Collusive Benchmark Rates Fixing

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Abstract

The fixing of the Libor and Euribor benchmark rates has proven vulnerable to manipulation. Individual rate-setters may have incentives to fraudulently distort their submissions. For the contributing banks to collectively agree on the direction in which to rig the rate, however, their interests need to be sufficiently aligned. In this paper we develop cartel theory to show how an interbank lending rates cartel can be sustained by preemptive portfolio changes. Exchange of information facilitates front running that allows members to reduce conflicts in their trading books. Designated banks then engage in eligible transactions rigging to justify their submissions. As the cartel is not able to always find stable cooperative submissions against occasional extreme exposure values, there is episodic recourse to non-cooperative quoting. Periods of heightened volatility in the rates may be indicative of cartelization. Recent reforms to broaden the class of transactions eligible for submission may reduce the level of manipulation, but can lead to more frequent collusive quoting.

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Trader RBS: “It’s just amazing how Libor fixing can make you that much money or lose if opposite. It’s a cartel now in London.”

Trader Deutsche Bank: “Must be damn difficult to trade man, especially if you are not in the loop.”

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1 Introduction

Financial benchmark rates such as Libor, Euribor and FX globally underlie large transaction values. The Libor and Euribor are calculated daily for various maturities and currencies as the trimmed average of a number of panel banks’ submissions that are to reflect their capacity to borrow unsecured funds in interbank markets. Between 370 trillion and 530 trillion dollars worth of interest rate derivatives, consumer and commercial credit—or between 4.5 and 6.5 times global GDP—are estimated to directly derive their value from these rates. The rise of the over-the-counter (OTC) derivatives markets from the late 1980s to early 2000s exponentially increased the volumes traded on the benchmarks. The Libors and Euribors are key variables in portfolio and risk management decisions and barometers of financial sector health.

Foreign exchange (FX or forex) rates are prices in the currency market, the largest market in the world, with transactions taking place around the clock. World Market/Reuters (WMR) provides the most widely used standardized forex benchmarks, including the 4 p.m. London close. They are determined as the median value of buy and sell transactions executed by forex traders, primarily large banks, for clients as well as their own accounts during specific short time windows. Trading in the foreign exchange market is worth in the trillions of dollars a day. The forex WMR rates allow fund managers to value holdings and are used in forwards and other multi-currency contracts.

The fixing of these financial benchmarks has proven vulnerable to manipulation. The rates are determined on the basis of contributions by market participants who also trade in the financial products that are valued on the benchmarks, giving them financial incentives to manipulate their contributions resulting from their trading exposure positions. Suspicion of manipulation of the benchmark rates arose when the Wall Street Journal reported that in the gathering of the global financial crisis


the Libors appeared to diverge periodically from other proxies of bank borrowing costs and risk, in particular credit default swaps (CDS) spreads.⁴

A few banks admitted early on to misreporting, and numerous major banks in the panels have been prosecuted since. As the individual submissions of banks used to be published publicly together with the calculated rates, panel members could try to come across as more creditworthy by underreporting their true borrowing costs in their submissions, in an attempt to avoid being charged higher risk premiums for suspicion of liquidity problems—so-called ‘low-balling’. In some instances, the practice appeared to be endorsed by senior management—as a senior treasury manager at Barclays instructed a submitter:

“[S]tick within the bounds[,] so no head above [the] parapet.”⁵

Vaughan and Finch (2017) even suggest that the Bank of England permitted, if not told, the British panel banks to low-ball Libor and thereby break the code of conduct to maintain financial stability.⁶

Subsequent government investigations into Libor and Euribor worldwide uncovered evidence of communication between panel bank employees aimed at manipulating submissions on a large scale, as well as colluding to increase trading profits.⁷ Bloomberg reporters discovered that also the foreign exchange rates had systematically been rigged by colluding traders. Many of them later turned out to have worked for the same large banks involved in the Libor and Euribor manipulations as well.⁸

In by invitation only chatrooms with names such as ‘The Cartel’ and ‘The Mafia’, senior currency traders met on a regular basis to agree on their strategies.⁹ These included front running client orders and pushing through trades in the 60-second

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⁴C. Mollenkamp and M. Whitehouse, “Study Casts Doubt on Key Rate; WSJ Suggests Banks may have Reported Flawed Interest Rate Data for Libor,” Wall Street Journal, 29 May 2008.


⁶Vaughan and Finch (2017), Chapter 11 details contacts between Barclay’s CEO Bob Diamond and Paul Tucker, deputy governor of the Bank of England. During a call on 29 October 2008, Diamond internally reported: “Mr Tucker states the level of calls he was receiving from Whitehall were ‘senior’ and that while he was certain we did not need advice, that it did not always need to be the case that we appeared as high as we have recently.” (Op.cit., page 97).


window when the benchmarks were set, so-called ‘banging the close’, by which forex traders could sell (buy) their own currency holdings at a high (low) price while pushing down (up) the rate at which to later buy (sell) currency from (to) clients. They would also withhold bids or offers for certain currencies in order to avoid moving the exchange rates in directions adverse to open positions by the cartel members, and place fake orders for currencies that were not intended to be actually executed, seeking to push rates prior to the closing period—so-called ‘painting the screen’—and ‘wash trades’ to pocket broker commissions.\(^{10}\)

Despite evidence of wider coordination in the benchmark rates fixings, most investigations so far have focussed on fraudulent misreporting in breach of the banking code of conduct and client confidentiality by individual traders trying to influence rate-setters during the financial crisis, generally within a bank and occasionally also between. The cases were handled for misconduct threatening the integrity of the Libor and Euribor, in the US by the Department of Justice’s Criminal Division’s Fraud Section and the Commodity Futures Trading Commission (CFTC), and in the UK by the Financial Service Authority (FSA)—the later Financial Conduct Authority (FCA). They were portrayed as incidental favors done between rogue traders for their own benefit, possibly against the interests of their employers.

Antitrust cases in financial benchmark setting are few so far. While the DoJ’s Antitrust Division was involved in the fraud investigations, it did not prosecute for collusion. Initially, in a private antitrust damages action, the Federal Court of New York ruled that the Sherman Act would not apply to the Libor setting mechanism, as it was deemed a cooperative rather than competitive process.\(^{11}\) This ruling got overturned on appeal, however, by the United States Court of Appeals for the Second Circuit in Manhattan, which determined that Libor manipulation could constitute price-fixing as a per se antitrust violation under Section 1.\(^{12}\)

European cartel investigations into Libor and Euribor started with leniency applications by UBS, RBS and Barclays. In a hybrid settlement and subsequent decisions, the European Commission established Article 101 TFEU cartel violations, in euro, Japanese yen and Swiss franc interest rate derivatives against nine of the largest panel banks and two brokers that had facilitated the cartel for record fines.\(^{13}\) In the Eu-
ribor case, collusion was established to have started in September 2005, well before the financial crisis.

Forex market prosecutions by UK and US authorities have been concluded with settlements as well. The European Commission has not disclosed anything to date about its ongoing cartel investigations into the foreign exchange market. As a result, little or no information about the supposed inner workings of the benchmark rates cartels is public.

There have been some notable cases of collusion in banking and financial markets. Christie and Schultz (1994) uncovered in a study of pricing patterns that NASDAQ market makers avoided quoting prices in odd eights of a dollar, suggesting implicit collusion to maintain wide bid-ask spreads, that led the DoJ to intervene. VISA, MasterCard and American Express have been prosecuted for restricting price competition and sued for antitrust damages in the US in various cases since the mid 1990s. The European Commission brought cartel cases also against large cooperatives of international banks in objection to the joint setting of multilateral interchange fees (MIFs) in determining credit card charges. In 2002, it fined eight Austrian banks participating in the ‘Lombard Club’ for fixing interest rates and service fees.

Commentators have argued that the benchmark systems would be too complex to conspire against, due to the size of the market, misaligned interests and the trimming mechanism in the rate setting processes. It is indeed not obvious that a for-profit cartel in the fixing of the benchmark rates could have worked. Contrary to conventional cartels, in which all members typically want to increase product prices, often

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18 Euribor-EBF officials believed that the composition of its bank panel made it impossible for a cartel to fix the index. Director Cedric Quéméner claimed that too many banks, certainly more than the 9 investigated, would need to be involved for manipulation of Euribor to be possible. Source: “Euribor cuts Libor Adrift in Scandal Storm,” EurActiv, 26 July 2012, in which an industry source is quoted to have added: “[I]f these banks were to convince each other that they want the same rate level, they would need to find at least another 15 different banks with the same interest as their own.” University of Florida professor of finance Andy Naranjo, for example, was skeptical of the ability of traders to manipulate foreign exchange rates due to its large size. See L. Vaughan, G. Finch and A. Choudhury, “Traders Said to Rig Currency Rates to Profit off Clients,” Bloomberg, 12 June 2013. Scopino (2016) at page 639 does recognize collusive schemes in the financial benchmark manipulations as a possibility and argues that the CFTC should be given broad authority to combat such anticompetitive conduct.
the interests of the panel banks in manipulating the benchmark fixings will not be aligned, as their exposures to the rates fluctuate partly unpredictably over time. Different banks will regularly find themselves on opposing sides of the market, where some gain from an increase in one or more of the rates, while others benefit from a decrease. Such diverse and constantly changing payoff incentives are a challenge to cartel stability. Simply swapping portfolio exposure positions within the cartel alone to bring interests more in line would not make collusion worthwhile, as the panel’s overall net portfolio position generally revolves around zero. Sub-coalitions of panel members with common interests colluding in a certain period would require panel-wide communication to form, and then not be tolerated by banks with a diverging interest knowing—or induce offsetting manipulations. The trimming of the higher and lower Libor and Euribor submissions adds to the difficulty. In addition, the proposed reforms are aimed at basing the interbank rates partly on actual transactions. Manipulation often requires suboptimal transactions against other than the going rates in order to move them, so that collusion can be costly.

In this paper we show how a full cartel in the fixing of interbank lending rates can work, despite conflicting and time-varying interests and the trimming, and without the need for side-payments. We develop a model in which panel banks exchange information and use it to agree on their contributions to the fixing and subsequent adaptations to their exposure positions. Two complementary mechanisms, inspired by evidence of the fraud investigations make this possible. First, in order to distort the rates in the joint-profit maximizing direction, designated banks engage in eligible transactions rigging. These are transactions of the type that is to justify a panel member’s contribution to the rate calculation process. Second, their self-created inside information about the new rate allows all colluding banks a time window in which to engage in lucrative front running and adjust their trading books. We establish stability of a continuous collusion strategy based on Fershtman and Pakes (2000), in which all the cartel members revert temporarily to independent quoting to stabilize the cartel against the occasional extreme exposure value that gives one (or more) of the panel banks incentive to deviate.

The emerging literature on benchmark rates focuses almost exclusively on manipulation by one or a few rates-setters. Abrantes-Metz et al. (2012) see episodes of suspicious Libor submissions by individual banks in comparison to the federal fund effective rate and 1-month T-Bill rates, yet no evidence for widespread manipulation. Abrantes-Metz and Sokol (2012) suggest that screens could have detected interbank rate manipulation and collusion earlier. Monticini and Thornton (2013) find more material anomalous patterns for the same period when using the relationship between Libor and large, unsecured certificate of deposit rates. Kuo et al. (2012) compare Libor quotes to bank bids in the Federal Reserve Term Auction Facility, deduced borrowing costs to find that Libor submissions were significantly lower than comparison rates during the crisis, which could indicate such low-balling. Gandhi et al. (2017) estimate monthly Libor-related positions and find a relation between the positions
and banks’ submissions, which is initially stronger for banks that were sanctioned by
the regulators.

Snider and Youle (2012) study the incentives behind portfolio based manipulation
of strategic Libor quote submission as signals of creditworthiness between individual
banks that each maximize their own trading profits. Youle (2014) uses the model to
estimate banks’ exposures and finds evidence suggesting that Libor was downward
biased during the recent crisis. Chen (2017) finds in a signaling game that banks’
individual manipulations decrease with the panel size and number of quotes used
in the calculation. His result of a distribution-free bias does not hold under collusion
however. Diehl (2013) models portfolio and reputation incentives and compares
the performance of different aggregates, such as the mean and the median, under
individual manipulation.

A few papers raise the possibility of agreements between two or several panel
members, but neither of them analyses how collusion would have worked. Eisl et al.
(2017) calculate how Libor misreporting by one or several banks together could have
moved the average, but do not analyze incentives. Using a time-varying threshold
regression model, Fouquau and Spieser (2015) argue that the breaks they find are not
consistent with exogenous money market shocks, suggesting manipulation by small
groups of panel banks that they propose to identify using a hierarchical clustering
method.

