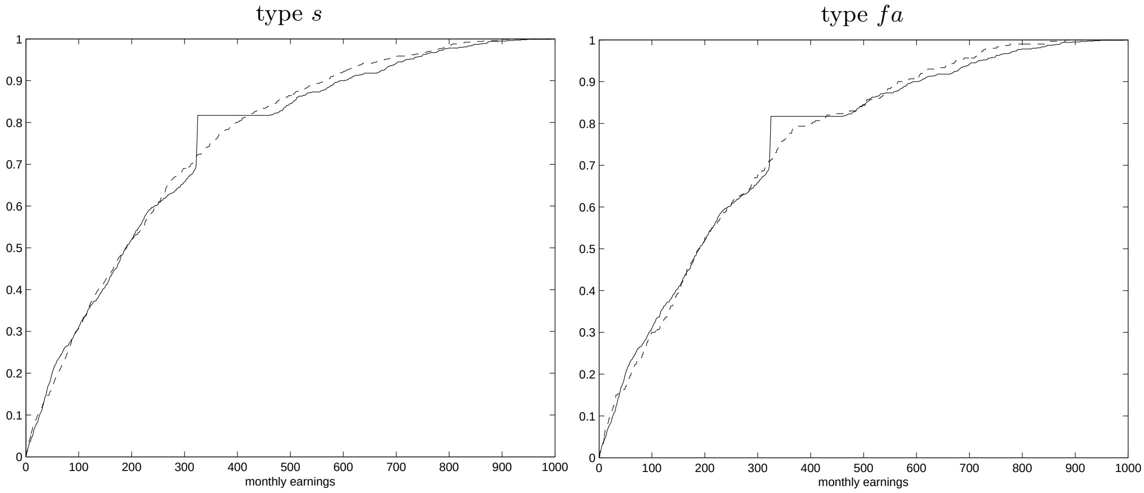
H Low-paid market with one worker type

Imagine all workers in the low-paid market are of type fa. That is, all workers are eligible for a potential tax exemption but would work when subject to full taxes. Counter-factual simulations in such a market are insightful because they abstract from differential changes of incentives between worker types and compositional changes in the workforce.

When the tax exemption is removed, firms which offered jobs at the threshold in the status quo now mostly offer jobs with higher earnings (left panel of figure 9). This is not surprising as jobs with high earnings are relatively more attractive when they are not subject to a disadvantaged tax treatment. Some firms reduce their earnings offer, though. The likelihood of earnings farther below the threshold is not affected. Removing the tax exemption has a positive effect on average (and total) gross earnings (table 14). One reason is the higher likelihood of a high earnings offer, another that workers more likely accept a high earnings offer.

Another way to smooth the minijob discontinuity is a gradually increasing tax rate for earnings between 325 and 800 €. The full tax exemption below 325 € remains. Firms which offered jobs at the threshold in the status quo increase their earnings offers. Relative to the status quo the distribution above the threshold is compressed, though. Firms account for the change in workers' incentives due to the high implicit marginal tax rate introduced by the reform.

Figure 9: Cumulative earnings offer distribution - status quo vs. reform - only fa workers



Source: own calculations.

I Robustness analysis with respect to market definition

Classifying individuals as job seeking in the main text underlies several assumptions. A spell needs to fulfill two conditions to be classified as type s. First, the respective individual needs to be employed with earnings exceeding 1000 Euro/month. Second, the individual must have had at least one low-paid second job within the analysis period (that is April 1999 to February 2002). On the one hand we disregard individuals seeking for a second job who haven't had one in the analysis period (implying that they have not been able to find one). On the other hand we might mis-classify some individuals as seeking a

Table 14: Effects of smoothing the notch - only fa workers

Change in	Removal	Kink
$\bar{w}$	0.50	0.33
$\bar{h}$	-0.56	-2.60
$\sum h$	-0.58	-2.71
$\bar{z}$	40.73	6.93
$\sum z$	8.67	1.48
$\bar{c}$	-10.56	25.28
$\bar{v}$	-7.67	17.43
jobs	0.00	0.00
taxes	269.64	-76.41

Notes: Removal: Complete removal of tax exemption; Kink: Notch replaced by kink; $\bar{w}$, $\bar{z}$ and $\bar{c}$ in €; $\sum h$ and $\sum z$ in %; jobs in 1000s; taxes in million €; jobs and taxes extrapolated to population; $\bar{w}$, $\bar{h}$, $\bar{z}$, and $\bar{c}$ conditional on employment, $\bar{v}$ based on all individuals
Source: SIAB; own calculations.

second job. The high unemployment ratio of type s workers suggest the latter being particularly important. In this section we further restrict the market of second jobs to spells of individuals with a future low-paid second job spell (which starts within the analysis period). The number of unemployment spells of type s workers decreases by 10,000 to less than 40,000 (table 15).