Several papers on forex rigging consider collusion more explicitly, but offer no
cartel theory. Evans (2016) examines trading patterns around the fixing-window in a
model of competitive trading to find that price changes display volatility and negative
serial correlation around the fix that is more consistent with collusive manipulation
than competitive trading. Michelsberger and Witte (2016) and Ito and Yamada
(2017) confirm these findings on the volatility, to which the latter note that widening
the transaction window eliminated the volume spike in the fixing window, suggesting
that banks now incur the costs of executing customers’ orders.

While Abrantes-Metz (2012) suggests changes to reduce the risk of collusion in
the interbank lending benchmark rates, this has not been the objective of the reforms.
Duffie and Dworczak (2014) propose a mechanism and Duffie and Stein (2015) reforms
against manipulation, not collusion, for both types of benchmark rate. Coulter et al.
(forthcoming) also use mechanism design to obtain unbiased estimates of the true
rates, basing the benchmark on bank transactions. Collusion is briefly discussed, but
their focus is on preventing unilateral manipulation.

We develop novel cartel theory to specific features of benchmark rate-setting.
Whereas in a classic cartel, the attraction of defecting is to steal the full cartel profit,
deviation from a benchmark cartel only affects the final rate to the extent of the
deviator’s submission—and not the demand or portfolio exposure position of the
other banks. When a bank draws a private extreme value portfolio position it has
incentive to deviate, as during booms in Rotemberg and Saloner (1986). However,
in our model there is no obvious ‘competitive’ strategy the cartel could fall back to
in order to avoid defection and assure continuous collusion. Instead, if agreeing on a collusive submission is not possible for the period, there is episodic recourse to non-cooperative quoting, as in Fershtman and Pakes (2000). Such ‘price wars’ are short run unprofitable, as in Green and Porter (1984), but an integral part of the collusive strategy, not punishment. Each cartel member incurs occasional losses as a part of the cartel strategy, but randomly and not by a history-dependent favoring of certain players based on productive efficiency, as in Athey and Bagwell (2001).

The remainder of this paper is organized as follows. Section 2 provides more detail on the institutional context, benchmark rate manipulations, and evidence for the existence of the cartel mechanisms we propose. Section 3 lays out the model and develops theoretical results on cartel stability. In Section 4, simulation exercises give insight into collusive rate patterns. In Section 5, we discuss possible damages resulting from collusion in the benchmark rates. Section 6 discusses extensions and concludes. The source code of a software that calculates optimal cartel strategies is given in an appendix.

2 Benchmark Rates Fixing

The Libor and Euribor interbank rates are produced with quotes from panel banks. Libor used to be compiled for 10 currencies and 15 maturities, ranging from overnight to 12 months, with panel sizes varying for different currencies between 6 to 18 banks. Barclays Bank, Deutsche Bank, Lloyds TSB Bank and The Royal Bank of Scotland have been in all currency panels. Euribor was calculated for 15 maturities by a panel of over 40 banks. A large number of banks, including those who had been part of all Libor currency panels, were in both the Libor and Euribor panels.

On every trading day before a certain time in the morning, a panel of banks submits quotes for a number of maturities that are intended to reflect the rate at which they could borrow these funds on the interbank market. Before reforms in response to the recent scandals, the Libor quotes were a prediction of the submitters’ own rates and the Euribors what each bank believed to be “the rate at which interbank term deposits are being offered within the EMU zone by one prime bank to another.”

Each rate is calculated as the trimmed average of the panel banks’ submissions. In the setting of the Libor, the middle 50% of the quotes are averaged, for Euribor the middle 70%. The Libor quotes are to be submitted at 11.00 a.m. GMT and published at 11.55 a.m.. Euribor quotes are submitted at 10.45 a.m. and published at 11.00 a.m. CET. The individual submissions are disclosed as well, albeit for Libor with a 3 months embargo as of 2013.

19Contrary to Libor, the Euro OverNight Index Average (EONIA) is not determined by panel submissions, but calculated by the European Central Bank on the basis of all overnight interbank assets created before the close of Real Time Gross Settlement systems at 6 p.m. CET.

20European Money Markets Institute, Euribor Code of Conduct, June 2016. On this point, the formulation has not changed from the original code.
The forex WMR rates are computed half-hourly for 22 currencies, and hourly for over 150.\textsuperscript{21} The most important of them is the ‘London 4 p.m. fix’ or ‘WMR Fix’ which is calculated as the median value of a subset of foreign exchange transactions that occur during a short time window at 4 p.m. CET—of 1 minute around before the reforms and 5 minutes since. Another major daily fix is the 1.15 p.m. CET European Central Bank fix produced in a similar way by the ECB. The forex benchmarks are meant to be a snapshot of going rates.

WMR collects bid and order rates from actual trades every second during the fixing-window on the three highly liquid trading platforms Thomson Reuters Matching, EBS and Currenex. Trading occurs every millisecond and therefore only a sample of trades is captured. Valid trades are pooled together and their rates used for the fix. WMR further has the discretion to exclude trades if it deems them non-representative, before publishing the rates. Even though the method of calculation takes no account of trading volume, the larger banks are likely to have a big impact on the rate. In 2016, eight banks traded close to 60\% of total trading volume in the market.\textsuperscript{22}

2.1 Mechanisms for Collusion

Financial markets are highly transparent and market participants can monitor each other closely. The designs of the interbank benchmark rates make them particularly vulnerable to cooperative manipulation. The Libor and Euribor panels consisted for long periods of time of a fixed number of known members, leading financial institutions that would have been in contact with each other and share information on various platforms and in different circles. The major traders on the foreign exchange market are also commonly known. Various investigations revealed evidence of mechanisms at work in the benchmark manipulation cases that may also have induced, facilitated, or be indicative of widespread collusion.

2.1.1 Coordination

The FSA concluded from its fraud investigation that Barclays had “acted in concert with other banks.”\textsuperscript{23} Traders texted messages such as this one, from a Barclays trader to a Deutsche Bank trader on Euribor: “[T]oday we need a low 3 month fixing, could you tell your guys as well if it suits you.”\textsuperscript{24} Or a Barclays trader telling a

\textsuperscript{22}“Citi tops Euromoney global FX poll again, but big banks lose grip,” Reuters, 25 May 2016.
\textsuperscript{23}Financial Services Authority, “Final Notice: Barclays Bank PLC,” 27 June 2012, recital 11. Vaughan and Finch (2017) narrate that to Tucker asking why Barclays submitted such higher Libors than other panel banks, Diamond had responded that it was: “[B]ecause it was the only bank being even vaguely honest about its borrowing costs.” (Op.cit., page 96).
\textsuperscript{24}Commodity Futures Trading Commission, “Examples of Misconduct from Written Communication,” 23 April 2015, page 4, recorded 29 December 2006.
trader of another panel bank after they had coordinated on their Euribor submissions: “[T]his is the way you pull off deals like this (...) the trick is you must not do this alone.”

Many of the panel banks were involved in the manipulations and coordinated potentially between all of them. The U.S. Department of Justice (DoJ) had identified at least five banks involved when it concluded:

“Barclays Euro swaps traders communicated with swaps traders at other financial institutions that were members of the EURIBOR Contributor Panel about requesting favorable EURIBOR submissions from the EURIBOR submitters at their respective banks.”

CFTC officials found that “Everybody was false reporting” and that:

“Libor was routinely being gamed by the banks that set it.”

A broker from ICAP was nicknamed ‘Lord Libor’ for sending a daily email with Libor predictions at 7 a.m. to more than a hundred traders and brokers, including representatives of almost all of the Libor panel banks. The broker proved sensitive to manipulation requests by traders and ICAP was implied in the European Commission cartel case. In forex chat groups, the main currency traders in the market discussed rates, trading positions and intentions on a regular basis.

Abrantes-Metz et al. (2012) interpret certain patterns in the Libor submissions as indicative of coordination. Figure 1 displays the intraday cross-sectional coefficient of variation in quotes, which is the standard deviation divided by the mean of the Libor quotes on a certain trading day. The authors finds it suspicious of collusion that the panel submits much more similar quotes before August 2007 than after, but do not find that the Libor is significantly different from its predicted level when comparing it with other measures of bank borrowing costs.

Arguably, manipulation other than by mass collusion was difficult, because of the way the Libor and Euribor are calculated. By discarding the top and bottom parts of the quotes, individual outliers would be discouraged. Even though misreporting by one bank alone could have moved the average, the effect on the rate of a group of

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26 CFTC lawyer Steve Obie and head of enforcement Greg Mocek quoted in Vaughan and Finch (2017), pages 42 and 76 respectively.
28 The EU General Court annulled in part the Commission’s decision against ICAP in the cartels relating to Yen Interest Rate Derivatives on 10 November 2017.
colluding banks theoretically becomes unbounded once the group consists of strictly more banks than the number of submissions that are trimmed from each side. In the case of Libor, of a panel of 16, the top and bottom 4 are trimmed, so that a minimum of 5 banks would be needed to have an unbounded effect on the rate. For the full original Euribor panel of 44 banks, the top and bottom 6 or 7 would be trimmed, so 7 or 8 would do. In practice, the members would probably not submit quotes that are too extreme, as that might have drawn attention from regulators. Larger groups of colluding banks can have a smoother and much larger influence on the final rate than individually manipulating banks. Likewise, in the case of the forex’ median value, it would have been difficult to affect the rates without pushing many transactions through in cooperation, because of the large size of the currency market.

2.1.2 Episodic Break-up

Whereas coordination of the rates was pervasive, it appears not to have been continuous. For example does the European Commission in *EIRD* stress how:

“On occasion, certain traders employed by [...] communicated and/or received preferences for an unchanged, low or high fixing of certain Euribor
tenors. Those preferences depended on their trading positions/exposures.”

Attempts to coordinate the rates were not always successful, for example when a co-conspirator was unable to accommodate another’s trading position. For such events, the bank employees involved made an effort to explain themselves, apparently in order not to damage the cooperative relationship. For example does a trader apologize for mistakenly failing to accommodate a manipulation request from a submitter at another bank:

“just b4 you beat me up... .. I was in meeting so didn’t do me libors today...thk they put .52 for Is...”

to which the submitter answered:

“hahah no thats fine - thats what i set too cheers skip.”

There would be understanding for inability to comply with requests if it would create a significant conflict with a bank’s portfolio position. On another occasion involving Rabobank, a submitter was asked to submit 3-month Euribor “at the ceiling” because, as another bank’s swaps trader explained “I am long in fixings against Dec futures it cost me a fortune yesterday,” upon which he was informed that Rabobank couldn’t because:

“long swaps need it low.”

Such incidents of independent quoting happened apparently only episodically. For example, on March 22, 2007, the Yen Libor submitter of Rabobank emailed the Lloyds TSB Yen submitter, requesting a “high lmt hpy libor set tomorrow please”. The Lloyds submitter forwarded the email to two colleagues who were making the Yen Libor submissions for Lloyds TSB in his absence and explained:

“We usually try and help each other out. .. but only if it suits .. .!”

There was recognition that even though coordination would not be possible every period, the longer term collusive arrangement was valid and valuable. One submitter preemptively contacted a trader at another bank on March 28, 2008 to excuse that he could not follow in manipulation of the rate that day:

31 European Commission, Case A.39914— Euro Interest Rate Derivatives, 4 December 2013, recital 32, emphasis added. The recital lists seven of such occasions. All bank names have been redacted.
“Submitter-4: ‘morning skip - [Trader-5] has asked me to set high libors today - gave me levels of lm 82, 3m 94....6m 1.02.’
Trader-B: ‘sry mate cant oblige today...i need em lower!!!'
Submitter-4: ‘yes was told by [a third party]... just thought i’d let you know why mine will be higher ..and you don’t get cross with me.’”

The evidence suggests that longer term collusive arrangements were maintained, yet not continuously consummated. Agreeing episodically to break-up coordination of submissions when the circumstances were not quite right appears to have been part of ongoing collusive fixing.

2.1.3 For-profit

Manipulation aimed at enhancing trading results, depending on their trading positions, sometimes accepting current short-run losses for long run future gains. A bank with a net lending position would profit from a higher Libor or Euribor, while a bank with a net borrowing position would prefer a lower one, and the cases provide illustrative quotes of both incentives. Derivatives traders at numerous panel banks were found to have requested submissions aimed at benefiting their trading positions. A Bank of Scotland trader explained:

“Many institutions set their Libors based on their derivative reset positions.”

In a conversation on 6 July 2006, a Frankfurt Eurodesk manager of Deutsche Bank checks whether the manipulated rate would also benefit his colleagues in London:

“Hihi [London MMD Manager], I just want to check whether we have conflicting interests in the June06 settlement. It doesn’t make sense if we try to push one way and u wld like to have it the other way around. We wld prefer a low 3me fixing to push June06 high. Is this ur preference as well?”

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36 European Commission, Case A.39914—Euro Interest Rate Derivatives, 4 December 2013, recital 32.
38 Quoted in Vaughan and Finch (2017), page 163 from an email of a Bank of Schotland trader to British Banking Association’s Libor Director John Ewan that was shown to the jury in the Tom Hayes’ trial.
Contributing banks’ money market desks would have been in a position to know the bank’s overall net expose to the various rates and how they would gain or loose from movements in them. Internal documents from Deutsche Bank, for example, show that on 30 September 2008 Deutsche Bank tallied that it could gain up to €68 million for each basis point change in Euribor and Libor. At UBS and later Citigroup, derivatives trader Tom Hayes, who was sentenced in the US to fourteen years in prison for dishonestly driving manipulation of Libor, stated the bank had software in place that calculated the exact effects of a change in Libor in each currency and maturity on trading profits. At his trial, Hayes asserted that his managers had condoned his actions and UBS instructed company-wide that submissions be based on the bank’s derivatives position spreadsheets.

Hayes’ hearing also gives insight into the volatility of trading exposures:

“At various times you referred, when talking to others, to the difference that a movement of 1 basis point would make in respect of the substantial nominal sums you traded. Those figures varied from $500,000 a basis point to $750,000, $1m a basis point to as much as $2.5m a basis point.”

The exposure position a trader of bank faces on any given day is uncertain and largely stochastic, as it is the sum total of a vast number of transactions done by the banks’ various trading desks worldwide. Around a smaller kernel of longer-term contracted money in- and outflows, exposure positions are largely driven by positions in OTC derivatives that are highly volatile.

### 2.1.4 Monitoring

The protocols by which the benchmark rates were produced by the British Banking Association (BBA) and the European Banking Federation (EBF) were vulnerable to collusion. The nature of the benchmark rates is that the fixings become public almost instantaneously, so that adherence to the agreement can easily be monitored. In a complete cartel, observing the averaged benchmarks suffices to detect defection by at least one cartel member. In addition, by publishing the individual submissions of each panel member identified together with the new rates, all cartel members could see what each of the others had submitted. Perfect monitoring is known to support the design of punishment strategies to stabilize collusion against individual cartel member’s

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41 Vaughan and Finch (2017), page 23.
temptation to unilaterally defect from the cartel agreement. Constant auditing of the individual bank submissions is standard practice for panel banks.

The Libor and Euribor rates were discussed upon publication. Messages around their manipulations reveal instantaneous observation, as in this example from Deutsche Bank:

“On 6 September 2006, Manager B contacted External Trader A and requested a low one month EURIBOR submission [...] On 7 September 2006, after the day’s EURIBOR rates were published, the following exchange took place between Manager B and External Trader A:

Manager B: ‘3.08!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!! thaaaaaaaaaankss’ External Trader A: ‘u see u see’ ”

Even though the fixing of the forex does not feature the same level of centralized information, the outcome of the forex rate could give it away if not the entire cartel respected the agreement. The effects of their manipulations, the conspirators monitored live in chatrooms, as this conversation between HSBC and RBS Traders transcribed by the CFTC makes clear:

“15:51:57 Bank P Trader: good luck
15:52:20 Bank T Trader: load up your 50 offrs...
15:53:14 Bank P Trader: ill do those ones if you want
15:53:19 Bank V Trader: haah
15:53:20 Bank T Trader: ur fkg [Bank V Trader], ramp it
[15:59:30 Fixing window opens]
[16:00:30 Fixing window closes]
16:00:41 Bank T Trader: nice one *****
16:00:56 Bank P Trader: look at you!...-well done mate...”

To the extent that WMR exercised it discretion to exclude some trades as non-representative, and if not all traders on the platform were involved in the cartel, monitoring from the fix values would not have been perfect.

44European Commission, Case A.39914—Euro Interest Rate Derivatives, 4 December 2013, recital 33.
2.1.5 Eligible Transactions Rigging

A challenge to manipulation in forex is that banging the close is done with actual trades, and is therefore potentially costly. Before the reforms, the Libors and Euribors were predictions for which there was no prescribed method of estimation, or even a basis in actual transactions or historical values, so that it was easy and gratis to change the quotes. After full implementation of the reform proposals, they will be based on certain eligible actual transactions, but it is too early to tell patterns of manipulation.

However, the transactions that qualify as eligible have a small volume compared to the total exposure positions of panel banks. In particular, OTC derivatives, which are not eligible transactions for the calculation of the rate, are by far the largest asset class directly related to these interbank benchmark rates. The Financial Stability Board (FSB) reported in 2014 that over 170 trillion dollars in OTC derivatives are tied to the USD Libor, and over 197 trillion dollars to the Euribor. In comparison, the second highest and most USD Libor-related asset class are syndicated loans with an estimated 3.4 trillion dollars. As banks’ Libor or Euribor-related portfolio is so large compared to the volume of eligible transactions, banks could manipulate the latter to justify their rate submissions at relatively low costs for very large gain. Panel members could potentially have done eligible transactions between themselves in order to built a submission basis, without sustaining the costs of suboptimal trades with outsiders.

Apart from the direct transactions costs involved in these trades, there would be indirect costs as well. The transactions that are now to underlie the Libor and Euribor submissions by construction trade against different rates than the current ones: the intended future ones, after all. In case of forex, pushing transactions through the fixing-window carries currency risks. An anonymous trader involved explained:

“It could still backfire if another dealer with a larger position bets in the other direction or if market-moving news breaks during the 60-second window.”

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48 There is evidence of such trades that served no commercial purpose in Vaughan and Finch (2017). For example in a phone call to broker Farr at RP Martin, Hayes proposed: “If you’ve got any mates, mate, who will do you like a net trade, I can like basically give you like, fucking I don’t know a trillion three-month Libor/Tibor and take back a trillion three-month Libor/Tibor. Obviously you’ll net it with the other guy.” (op. cit., page 63). These practices are explained as so-called ‘wash trades’, with the purpose of paying commissions to brokers who helped influencing submissions of other banks. It would also be a means of carrying out matching eligible transactions at the intended cartel rate, avoiding costs of collusion.
Dealers therefore would only employ the strategy if they knew enough about the other banks’ positions and had sufficiently large client orders:

“Typically, that would need to exceed 200 million euros to have a chance of moving the rate, two of the traders estimated.”

For these reasons it was attractive for traders to collude on front running client orders and concentrating trade when the benchmark rates are set—note that in and of itself, concentrating high volumes of trades around the fixing-window was not illegal. Likewise would lone attempts to support a different interbank rate be costly without much guarantee of being effective.

2.1.6 Front Running

After determining their actions collusively, and in the time window between when banks submit their Libor and Euribor quotes to the central administrator and when the new rates are being published, panel members would have inside information on their committed upcoming quote submissions before the new rate became public. This would allow the banks involved to adopt more favorable exposure positions through insider trading, at the expense of uninformed other market participants. It also would make alignment of panel members interests in colluding possible. There is no explicit discussion of front running in what has so far become public of the Libor and Euribor investigations. However, there is evidence suggesting that traders adjust their exposure to the rate depending on information on future quoting behavior. For example Hayes was quoted to have asked another submitter by email, referring to the bank’s decision to increase its Libor submissions:

“Do you talk to the cash desk and did we know in advance? We need good dialogue with the cash desk. They can be invaluable to us. If we know ahead of time we can position and scalp the market.”

In particular OTC derivatives are well suited for front running, since they determine most of a bank’s exposure position, are highly volatile, short and long, and non-eligible for calculation of the benchmark.51

Front running certainly was central in the forex manipulations.52 Traders exchanged confidential information on their own positions and clients’ trade orders, agreed beforehand in which direction they were going to push the rate in the upcoming fixing-window and planned how to transact to benefit from banging the close.

50 Vaughan and Finch (2017), page 114.
Suppose that a trader would get client orders large enough to move the market at 3.30 p.m. to sell dollars for euros at the 4 p.m. fix. Executing it would lower the value of the dollar. The trader with coconspirators would have incentive to sell their dollar positions—or even going short in dollars—before the start of the 4 p.m. window. In the fixing-window, the cartel would then sell as many dollar client orders as possible, lowering the dollar benchmark at which the member later buy back dollars to their recover their positions. Without this banging the close, the dollar benchmark would have been higher and the client would have received more euros for his dollars. The traders benefitted from first selling at higher price and then buying back at a lower price. The opposite strategy allowed for collusively benefitting from large or combined buy orders to push up the 4 p.m. fix, at which later to sell the own currency positions.53

In addition, traders could use their knowledge about impending client orders to build up positions ahead of the fix to align interests:

“Any members of the group with an opposing interest knew to offload their ‘ammo’ ahead of the fix to an unsuspected victim. Traders who weren’t in the club were steamrollered mercilessly.”54

Such a strategy constitutes a risk, as the price for dollars could increase after having created a short position, for example due to market-moving news or trading from other parties on the platform. By colluding with (enough) other banks, traders would have a larger effect on, and a better idea of the trading taking place in the fixing-window. Moreover, they would be able to profit from front running on any other cartel member’s large client orders.

### 2.2 Reforms

In response to the benchmark rate scandals, several reforms to the rate setting processes have been proposed and are being implemented. Following the 2013 FSA reforms, individual Libor quotes are no longer published simultaneously with the final rate.55 Maturities and currencies that were too thinly traded were discontinued, and a 3-month embargo was put on the publication of the individual submissions—Euribor quotes are still published simultaneously with the rates. Furthermore, the administration of Libor was transferred to ICE Benchmark Administration (IBA) in order to improve governance and oversight over the rate setting processes—and Euribor-EBF changed its name to the European Money Markets Institute (EMMI). The FSB

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53 In L. Vaughan, G. Finch and A. Choudhury, “Traders Said to Rig Currency Rates to Profit off Clients,” Bloomberg, 12 June 2013, a trader interviewed estimates that a move in the benchmark of 2 basis points would be worth over $200.000 on a $1.3 billion client order.


subsequently called for underpinning the benchmarks with actual transaction data, which the IBA and EMMI are implementing.\textsuperscript{56}

Libor contributions are now to be supported by actual trades at the submitted rates in unsecured deposits, commercial paper and certificate of deposit, where the submitting bank received funding from wholesale market counterparties such as other banks, central banks and large corporations.\textsuperscript{57} In principle only actual transactions done in the 24 hours prior to the submission deadline are eligible, but in case of insufficient transactions, a somewhat wider range can be used. Of all eligible transactions, the volume weighted average rate constitutes the rate submission, unless it is deemed unrepresentative by IBA, in which case it can adjust it.\textsuperscript{58} The Libor remains to subsequently be calculated as the average of 50\% of the submitted rates after trimming.

The Euribor is also to evolve into a transaction based rate.\textsuperscript{59} Only transactions of unsecured cash deposits from specified counterparties and short-term securities, such as commercial paper and certificate of deposits, traded in the wholesale unsecured money markets in the past 24 hours are eligible in the volume-weighted average rate submissions. In case of insufficient transactions, eligible transactions from previous days can be included as well. As a last resort, EMMI can take responsibility for setting the benchmarks. Contrary to Libor, banks have no residual, non transaction-based, discretion over their submissions. Euribor would subsequently be calculated as the average of the middle 4 or 5 quotes, or with the current 20 banks to which the Euribor panel has gone down since the scandals, 75\% trimming. Yet EMMI intends to grow the Euribor panel again to approximately 40 members.\textsuperscript{60}

While these reforms make the Libor and Euribor more robust, the forex manipulations have shown that even an entirely transaction-based rate could be colluded to increase trading profits. The eligible transactions are only a small part of the bank’s portfolio exposures to the benchmarks.\textsuperscript{61} Banks could transact those at rates that are suboptimal on their own, but support manipulations of the benchmarks that are


\textsuperscript{58}IBA also studies the feasibility of a centrally calculated rate, where banks would only submit raw data on eligible transactions. See ICE Benchmark Administration, “Roadmap for ICE Libor,” 18 March 2016, pages 20-21.