Similarly, we classify formal unemployment spells as type f when the respective individual have had a low-paid job spell in the analysis period. Not all low-paid job spells, however, are preceded by a formal unemployment spell. In this section we additionally classify individuals as seeking for a low-paid job who are out of labor force. Spells of being out of labor force are specified as the gap between two observed spells of formal (un)employment. The amount of unemployment spells of type f workers more than doubles to 116,182 (table 15).

Table 15: Total number of spells by type and employment - alternative definition of market

Status \ Type	s	f
Unemployed	38,048	116,182
Employed	30,121	287,255

Notes:

Source: SIAB; own calculations

As expected the offer arrival rate of type f workers is estimated to be considerably lower than in the main text. By contrast, the offer arrival rate of type s workers is almost constant. The value of θ appears rather sensitive and almost halves vis-à-vis the basic specification. The amount of jobs created by the minijob regulation should thus not be over-interpreted.

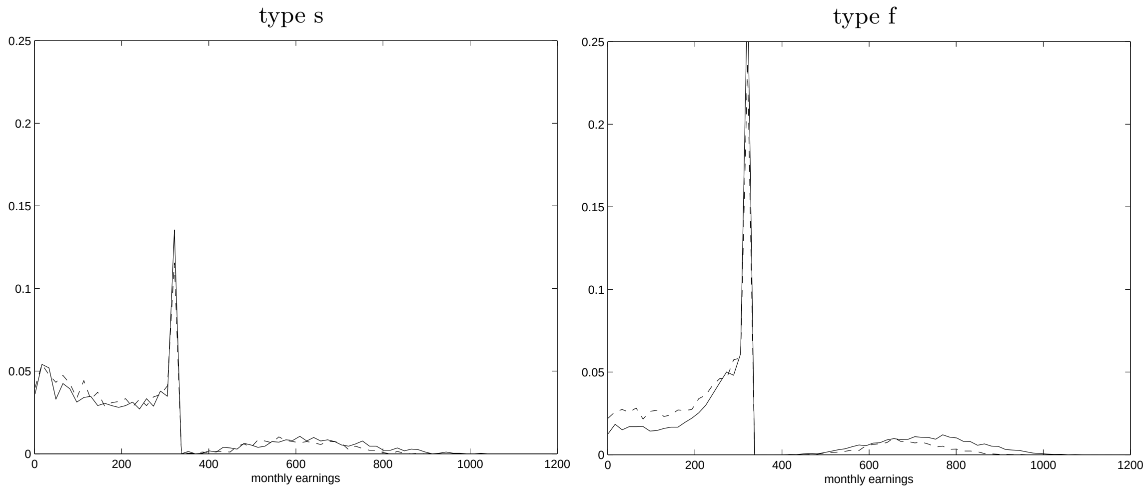
Table 16: Parameter estimates - alternative definition of market

Parameter	p.e.	s.e.
δ_s	0.0474	0.0005
δ_f	0.0451	0.0002
λ_s	0.0296	0.0002
λ_f	0.1571	0.0028
θ	0.2064	0.0775
α	0.8843	0.1121
σ	67.500	6.963

Notes: p.e. – mean point estimate of 50 bootstrap repetitions, s.e. – standard deviation of 50 bootstrap repetitions; length of period is one month
Source: SIAB; own calculations.

The predicted realized earnings distributions are very similar, though (figure 10). For type f workers there is slightly more mass to the left of the cap and slightly less at and to the right of the threshold.

Figure 10: Predicted realized earnings distribution - basic and alternative distribution



Notes: Solid curve pertains to basic specification, dashed curve to alternative specification. Specifications differ in the definition of the labor market. In the basic specification *Source:* SIAB.