\textsuperscript{60}European Money Markets Institute, “The Path Forward to Transaction-based Euribor,” 21 June 2016, page 13.

\textsuperscript{61}By far the largest volume of transactions tied to Libor and Euribor is from over-the-counter derivatives, which are not eligible for the calculation of the rate. See Financial Stability Board, “Reforming Major Interest Rate Benchmarks,” July 2014, page 348.
beneficial to the larger rest of their trading books.\textsuperscript{62} Manipulation with transactions remains possible with the reformed forex as well, to which the main adaptation has only been to base the fixes on transactions collected from a somewhat longer time window: between 2.5 minutes before and after 4 p.m., instead of a 1 minute window around the hour.\textsuperscript{63} Monitoring of individual submissions remains perfect in Libor and Euribor, even though the delay in publishing potentially makes deviations more difficult to detect. Despite implemented and proposed reforms, the benchmark rate setting processes remain vulnerable to collusion.

Most recently, the U.S. Alternative Reference Rates Committee (ARRC) has recommended the phasing out of Libor as a benchmark rate in a transition to the Secured Overnight Funding Rate (SOFR) in the next five years.\textsuperscript{64} The SOFR is a wider applied rate than just interbank lending, so that activity in the rate is in better balance with the amount of contracts written on it. The secured funding market also is more liquid, so that a larger volume of eligible transactions would be needed to rig the rate, making manipulation more costly. Nevertheless, the OTC market is still much larger in volume and the SOFR would also be susceptible to collusion.

3 A Model of Benchmark Rates Collusion

Consider a panel of \( N \) banks \( i = 1, \ldots, N \) that play an infinitely repeated simultaneous move game. On day \( t \), let \( v_{0it} \) be bank \( i \)'s baseline portfolio position by which it is exposed to changes in the interbank rate for a certain maturity.\textsuperscript{65} The true borrowing costs, against which bank \( i \) can do transactions on the day are \( c_{0it} \). A bank's eligible transactions are not part of its exposure to the benchmark rate. Both \( v_{0it} \) and \( c_{0it} \) are private value daily draws. Variations in \( v_{0it} \) reflect changes in the bank's net trading book exposure to all its Libor-related activities. Changes in \( c_{0it} \) reflect variations over time of the bank's ability to borrow on the money market, which is affected by bank-specifics such as capital structure and liquidity position.

At the start of each trading day, the valid interbank lending rate is \( L_{t-1} \), published the day before. The new rate, \( L_t \), is to be fixed on the basis of all new rate submissions \((c_{11t}, \ldots, c_{1Nt})\). During the day, the banks engage in various business transactions. If

\textsuperscript{62}The possibility that eligible transactions would be rigged for the purpose of unilateral manipulation was suggested in Duffie and Dworczak (2014).


\textsuperscript{65}We develop the model in the text for one benchmark rate, whereas there are many rates set, partly simultaneously, for various maturities on a daily basis. Since the exposures to each of these rates are in principle unrelated, also the rates on various rungs of the maturity ladder can have been so manipulated independently.
a bank intends to submit a rate $c_{1it}$ that is different from $c_{0it}$, it will have to engage in eligible transactions against the intended rate, rather than the true rate. In addition, banks can adjust their portfolio position with an eye to the new rate. Bank $i$’s eligible transactions rate submitted is $c_{1it} = c_{0it} + \Delta c_{it}$, and at the time the new benchmark rate is published, its realized exposure position is $v_{1it} = v_{0it} + \Delta v_{it}$. We refer to choice variables $\Delta c_{it}$ and $\Delta v_{it}$ as ‘eligible transactions rigging’ and ‘front running’ respectively.

It is assumed that the panel banks can always find counterparties for their intended trades in the derivatives markets. Front running takes place at the going prices in vast and liquid markets. Eligible transactions rigging happens in a thinner market, yet against terms that can be profitable for outsiders. The intended rigged future benchmark rate for which a panel bank seeks eligible transactions is different than the valid one. Either it borrows at a higher rate, in which case the counterparties gain directly, or the panel bank would need to offer conditions to a loan it requires at a lower rate in order to make it an interesting proposition. In either case, even if a part of the other market participants had knowledge of the panel banks colluding, the panel banks’ offers would still go through at their conditions.

Assuming that each panel bank always submits a quote each period, the new interbank rate $L_t$ is determined as the trimmed average of all $N$ quotes.\footnote{The rules for calculating the Libors and Euribors allow for a minimum number of submissions to calculate the rate (in the case of Libor: 5 quotes, for all of the tenors for a particular currency; in case of Euribor: 50% or 12 banks, from at least 3 different countries), so that in practice a cartel could save on manipulation costs by agreeing the that banks with the highest cost to do submit a quote at all. In practice, the minimum rule is meant for exceptional circumstance and banks systematically not submitting would raise suspicion with the authorities, even prior to the scandals.} We call the set of submissions from which the upper and lower share of ranked quotes are discarded the ‘trimmed range’ $T$ consisting of $n$ banks. Hence,

$$L_t = \frac{1}{n} \sum_{j \in T} c_{1jt}. \quad (1)$$

Note that before the reforms, the individual submissions $c_{1it}$ were published simultaneously with $L_t$—since the Libor embargo, this is with a delay.

Since the majority of financial contracts, such as swaps, futures and corporate loans, have linear payouts, bank $i$’s gains from changes in the rate from the current to the next trading day are

$$\pi_{it} = v_{1it} (L_t - L_{t-1}) - C_i (\Delta c_{it}, \Delta v_{it}), \quad (2)$$

where $C_i (\Delta c_{it}, \Delta v_{it})$ are any costs associated with bank-specific changes in exposure and rate.

Manipulation is costly since eligible transactions rigging requires banks to borrow on otherwise suboptimal terms and front running involves direct transaction costs, trade risks and liquidity constraints. Extreme adjustments may further be constrained
by the risk of raising suspicion of manipulation with other market participants or regulators increasing in the degree of front running and eligible transactions rigging respectively. In the following, we simplify to the symmetric case $C_i(\Delta c_{it}, \Delta v_{it}) = C(\Delta c_{it}, \Delta v_{it})$ for all $i = 1, \ldots, N$. It is natural to assume that $C(\Delta c_{it}, \Delta v_{it})$ is strictly convex in both $\Delta c_{it}$ and $\Delta v_{it}$. This assures that a global maximum for each bank $i$’s objective function $\pi_{it}$ exists and is unique if also $C''_{\Delta v_{it} \Delta c_{it}}(\cdot)$ is small enough, which is a mild assumption since the two manipulation mechanisms relate to very different classes of transactions.\(^{67}\)

Through $L_t$, the payoff function of each bank depends not only on its own exposure and eligible transactions, but also on the eligible transactions of the other banks in $T$. As a result, there is an incentive to coordinate behavior. If the panel colludes, the baseline values $(c_{0t}, v_{0t})$ are shared and the cartel determines the joint-profit-maximizing front running and eligible transactions rigging strategies collectively.

Figure 2 illustrates the timing of cartel events in the Libor rate-setting process relative to the opening and closing bells of the trading day at the London Stock Exchange—$OB$ and $CB$. At opening, $L_{t-1}$ is the current Libor rate. Suppose that at time $0_t$, shortly into day $t$, all banks learn $c_{0t}$ and $v_{0t}$.\(^ {68}\)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{A trading day in the life of Libor.}
\end{figure}

Subsequently, the banks consider alternative eligible transactions rigging $\Delta c_{it}$ and front running strategies $\Delta v_{it}$: either none, non-cooperatively or cooperatively. If they collude, the panel banks share their private information at cartel meeting $C_t$ in which the designated joint-profit-maximizing front running and eligible transactions rigging

\(^{67}\)Since the first part of $\pi_i$ is linear in both $\Delta v_{it}$ and $\Delta c_{it}$, together with positive and increasing marginal costs, a necessary and sufficient condition for global maximum is that $C''_{\Delta v_{it} \Delta c_{it}}(\cdot) \times C''_{\Delta c_{it} \Delta c_{it}}(\cdot) - C''_{\Delta v_{it} \Delta c_{it}}(\cdot)^2 > 0$.

\(^{68}\)Note that although illustrated in Figure 2 at a specific point in time ($0_t$, shortly after $OB$), in practice the banks see their baseline values change continuously, as OTC trading in particular takes place around the clock worldwide and unforeseen events or market-moving news could constantly affect the baseline values. The cartel can accommodate such multiple changes by sharing the relevant information and updating the cartel strategy throughout, as long as the windows for manipulation are open.
is determined for each member. Latest at 11.00 a.m., moment $S_t$, all banks submit their Libor quote based on $c_{1t}$, which closes the window for eligible transactions rigging.\(^69\) The window for front running remains open longer, until publication at 11.55 a.m. of the new rate at $L_t$ when the new rates become known to all market participants.\(^70\)

Both $v_{0t}$ and $c_{0t}$ are assumed to be independent and identically distributed, each according to a symmetric and commonly known continuous distribution. The overall exposure position of the panel banks to the rate is assumed to fluctuate around zero, i.e. $E[v_{0i}] = 0$. This setup captures that exposure in large part stems from transactions in OTC derivative markets, which have a buyer and seller for every contract and are volatile and liquid enough for all banks to regularly find themselves flipped from one side of the market to the other, while allowing part of a bank’s exposure profile may be relatively stable on the books, more or less symmetrically. It limits the analysis to myopic profit maximization.

The baseline eligible transaction rates $c_{0it}$ are drawn from a common distribution with mean $E[c_{0i}] = L_{t-1}$, reflecting that Libor is a main signal to creditors, who would not have known about any manipulation. The mean is assumed to be equal across panel banks, which are all global systemically important banks (G-SIBs) facing similar regulatory requirements.\(^71\) Shocks to the panel banks’ respective borrowing capacities are assumed to be non-persistent.

We study collusion without side-payments, either as explicit transactions or in more sophisticated forms, such as partially swapping positions internally, in which a cartel member with a major profitable position to the cartel strategy would trade with other members to mitigate their positions opposite to the general cartel interest. Such internal alignments of interests would further facilitate collusive manipulations. The following subsections describe banks’ strategies in case of independent behavior, collusion and defection.

\(^69\) Note that eligible transactions for Libor submissions on day $t$ are those executed between the previous submission and the new submission at $S_t$. Collusive eligible transactions rigging can only be done after information has been exchanged at time $C_t$. For Euribor submissions on day $t$, all transactions executed on trading day $t-1$ are eligible. Therefore, the eligible transactions rigging window for Euribor is somewhat different from the one in Figure 2. The Euribor cartel would use earlier baseline information and need to meet earlier, so as to manipulate eligible transactions the day before.

\(^70\) For Libor, the difference between the eligible transactions and front-running windows is less than an hour: quotes are submitted by the end of the eligible transactions window at 11.00 a.m. and the rate is published at 11.55 a.m. In the case of Euribor, the difference between the eligible transactions rigging- and front-running windows is the time between the end of trading day $t-1$ and when the rate is published shortly after 11.00 p.m. on day $t$.

\(^71\) The daily rate drawings can alternatively be assumed from an unmanipulated mean, in particular a daily rate that follows from honest reporting only. If manipulation was indeed widespread and commonly known, a ‘shadow Libor rate’ may have accounted for the actual borrowing standard. The effects of alternative means on rate patterns and screening are discussed in Section 4.
3.1 Independent Quoting

If the panel banks formulate their contributions independently, they determine their portfolio changes and submissions with incomplete information. If all banks follow the banking code of conduct and accordingly honestly submit their true borrowing cost and not front run, the strategy of bank $i$ at period $t$ is $\Delta v_{it} = \Delta c_{it} = 0$ with payoff $\pi^*_{it}$. It then follows directly from the distribution assumptions that $E_{it}[\pi^*_{it}] = E[\pi^*] = 0$ for all $i = 1, \ldots, N$ and $t = 1, \ldots, \infty$—the $*$ indicating honest quoting.

To follow the banking code of conduct is not individually optimal, however. Instead, maximizing own expected gains in the benchmark-setting gives:

$$\pi^{BN}_{it} : \max_{\Delta v_{it}, \Delta c_{it}} E_{it}[\pi_{it}] \quad \forall i = 1, \ldots, N,$$

(3)

potentially induces each bank independently to engage in front running or eligible transactions rigging. Manipulation is unilateral, under the assumption that the other panel members report honestly, since banks have no information on the baseline positions of the other banks. Equilibria are in pure strategies. Let $\pi^{BN}_{it}$ be the payoff of bank $i$ in period $t$ in the static Bayesian-Nash equilibrium, with expected payoff $E_{it}[\pi^{BN}_{it}]$. Under the symmetry and distribution assumptions, $E_{it}[\pi^{BN}_{it}] = E[\pi^{BN}] \geq 0$ for all banks $i$ and each period $t$.

3.2 Collusive Quoting

If the panel banks would form a cartel instead, they share the baseline information $v_{0it}$ and $c_{0it}$ on all banks $i = 1, \ldots, N$ at the beginning of each trading day. With this inside information, the joint-profit-maximizing new rates are established, as well as by how much each member is to engage in eligible transactions against their target submission in order to support the target rate. Additionally, the changes to the portfolio positions of the banks are determined that reduce any misalignment of exposures between them and give the cartel optimal exposure to the future rate.

The complete information cartel strategy in period $t$ follows as

$$\pi^C_{it} : \max_{\Delta v_{it}, \Delta c_{it}} \sum_{i=1}^{N} \pi_{it},$$

(4)

where $\Delta v_{it}$ and $\Delta c_{it}$ are vectors of the front running and eligible transactions rigging targets. We denote the vector of $N$ realized payoffs by $\pi^C_{it}$ and offer the following result.

**Proposition 1** There exists a per-period unique globally optimal cartel strategy.

**Proof.** As part of the equilibrium conditions, the marginal bank-specific costs of changes in the eligible transaction rate are assumed to increase in $\Delta c_{it}$, i.e. $C^C_{\Delta c_{it}} > 0$. Therefore, if the cartel would change the ranking of the eligible transaction rates, the
same set of final rates \((c_{1t}, \ldots, c_{Nt})\) could have been achieved at lower total eligible transaction rate rigging costs by retaining the ranking. This implies that the following inequality constraints hold

\[ c_{0(i+1)t} + \Delta c_{(i+1)t} \leq c_{0it} + \Delta c_{it} \quad \forall i = 1, ..., N - 1, \tag{5} \]

where bank indicator \(i\) is now equal to its rank based on the baseline eligible transaction rates \((c_{01t}, \ldots, c_{0Nt})\). Since the baseline eligible transaction rates are drawn from a continuous distribution, there exist various possible strategies where the ranking does not change and all constraints hold with inequality—an obvious candidate is the strategy of no manipulation, \((\Delta c_t, \Delta v_t) = (0, 0)\). These are Slater points, the existence of which is both necessary and sufficient for the existence of a global optimum in a non-linear optimization problem with inequality constraints. See, for example, Brinkhuis and Tikhomirov (2005), pages 210-211.

Let \(\pi^C_{it}\) be the realized payoff of bank \(i\) in the cartel optimum in period \(t\), following the optimization. Ex ante, the per-period expected payoffs from participating in the cartel are \(E_t[\pi^C_{it}] = E[\pi^C]\). While \(\pi^C_{it}\) may be negative, and even lower than under independent behavior as cartel members in turn ‘take one for the team’ by submitting quotes that are not optimal given their baseline exposure position, over time all banks can expect to profit from colluding equally. This ensures that no explicit side-payments are necessary. Further note that \(E[\pi^C] > E[\pi^{BN}] \geq E[\pi^*] = 0\), since it is always possible for a bank participating in the cartel at least to front run. The ability to create movement in the rates that is predictable with inside information makes a cartel even more beneficial. As counterparties trading in financial products tied to the rate are less well-informed of where the future rate will go, cartel members can profit at their expense.

The cartel is efficient in the sense that the order of the baseline transaction rates is preserved in the submissions that are asked of the members, which minimizes the cartel’s total eligible transactions rigging costs. Apart from the \(n\) banks in the trimmed range \(T\), up to either all the banks that will end up in the lower or the upper ranked share of discarded submissions are asked to engage in eligible transactions rigging—between half and three-quarter of the panel in the case of Libor. the panel. Which banks are included in \(T\) and which are not varies with the daily drawings. Banks outside \(T\), even though their submissions are discarded in the determination of the interbank rate, may also be called upon to engage in eligible transactions rigging in order to move over and accommodate the rigging by banks within \(T\).

Figure 3 illustrates such a situation in the case of four panel banks, the middle two of which are in the trimmed range. Bank 1 moves over to the right, so that banks 2 and 3 together can drive up \(L_1\) as their average submission. Never, however, does a bank in the periphery (banks 1 or 4) cross over into \(T\) and move the rate instead of the bank(s) with an interior position, as this is always more costly. Banks both inside and outside \(T\) will always find it in their private interest to front run, independent of the cartel strategy.

25
Figure 3: Panel bank 1, not in \( T \), engaging in collusive eligible transactions rigging.

Note that the assumption that the panel members report their true borrowing costs and baseline exposure positions to the cartel truthfully is not that stringent, in the sense that it is not obvious how a bank would be better off lying—when the cartel requires all banks to report their position and rate simultaneously. In principle, banks reporting different than actual values would not be easily discovered, as long as they subsequently behave according to cartel instructions. A motive could be to try to avoid cost of collusion, by pretending to have relatively low or high borrowing costs, or increase chances of the optimal cartel quote being more favorable by overstating its exposure position. However, reporting other than true costs may just as well land a bank at the wrong side of true borrowing cost—ending up being assigned higher eligible transactions rigging costs than it would with the truth. Similarly, by overstating its exposure position a bank risks too extreme optimal cartel quotes, for which its manipulation costs would exceed the gains from trying to manipulate the cartel agreement, or even from collusion as a whole.

3.3 Defection

After banks have shared their private information and determined the optimal cartel strategy for the day, each bank may have the incentive to unilaterally defect. For example, one or more banks within the trimmed range \( T \) may have a negative exposure to changes in the interbank rate, but still be designated to facilitate upwards rigging of \( L_t \) for the benefit of the cartel. By unilaterally defecting, a bank in such a position would benefit from reducing the upward manipulation of \( L_t \) and forego the costs of its eligible transactions rigging—at the expense of the other cartel member banks.

The optimal deviation of bank \( i \) in period \( t \) follows from

\[
\pi_{it}^D : \max_{\Delta v_{it}, \Delta c_{it}} \pi_{it}(\Delta v_{C_{it}}, \Delta c_{C_{it}}),
\]

in which \( \Delta v_{C_{it}} \) and \( \Delta c_{C_{it}} \) refer to the front running and eligible transactions rigging...
of all panel members but bank \( i \) under the collusive optimum. We denote the optimal defection payoff of bank \( i \) following this optimization by \( \pi_{it}^D \).

The trimming limits the scope for deviating. For a bank in the trimmed range \( T \), defection always increases profits. A bank not in the cartel \( T \) can decide to position itself at any point within it to make its quote count, yet this need not be optimal, depending on its position. For example in Figure 3, if bank 4 had a negative exposure it would want to see the new rate as low as possible, whereas positioning itself within \( T \) would only result in a (weakly) higher rate and positive eligible transactions rigging costs. Therefore, a deviating bank will position itself in \( T \) in order to attempt to influence the rate, or does not engage in eligible transactions rigging at all, whichever gives higher payoff.

Internal monitoring of quotes is perfect. Once the interbank rate is published, all the cartel members can immediately infer from the rate whether there has been defection from the collusive eligible transactions rigging strategies—in addition, with a delay the individual submissions are published as well. While it is not obviously possible to observe whether a bank has deviated from the agreed collusive exposure position changes, these are individually optimal for each cartel member to carry out, given the agreed submission strategy. Also deviations in \( \Delta v_{it} \) have no effect on the profits of other cartel members.

### 3.4 Cartel Stability

The cartel would need to stabilize adherence to its agreements against incentives to deviate. That is, it plays the per-period strategy that maximizes joint profits, subject to the constraints that for each bank \( i \) in period \( t \) the expected value of collusion \( V_{it}^C \) is at least as high as the expected value of defection \( V_{it}^D \). Note that the effects of deviating here are different from a conventional cartel, in which defection profits are higher relative to collusive profits, the more cartel members there are to share the pie with. In a benchmark cartel, more cartel members means that every individual bank has a smaller impact on the rate, and therefore defection profits instead are smaller. As a result, benchmark collusion can be more easily sustained for larger numbers of participating banks.

Using \( \pi_{it}^C \), \( \pi_{it}^D \) and \( \pi_{it}^{BN} \) and discount rate \( \delta \in (0,1) \), we can specify for bank \( i \) in period \( t \) the expected value of collusion as the sum of current-period payoffs and discounted continuation values, i.e.

\[
V_{it}^C = \pi_{it}^C + \delta E[V^C].
\]  

\( E[V^C] = \sum_{t=0}^{\infty} \delta^t E[\pi^C] \) is the expected discounted continuation value of collusion.

The instantaneous payoff from deviating plus the expected discounted value of its consequences when discovered and punished is

\[
V_{it}^D = \pi_{it}^D + \delta E[V^P].
\]
For every punishment strategy in which defection triggers $T \geq 0$ periods of reversion to non-cooperative contributions, the off-equilibrium occurrence of punishment means that increasing $T$ only increases cartel stability, so that it is optimal to set $T \to \infty$ and specify $E[V^P] = \sum_{t=0}^{\infty} \delta^t E[p^{BN}]$. The grim trigger strategy is credible, since once banks revert to Bayesian-Nash punishment, that is a sub-game perfect equilibrium. Yet the cartel would be stable for any (possibly stochastic) $T$ sufficiently large.

To assure adherence to the cartel by bank $i$, in each period $t$ the panel banks maximize joint profits (4), subject to $V_{it}^D \leq V_{it}^C$. This solves as

$$\pi_{it}^D - \pi_{it}^C \leq \frac{\delta}{1-\delta} (E[\pi^C] - E[\pi^{BN}]) \quad \forall i = 1, ..., N,$$

in which the left-hand side payoff differentials vary between banks and periods, depending on the portfolio positions, and the right-hand side is a fixed critical cut-off value that decreases in discount rate $\delta$. Note that if these incentive compatibility constraints hold for Bayesian-Nash independent quoting, they certainly do for honest quoting, since $E[\pi^{BN}] \geq E[\pi^*]$. Using this supergame structure, we first identify the first-best continuous collusion strategy and explain why this strategy is not feasible. We subsequently identify a practical cartel strategy that involves episodic break-up.

### 3.4.1 Continuous Collusion

The optimal cartel strategy would be continuous collusion, in which the cartel adjusts the profit maximizing vector of eligible transactions rigging and front running each period, such that the incentive compatibility constraints resulting from individual banks’ baseline value draws hold. That is, each day the cartel is to keep each payoff differential $\pi_{it}^D - \pi_{it}^C$ below the critical value by potentially adjusting $\pi_{it}^C$, and thereby indirectly also $\pi_{it}^D$ in the incentive compatibility constraints (9) to

$$\max_{\Delta \pi, \Delta c} \sum_{i=1}^{N} \pi_{it} \quad \text{subject to} \quad \max_{i=1, ..., N} (\pi_{it}^D - \pi_{it}^C) \leq \frac{\delta}{1-\delta} (E[\pi^C] - E[\pi^{BN}]),$$

in which the bank that poses the tightest constraint is endogenously determined.

Continuous collusion on benchmark rates is considerably more complex than in conventional markets. Generically the payoff functions are asymmetric and provide $N$ different inequality constraints, each of which itself results from the optimization problem by which each bank determines its optimal defection strategy $\pi_{it}^D$ for its portfolio position and rate, given that all other panel banks behave according to the cartel agreement. Rotemberg and Saloner (1986) rely on the cartel having the option to fall back on marginal cost pricing, from which no cartel member would deviate, during booms, when the incentive to deviate is largest. By lowering the payoff differential from defecting, the cartel remains stable under infinite punishment.
Such a fixed fallback option does not exist in our model of benchmark rates collusion, since the incentives to deviate vary with individual positions and rates. For instance, if the cartel would instruct to revert to a case where the rate is not manipulated, each member would still have incentive to unilaterally manipulate and front run, given that the panel exchanged information. Portfolio position-specific stable collusive actions therefore need to be determined every day anew.

However, finding common ground in the daily cartel optimization problem is computationally demanding for several reasons. Solving (10) requires knowing the expected collusion payoff $E[\pi^C]$, which is not a priori determined. In addition, both the optimization and its constraints are endogenous, since $\pi^C$ and $\pi^D$ both follow from the solution of (10) and are part of the constraints used to obtain it. Defection profits $\pi^D$ even follow from a separate optimization by each bank, maximizing its own profits given that the other banks play the previously determined collusive strategy $\pi^C$. Furthermore, the optimal cartel strategy can include that a bank with a lower baseline eligible transaction rate is required to submit higher quotes than a bank with a higher rate that has an incentive to deviate, in order to keep cartel stability. Since the ranking of eligible transaction rates no longer needs to preserve the order of the baseline transaction rates, the proof of Proposition 1, which relies on the absence of cross-overs and thereby significantly reduces the strategy space, no longer holds. Note that one cartel member incurring higher manipulation costs to allow another a larger cartel profit is a form of side-payments and makes the continuous collusion strategy cost-inefficient. Finally, brute force calculations to derive all outcomes of each possible strategy set and identify the global optimum among the subset of outcomes for which the constraints hold would require discretizing and ex ante restricting the strategy space, as the choice variables are continuous and unbounded. The necessary high number of small bins, combined with the dimensionality of choice variables ($2N$) would yield a very large number (number of bins to the power $2N$) of strategies that would need to be checked.

### 3.4.2 Episodic Break-up

The benchmark cartel is feasible using an episodic break-up strategy, as in Fershtman and Pakes (2000). In this much simpler strategy, all panel banks choose the unconstrained joint profit maximizing strategy as long as it satisfies per period the incentive compatibility constraints of all banks, until at least one panel bank would deviate, in which case all banks revert to non-cooperative contributions for that period.\textsuperscript{72} The cartel is continuous in that each period information is shared, but also breaks up episodically in unstable periods to determine strategies individually. Only deviation from this strategy would be punished with reversion to non-collusive contributions forever after.

\textsuperscript{72}We are indebted to Joe Harrington for suggesting this.
During a break-up, the panel banks determine their contributions non-cooperatively with complete information as

$$\pi^N_{it} : \max_{\Delta v_{it}, \Delta c_{it}} \pi_{it} \quad \forall i = 1, \ldots, N.$$  \hfill (11)

Let the one-period static Nash equilibrium with full information of the panel banks be $\pi_{it}^N$, with $E_{it} [\pi_{it}^N] = E [\pi^N]$ for all $i = 1, \ldots, N$ and $t = 1, \ldots, \infty$. Since interests are typically conflicting, the Nash equilibrium need not be unique, nor exist in pure strategies. Note however that $E [\pi^N] \geq E [\pi^{BN}]$, since all banks are fully informed in formulating the break-up contributions and any information that helps a bank to better predict the new rate allows it to front run lucratively and increase expected payoff. Without agreement on the rate, banks can only front run in the direction of where they expect the rate to go. Generally, their portfolio changes will be more conservative than under full collusion.

To analyze the pattern of switching between full collusion and episodic break-ups, let $\rho \in [0, 1]$ be the probability that the unconstrained joint-profit-maximum violates one or more of the incentive compatibility constraints and the cartel reverts to one-period static Nash. Per-period expected payoff from colluding then is

$$(1 - \rho) E [\pi^C] + \rho E [\pi^N],$$  \hfill (12)

where $E [\pi^N]$ is conditional on there being a break-up, and $E [\pi^N]$ on not. Since break-up occurs at extreme value positions that the panel bank(s) can exploit with their shared information, it may well be that $E [\pi^N] > E [\pi^C]$.

Given infinite punishment, the net present value of all forgone future expected payoffs in case of cartel defection becomes

$$\frac{\delta}{1 - \delta} ((1 - \rho) E [\pi^C] + \rho E [\pi^N] - E [\pi^{BN}]) \equiv \Psi (\delta, \rho),$$  \hfill (13)

since only in a punishment phase is quoting truly non-collusive, resulting in $\pi_{it}^{BN}$—or possibly $\pi_{it}^*$ if the panel banks choose to follow the code of conduct. $\Psi (\delta, \rho)$ is the critical cut-off value for the value differential $\pi_{it}^D - \pi_{it}^C$, below which collusion is stable.

The probability $\rho$ of episodic cartel break-up is now defined implicitly by the tightest stability constraint through

$$\rho = 1 - Pr \left[ \max_{i=1, \ldots, N} (\pi_{it}^D - \pi_{it}^C) \leq \Psi (\delta, \rho) \right].$$  \hfill (14)

Given $\Psi(\delta, \rho)$, the value of $\rho$ is under the remaining tail of the probability density function of $\max_i (\pi_{it}^D - \pi_{it}^C)$, which derives from the distributions over bank $i$’s initial portfolio position $v_{0it}$ and eligible transaction rate $c_{0it}$. Figure 4 illustrates.

We can now establish conditions for the existence of stable continuous collusion with episodic break-up.
Proposition 2. For a continuous and sufficiently widely supported distribution of the maximum payoff differential, break-up probability \( \rho \) is strictly between 0 and 1.

Proof. The implicit definition of \( \rho \) in equation (14) is a continuous mapping from a nonempty, compact and convex set \( \rho \in [0, 1] \) onto itself, so that at least one fixed point solution exists. Let the support of the continuous distribution of \( \max_i (\pi^D_{it} - \pi^C_{it}) \) be \([a, b]\). For a lower bound \( a < \Psi(\delta, \rho = 1) = \frac{\delta}{1-\delta} \left( E[\pi^N] - E[\pi^{BN}] \right) \) and an upper bound \( b > \Psi(\delta, \rho = 0) = \frac{\delta}{1-\delta} \left( E[\pi^C] - E[\pi^{BN}] \right) \), the largest payoff differential can occur with positive probability for which the cartel always breaks up and for which the cartel never breaks up. Hence, \( \rho = 1 \) and \( \rho = 0 \) can not be a fixed point and \( \rho \) must lie strictly between 0 and 1.

For reasonable assumptions on the underlying stochastics, the cartel always exists to share information, regularly quotes collusively \( (\rho < 1) \), but occasionally reverts back to non-coordinated quoting with inside information \( (\rho > 0) \) to deal with extreme value exposure and eligible transaction rate drawings. Note that while switches between collusion and break-up are discrete, the cartel agreement itself is continuous in that actual deviation is off-equilibrium.

We say the cartel is more ‘steady’ if \( \rho \) is closer to 0, so that it breaks up less regularly. Equation (14) does not yield a closed-form solution for the effect of the discount factor \( \delta \) or manipulation cost \( C(\Delta c_{it}, \Delta v_{it}) \) on cartel steadiness \( \rho \) in general, which is probability distribution-specific. However, a negative relationship between \( \delta \) and \( \rho \) is to be expected, since the more patient the panel-banks are, the less tempted they are to deviate with a more extreme position.

Higher manipulation cost on the one hand make defection less attractive, so that higher extreme value positions can be sustained without the cartel having to break
up—alternatively, with very low manipulation costs the cartel breaks up constantly, to the point of being merely an exchange of information to play Nash rather than Bayesian-Nash. On the other hand do higher manipulation costs reduce cartel profits, making collusion less attractive. The combined effect on cartel steadiness ($\rho$) is ambiguous. Yet even if it breaks up less often, a benchmark cartel in operation will be manipulating the rates less extremely when manipulation costs are higher. Therefore, while likely reducing the extent of manipulation, the reforms to enlarge the class of eligible transactions—which increase the cost involved in manipulating the benchmarks—can increase the frequency of collusive quoting.

Decreasing the trimmed range $T$ by discarding more of the highest and lowest quotes, which is part of the Euribor reforms, also has opposing effects on incentives to collude. While fewer banks can influence the rate by deviating from the collusive agreement, each one has a larger individual effect on the published rate, as a smaller number of quotes are averaged, so that the overall effect on defection incentives is ambiguous.

If the cartel for some reason were to apply a finite $T$ punishment period, the effect of it is confined to a reduction of the critical cut-off value $\Psi(\cdot)$, increasing $\rho$, so that the episodic break-up strategy became less steady.

All elements combined, this feasible continuous benchmark rates cartel strategy with episodic break-up can now be fully specified as:

$$\max_{\Delta v_t, \Delta c_t} \sum_{i=1}^{N} \left[ (v_{0it} + \Delta v_{it}) \left( \frac{1}{n} \sum_{j \in T} c_{1jt} - L_{t-1} \right) - C(\Delta v_{it}, \Delta c_{it}) \right]$$

if $\max_{i=1,\ldots,N} \left( \pi^D_{it} - \pi^C_{it} \right) \leq \frac{\delta}{1-\delta} \left( (1-\rho) E[\pi^C] + \rho E[\pi^N] - E[\pi^{BN}] \right)$

and $\max_{\Delta v_{it}, \Delta c_{it}} \pi_{it} \forall i$ otherwise.

Note that while the cartel strategy of continuous collusion with episodic break-ups is not first-best—in the sense that total profits in Nash-quoting periods can be lower than under coordinated quoting—it does minimize total costs of collusion and uses no implicit side-payments in manipulation costs sharing. A stable cartel with episodic break-ups would also be sustainable under continuous adjustments, if it were feasible.

## 4 Collusive Rate Patterns

To illustrate how benchmark rate collusion could play out in practice and the type of empirical trail it may leave, we simulated a data generating process and determined the strategies of continuous collusion with episodic break-up using the cost function

$$C(\Delta c_{it}, \Delta v_{it}) = \alpha \Delta c_{it}^2 + \beta \Delta v_{it}^2,$$
where $\alpha$ and $\beta$ are the cost parameters for eligible transactions rigging and front running, respectively. The resulting linear-quadratic payoff function satisfies the conditions for a unique global maximum. Parameter values are: $N = 16$, $n = 8$, $\alpha = \beta = 1$, $v_{0it} \sim N(0, 0.1)$ and $c_{0it} \sim N(L_{t-1}, 0.1)$, with starting value $L_0 = 1.73$.

First, using Monte Carlo simulations the implicit probability of break-up was calculated for different discount rates. To assure mean convergence, we simulated 100,000 daily draws of baseline eligible transaction rates $c_{0it}$ and baseline exposures $v_{0it}$, derived payoffs in static Bayesian-Nash ($\pi_{iit}^{BN}$), collusion ($\pi_{iit}^{C}$), defection ($\pi_{iit}^{D}$) and static Nash ($\pi_{iit}^{N}$) in each draw, for each bank $i = 1, \ldots, N$, and determined the expected payoffs $E[\pi_{iit}^{BN}]$, $E[\pi_{iit}^{C}]$ and $E[\pi_{iit}^{N}]$. These identified the simulated distribution of the largest payoff differential $\max_i (\pi_{iit}^{D} - \pi_{iit}^{C})$ and the fixed point $\rho$ as a function of discount rate $\delta$. Second, with the elements obtained a 240-day time-series of the interbank rate was generated, looking separately at Bayesian-Nash, honest and optimal collusive behavior. The MATLAB source code of the cartel routine, including advised positions and submission targets, for $N = 4$ is given as an appendix.\textsuperscript{4}

### 4.1 Payoffs and Break-ups

Figure 5 gives the simulated payoff frequency distributions for independent Bayesian-Nash (grey) and collusive (black) quoting. Under independent quoting, payoffs are more closely concentrated around zero—the mean is slightly positive because of the manipulation benefits.\textsuperscript{74} The cartel materializes higher profits more often, but also losses: there are more instances in which cartel members take one for the team in the sense that they would have done better under independent quoting. Yet in collusion, both losses and profits are more concentrated on the right side of their spectra: losses are more often closer to zero and profits are more often large. As a result, the average expected payoff is almost forty times higher under collusion than under independent quoting. All panel-banks gain in expectation from participating in the collusion.

Figure 6 provides a frequency table for the $\max_i (\pi_{iit}^{D} - \pi_{iit}^{C})$ and identifies break-up probability $\rho$ as a function of $\delta$. In the left-hand panel, for $\delta = 0.90$ the critical cut-off value below which collusion is stable is $\Psi \approx 0.0028$. Together with the conditional expected collusion payoff this implies a break-up probability of around 0.38.\textsuperscript{75}

In the right-panel, the $\rho (\delta)$-curves are for three different levels of $\alpha$.\textsuperscript{76} Cartel steadiness increases in $\delta$ and the cost of manipulation: monotonic increases in the cost

\textsuperscript{73}Qualitatively similar results obtain for different values of $\alpha$, $\beta$ and the variances—in particular for $\alpha \gg \beta$ and the variance of $v_{0it}$ of a higher order than that of $c_{0it}$.

\textsuperscript{74} $E[\pi_{iit}^{BN}] \approx 0.000024$, $\sigma_{BN} \approx 0.000274$; $E[\pi_{iit}^{C}] \approx 0.000898$, $\sigma_{C} \approx 0.00679$; $E[\pi_{iit}^{N}] \approx 0.000218$, $\sigma_{N} \approx 0.00283$.

\textsuperscript{75} $E[\pi_{iit}^{C} | \text{no break-up}] \approx 0.00012$, $E[\pi_{iit}^{N} | \text{break-up}] \approx 0.000069$ and $\rho \approx 0.37901$.

\textsuperscript{76} A similar picture obtains if both $\alpha$ and $\beta$ vary.
of eligible transactions rigging decrease the probability of break-up for all discount factors. For this specification, that is, higher manipulation cost make defection more less attractive than the prospect of cartel profits, resulting in a more steady cartel.

4.2 Time-series

With the fixed point determined, we simulate time-series. Figure 7 displays a common interbank rate over time for $\delta = 0.90$, first when banks determine their submissions independently, respectively individually optimal and honest for 60 days each, and after that in continuous collusion with episodic break-up for 120 days. In the collusion period, the vertical shaded areas are episodes of non-cooperative quoting following an extreme value drawing.\footnote{This happened 49 out of the 120 days of collusion, which is in the neighborhood of the 38\% projected.}

While the rate pattern may seem somewhat different between the collusive and non-collusive periods, it is not evident from the simulated benchmark rates alone whether the banks quoted independently or collusively, nor which cartel periods were break-ups. Any drift in the mean is random hysteresis since the rate follows a random walk around 1 and the effects on volatility are not obvious.
Figure 6: Left-panel: Frequency table of $\max(D_{it} - C_{it})$ for $\delta = 0.90$. (Light-shaded area is $\rho = 0.38$ at $\Psi = 0.0028$.) Right-panel: Cartel break-up probability $\rho$ as a function of discount rate $\delta$ for different $\alpha$’s.

The intraday variance patterns are not statistically different between the regimes, either for the full panel or the banks that determine the rate.\footnote{On average, the intraday variance is 0.0103 for the full panel and 0.0022 for the trimmed range during the 60 honest days and 0.0099 for the full panel and 0.0018 for the trimmed range during the 120 manipulation days. These differences are not statistically significant. Also within the 120 manipulation days there is not significant difference between collusion days and temporary break-up days.} Figure 8 displays the intraday variance between the quotes on any given day of the 16 panel banks and the 8 banks in the trimmed range.

Colluding banks may be expected to ‘bunch’ together around one of the boundaries of the trimmed range, which would decrease the intraday variance of bank quotes. However, for the full panel this intraday variance decreasing effect is partially offset by a larger distance between the manipulating banks and the share of trimmed banks on the other extreme. Within the trimmed range, there is more bunching together around one of the pivotal quotes in the same direction than under independent quoting, so that a decreased intraday variance may be more likely found, but not in this illustration. Abrantes-Metz et al. (2012) conjecture on this expectation that reduced intraday variance is indicative of benchmark collusion, yet we do not find evidence for that in our model and simulations.\footnote{Low-balling could remain an explanation for the period of observed low intraday variance in Abrantes-Metz et al. (2012), for which they find no evidence. It should however also be noted that in the time window they analyse, the Summer of 2007, there was considerable market turmoil.}
The interday variance (or volatility) of the interbank rate over a certain time window does give distinct differences in some of the runs. Figure 9 shows the interday variance for two windows: 11 days and 5 days.

Clearly, the benchmark rate under collusion displays more extreme behavior than during independent quoting—while again it is not possible to tell apart optimal Bayesian-Nash from honest independent quoting. The average volatility under collusive quoting is about twice as high as under independent behavior. This difference is statistically significant with a $p$-value below 0.00001, for both windows, using a one-sided Wilcoxon ranked sum test.\footnote{The average interday variance is 0.0011 for the 11-day window and 0.0006 for the 5-day window during the 60 honest days and 0.0031 for the 11-day window and 0.0015 for the 5-day window during the 120 manipulation days.}

Also the average absolute change in the interbank rate is statistically significantly different between the break-up and full collusion regimes at the 1\% level.\footnote{On average, the absolute change in the interbank rate is 0.0224 during the 60 honest days and 0.0318 during the 120 manipulation days.} Moreover, within the collusion period it is statistically significantly higher in no break-up than during break-up.\footnote{The absolute change in the interbank rate is 0.0357 during collusion and 0.0253 during break-up.} Using a one-sided Wilcoxon rank sum test, the null that the mean of the volatility is the same during no break-up and break-up within the collusion
period is rejected with a $p$-value of 0.0049. These results are robust against changes in the length of the rolling window.

In line with our theory, the benchmark cartel benefits from more volatility in the rates over time, as that allows the panel bank members to better exploit their inside information about the rates movements in advance by adjusting their portfolio exposures, against non-initiated financial institutions and investors. During break-ups, these benefits are much smaller, as cartel members no longer take into account the externality effects of their behavior. It can also cause them to pursue conflicting directional changes, reducing volatility. Nevertheless, only part of the simulations return an identifiably different volatility pattern.

### 4.3 Screening

Our finding that the benchmark rate can fluctuate more during periods of collusion (no break-up or break-up) than independent quoting (Bayesian-Nash or honest) suggests a type of empirical screen that can help target deeper investigations. With the actual periods of collusive quoting unknown, Bai-Perron structural break tests can be used to identify cartel episodes more systematically.\(^8^3\) On the 5-day volatility of this section, the Bai-Perron test identifies one and only one break occurring at day

\(^8^3\)Bai and Perron (1998, 2003) provide a collection of tests that allow for identifying structural changes, break dates and magnitudes of change in time-series when both the number and the dates of the breaks are unknown. For an application to identifying the begin and end dates of cartel effects, see Boswijk et al. (2017).
Figure 9: Volatility in quotes, 11 days and 5 days window, including 5-day Bai-Perron structural break test results.

117, which is close to the actual break day 120. The fitted values are drawn in Figure 9. Similar results are found using the 11-day volatility.

For accurate application in practice, such collusion screens would need to be further calibrated and controlled for other drivers of volatility in benchmark rates, in order to avoid them falsely flagging as suspicious increased volatility between different days that is due to legitimate market events. However, banks’ quotes are difficult to rationalize with other measures of bank borrowing costs, as Snider and Youle (2010) show, even when including banks’ own quotes in other currency panels. Kuo et al. (2012) list several reasons why comparable measures of bank borrowing cost would follow quite different paths than Libor. The power of volatility screens to discriminate will be highly specific to the circumstances in which they are applied.

We note that the increased volatility under collusion in Figure 9 results in large part from the true borrowing costs following the published, manipulated Libor, i.e. that $E[c_{0i}] = L_t-1$. Upward (downwards) manipulation is followed by a higher (lower) baseline value draw in the next period, which combines with the collusive variance. If we use a more stable mean for the daily rate drawing instead, volatility alone remains a sufficient statistic to tell apart independent from coordinated quoting only rarely. If manipulation was indeed widespread and commonly known, possibly the initiated financial institutions accounted for an actual borrowing standard that would have followed from honest reporting only—as a ‘shadow Libor’. Yet even if there were purer determinants for the true cost of borrowing of the panel banks, it seems reasonable to expect those to have at least been somewhat contaminated by the Libor
manipulations.

Combination tests may perform better. Periods of collusion would in particular leave traces in transactions over time, as the banks involved change their exposure position in the same direction in which the rate is rigged—and also all of comparable size. A high correlation between a bank’s transactions in the front-running window and the subsequent change in the rate could be an indication of suspicious exposure alignment. A screen would look for a spike in these combined correlations in time or compared to non-panel banks.

Under the assumptions in our model, in collusion banks adapt their portfolio exposure position perfectly in the same direction as the future rate, so that the correlation is high—compared to around zero for independent trading, which also would typically be lower in volume due to the higher uncertainty that non-initiated traders face. Alternatively, a correlation screen based on binary variables would flag as suspicious frequent increases (decreases) in the rate accompanied by positive (negative) position changes. The advantage of such a screen is that larger banks with more extensive trading books or lower front running costs would not largely drive the correlations. Applying this screen on data simulated with our model would yield a correlation of \( 1 \) under collusion—since banks always front run in the right direction—and around 0.5 under individual behavior—where banks engage in minor front running but have no information on the future rate. In practice, however, the difference will be weaker with more heterogeneity among banks. For example will the exact moment of information exchange \((C_t)\)—that is, the opening of the front-running window—not be known outside the cartel. Also can other transactions that classify as eligible take place simultaneously for non-collusive reasons. The panel banks may not involve all of their trading activities world-wide—facing internal coordination issues or possibly lacking a complete picture themselves. While correlations will be different in magnitude as a result, in general they can be markedly higher even with a somewhat shifted window, transaction set, or general noise in the transaction data.

To complement, robustness checks for different length front-running windows up to the submission time \((S_t)\) can be used.

However, while data on the interbank rates and individual submissions of panel banks is readily available for anyone interested in screening for benchmark manipulation, transaction data are not. They simply are not (yet) collected systematically for that purpose. Eligible transactions, although limited to interbank loan data, could to a certain extent be retrieved from the TARGET2 real-time gross settlements system, using a method such as the Furfine (1999) algorithm, for transactions within Europe. A similar data set, the Fedwire Funds Service, is the large-value bank payments system operated by the Federal Reserve banks in the United States. Kuo et al. (2013) develop a methodology to infer information about individual term dollar interbank loans settled through this system. However, the real challenge lies in identifying banks’ overall exposure positions to the rate, as these are largely driven by OTC derivatives transactions, which take place without an exchange. Data on those
transactions is not currently available.

Recent initiatives to construct Trade Repositories (TRs) aim at maintaining electronic records of all transactions data, including OTC derivatives transactions in which one of the counterparties is of the same nationality as the repository. If sufficiently developed in the future across different countries, these repositories could provide authorities with the necessary transactions data on a sufficiently detailed level to be useful in screening for collusive benchmark rates fixing.

Screening for increased intraday variance patterns may deter manipulation if panel banks are aware of this. It would be hard to gain from collusion and not raise suspicion, because the volatility the cartel generates with inside information foresight is the same as the source of the cartel profits. While it is possible to dodge the screens we propose, doing so marginalizes cartel profits, thereby affecting stability. Interday variance can be lowered artificially by having more panel banks engage in eligible transactions rigging, including trimmed members, which however raises the cost of collusion.

5 Damages

Collusion in the fixing of the benchmark rates can have substantial consequences, as the Libor, Euribor and forex rates are the fundament of financial and real markets around the world. As a result of the manipulation scandals, the benchmarks likely lost part of their trustworthiness as foundations of value and signals of underlying risks. The adverse effect on overall trust in the financial system will likely have had consequences for financial market stability.\textsuperscript{84} Arguably, as long as the panel banks would have succeeded to create the illusion of creditworthiness and thereby masked the severity of the financial crisis at the time, this may have kept the crisis at bay.\textsuperscript{85} Upon coming to light, however, the machinations may have deepened the financial crisis instead. Moreover, part of the benchmark manipulations preceded the crisis, and was therefore not with first intent to appear more credit-worthy or prevent systemic crises.

To the extent that movements in the pricing of futures, options and derivatives are zero-sum games, where there is a winning contract for every correlative losing one, the overall effect on welfare of a benchmark rates cartel would be limited to primarily rent shifting, with little or no deadweight loss. Yet even with off-setting winners, each downside on a deal still potentially has an antitrust damage claim for being overcharged. The cartel gains would be at the expense of uninformed counterparties who lent or borrowed at distorted rates tied to the Libor or other interbank benchmarks.


\textsuperscript{85}See Vaughan and Finch (2017), page 97.
such counterparties may include non-panel banks and traders such as insurance companies, municipalities, corporations and investors. Using inside information to front run on self-created variation, the cartel banks could shift their exposure position in their own favor, at the expense of these other market participants.\textsuperscript{86} In part, this can have taken place through hedging strategies that are hard to reconstruct.

The loss of a reliable benchmark could have induced different borrowing behavior and could thereby have affected countless underlying markets. By affecting market values, both of financial products and in underlying markets, the manipulations likely also impacted the efficient allocation of resources.\textsuperscript{87} Too low a rate could have led lenders to withdraw funding, for example from mortgage markets or for small business, affecting real family well-being and entrepreneurial activities that could have brought important benefits. Similarly, in the case of foreign exchange rates collusion, international trade and foreign direct investments are likely to have been harmed, as the forex cartel aimed at making transactions in foreign currencies more expensive through spreads, as well as postponing, or sometimes not executing, client orders that did not fit their own collusive profit objectives. These distortions could potentially have had rippling real effects on businesses and consumers worldwide.

Finally, we note that the schemes in our model would also introduce plain costs of manipulation, transaction costs and foregone profits, for example, in the case of eligible transactions rigging, from trades at different rates than the going ones—or at different times than the commissioned ones. In some scenario’s, part of the cartel strategy could be to inflict high costs of collusion on designated cartel members, in order to keep internal stability. Yet, trades in eligibles would typically be small relatively to the total portfolio exposure. Some cartel agreements were found to be aimed at reducing transaction risks, maintaining narrower spreads for trades in order to lower the members’ own transaction costs and maintaining liquidity between them.\textsuperscript{88} The assessment of damages due to the benchmark rates cartels will be highly case-specific and often complex.

\section{Concluding Remarks}

Despite evidence of wider coordination in the benchmark rate fixings, the Libor and Euribor scandals have been passed off mostly as incidents of bilateral manipulation by a few rogue traders for their own benefit, quite possibly against the interests of their employer banks. In this paper we develop novel cartel theory to show how a full stable for-profit benchmark rates cartel could work, despite the design of the rates and the panel banks’ interests typically not being aligned—and without a need for side-payments. We reveal two mechanisms, observed in regulators’ investigations,\textsuperscript{86}\textsuperscript{87}\textsuperscript{88}

\begin{thebibliography}{9}
\bibitem{86} See Foster (2014).
\bibitem{87} See Abrantes-Metz et al (2012).
\bibitem{88} European Commission. “Antitrust: Commission settles cartel on bid-ask spreads charged on Swiss Franc interest rate derivatives; fines four major banks € 32.3 million,” 21 October 2014.
\end{thebibliography}
that can facilitate collusion: front running and eligible transactions rigging. By creating inside information, the panel banks are in a position to front run and create a more beneficial exposure position to the upcoming rate, thereby reducing conflicting interests in their trading books. Some cartel banks would also have to engage in eligible transactions rigging, placing transactions at rates required to allow the cartel to justify the collusively optimal quotes.

Collusion is costly, nevertheless worthwhile. Occasionally, participating banks may be required to incur manipulation costs exceeding the period cartel gains. Even though these can be substantial, a panel bank’s average expected collusion payoff is substantially higher than under independent quoting. Panel banks would often bear costs on the smaller book of eligible transactions for the occasional large gain on the exposure when the rate is manipulated particularly favorably for them. This picture seems broadly consistent with evidence on the money markets involved, in which panel members are in multi-market contact over a variety of financial products linked to different maturities and currencies, typically meeting short-term inside liquidity demands also sometimes at a small loss, in order to maintain longer-term banking relationships and benefit from large outside business gains. A cartel can reduce total manipulation costs by doing internal eligible transactions between the members, as well as align portfolios partly by swapping positions—although the latter does not increase overall cartel profits.

Consistent with the evidence found, our benchmark rates cartel is characterized by episodic recourse to independent quoting. We explain these temporary break-ups as part of an ongoing collusive strategy, to which the cartel reverts in response to occasional extreme exposure values that give incentive to deviate. These reflect that payoffs in financial markets can be volatile and cartel deviation is nonstandard. We describe in detail how the cartels can be administered. Collusion leaves no obvious traces in the benchmark patterns over time, nor in intraday variance in the quotes. It does markedly increase the volatility in quotes between trading days. On this basis, we propose volatility screens, possibly supplemented with transactions data to collect, to monitor submissions for periods of collusive manipulation.

The mechanisms revealed in our model apply with minor modifications to the collusive rigging of foreign exchange (forex) rates, which similarly relies on exchanging inside information, aligning exposure positions and planning eligible transactions. Even though the number of traders is larger and can vary, and their individual impact on the forex rates is asymmetric, banging the close is essentially eligible transactions rigging, as those trades in the window are eligible for calculation of the rate and a cartel is able to exercise more influence on the rate jointly than any individual bank. Different is that forex manipulations with actual transactions are typically not suboptimal, but rather the source of cartel profits—although there are rigging costs in the form of missed commissions in case of withholding trades, potential transactions being pushed through at a sub-optimal moment and risks. Exchanging information on large client orders to be executed in the future and on manipulation
strategies towards them, banks in the forex cartel were able to front run as they had inside information on the direction in which the rate would move in the future. Other possible applications include insider trading in benchmarks and price reference points in gold, energy and commodities markets—some of which have been subject to allegations of misconduct.

Several of the assumptions we make warrant some further discussion. We model portfolio positions as independently distributed around zero, so that there is no accumulation and expectations on future positions are unrelated to current positions. Even though trade in OTC derivatives is fast-changing and vast in comparison, banks may have a relatively stable exposure profile of the same sign, such as long-term mortgage contracts with Libor-based rates. Position fluctuations can be modelled also around a steady bank-specific exposure profile, positive or negative, as long as their variance is large enough—although this will introduce a drift in the rate manipulation in the direction of the sign of the panel’s overall mean. Our symmetric model can then be seen as an approximation on the larger part of the portfolio, or alternatively as being about a desks or traders cartel maximizing joint profits on their liquid trading books only, instead of their employer banks’ overall exposures. In that case the panel banks as corporation would possibly be damaged on their remaining smaller portfolio parts.

We assume that panel banks’ borrowing capacities fluctuate around the same mean and that shocks to baseline borrowing costs or exposure positions are not persistent. In practice, some banks may be able to borrow at lower rates than others, due to reputation or scale for example. For the cartel, this introduces serious problems in getting interests aligned, as some banks would on average be more likely to gain than others. Persistent shocks to true borrowing costs or trade book building would for example lead to Bayesian updating upon break-up, as banks would be able to use information from previous periods. These introduce complex dynamic optimization, as do expectations about future demand, correlation of demand shocks and other features of business cycles in, among others, Haltiwanger and Harrington (1991), Kandori (1991) and Bagwell and Staiger (1997). In our model the effect of such extensions on cartel stability or cartel formation incentives is not obvious. These dynamics are likely to introduce the necessity of side-payments or rotation on which banks are supposed to pay the eligible transactions rigging costs, for which there is some evidence. It may also lead to longer break-up periods, as it may take time to get trading books incentives aligned again and find a stable cartel strategy. While the literature thus has extended Rotemberg and Saloner (1986), dynamic collusive benchmark rates fixing is left for future research.

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89 In Commodity Futures Trading Commission, “Order Instituting Proceedings: Deutsche Bank AG,” 23 April 2015, on page 27 it is reported that: “The UBS Senior Yen Trader also offered to enter into trades at rates detrimental to him but beneficial to the Senior Yen Trader-Submitter to ensure the Senior Yen Trader-Submitter’s involvement in his plans and to entice him to make Deutsche Bank’s Yen LIBOR submissions in the manner he desired.”
We model manipulation costs equal across all the panel banks, which is reasonable to assume for the main cost components, in particular raising suspicion and suboptimal transactions in eligible transactions rigging. In front running, however, certain panel banks may face lower costs than others, depending on their core activities and size. Our proof of existence of a one-shot collusively optimal set of submissions relies on the fact that with equal manipulation costs, banks with the highest baseline true cost parameter submit the highest quote and that the cost parameters are equal across banks. With heterogeneous costs, this order may be broken and a certain set of collusive submissions in theory may be achieved in different ways: either by choosing the minimum amount of eligible transactions rigging or by letting banks with lower manipulation costs engage in more eligible transactions rigging. Given banks’ heterogeneous cost functions, the probability of a collusive outcome not being unique—which would require the exact occurrence of certain draws—is zero, so that a unique global cartel optimum remains, provided all individual cost functions satisfy the existence conditions.

Instead of a full panel cartel, one or various smaller combinations of banks might have colluded unbeknownst to the others. However, establishing the common interests needed to form sub-coalitions requires panel wide communication, and no panel bank knowing that some collusion was going-on would tolerate manipulation in directions adverse to its own interest. Partial arrangements involving all panel banks could be scenario’s in which a sub-set of the panel carries on colluding for the break-up period, not including those for whom the incentive constraints do not hold. Such a partial cartel as part of the continuous collusion strategy can allow all panel banks to benefit from front running and can increase expected cartel profits. An individual bank may also want to skip a period of cartel participation, for example when it drew a position close to neutral. Allowing for sub-coalitions can make full collusion less stable, however, as banks could temporarily freeride on a partial cartel. The benefits of not participating a period, such as saving manipulation costs, may be larger than the effect of not having the bank’s interests internalized by the collusive coalition. At the same time does the trimming mechanism dictate a minimum size cartel coalition, as contributions to and benefits of the manipulated rate are high for the first couple of banks that get inside the trimmed range. Although partial benchmark rate cartel theory remains to formally be developed, cartel clusters in partial collusion could be identified by a high correlation between their members’ exposure position trades and rates.

Information sharing in the cartel is modeled to be perfect, which reveals to all members everyone’s options to deviate. Alternatively, the colluding banks could employ an independent administrator who collects the information, runs the cartel software and then only provides personal instructions to each bank. If the panel banks would manage to all commit to such a central ringleader, it would significantly increase cartel stability by largely taking away the incentives to deviate.

The benchmarks remain vulnerable to the cartel mechanisms we suggest, also af-
ter the implementation of recent and proposed reforms. Moreover, further reforms to make the rates setting processes more resilient to collusion are not obvious. As shown, an extension of the class of transactions that are eligible to support rate submissions would increase the cost of manipulation, but at the same time reduce defection gains, leading to less break-ups and so potentially stabilize the cartel. On the other hand, higher manipulation costs may reduce the total extent of manipulation, thereby leading to a Libor that is closer to its true value. Also the embargo on individual banks’ Libor submissions does not affect a full cartel’s ability to infer adherence to the collusive agreement from the rate itself—although it would be impossible to detect deviation in partial cartels that consist of fewer members than the trimmed range plus the number of banks trimmed on one side. As done in Coulter and Shapiro (forthcoming) and Duffie and Dworczak (2014) for individual manipulation incentives, a mechanism design approach could also make benchmark rates fixing more collusion-proof.

References


A MATLAB® Cartel Routine

The following MATLAB® script calculates the optimal cartel strategies. Each bank inputs its daily baseline values, with which the software derives the optimal collusion and deviation strategies and their associated payoffs. The routine also determines whether all of the $N$ cartel stability conditions hold, and dictates break-up as a strategy to all cartel members when one or more do not. The script provides all banks with the exact front running and eligible transactions rigging strategies. The kernel is provided below—for the condensed case of $N = 4$.

```matlab
% Parametric assumptions
N = 4; % Number of panel banks
n = 2; % Share of banks within trimmed range
a = 1; % ETR cost parameter
b = 1; % FR cost parameter
sc = 0.1; % Standard deviation transaction rates
sv = 0.1; % Standard deviation exposure
delta = 0.9; % Discount rate

% Derive critical cut-off level Psi
psi = fpsi[N,n,a,b,sv,sc,delta];

%% Step 1: Prompt input baseline values
dlg_title = 'Enter baseline exposures'; num_lines = 1;
prompt = {'Bank 1','Bank 2','Bank 3','Bank 4'};
defaultans = {''='',''='',''='',''};
V0 = str2double(inputdlg(prompt,dlg_title,num_lines,defaultans,'on'))';

dlg_title = 'Enter baseline transaction rates'; num_lines = 1;
prompt = {'Previous interbank rate','Bank 1','Bank 2','Bank 3','Bank 4'};
defaultans = {'','','','',''};
C0 = str2double(inputdlg(prompt,dlg_title,num_lines,defaultans,'on'))';

%% Step 2: Calculate collusion and deviation payoffs
% Collusion payoffs
fJointProfit = @(DC)-((sum(V0)+sum(DC(2,:)))*(trimmean(C0+DC(1,:),...
    n/N*100)-L0)-a*(sum(DC(1,:).^2))-b*(sum(DC(2,:).^2)));
CStrategy = fminunc(fJointProfit,zeros(2,N),options);
PCol = fpayoff(V0,C0,N,n,L0,a,b,CStrategy);

% Deviation payoffs
PDev = zeros(N,1);
for i = 1:N
```
Cj = CStrategy; DCj(:,i)=[]; C0j = C0; C0j(:,i)=[];

\[ f_{\text{OwnProfit}} = @(DD) - \left( (V0(1,i) + DD(2,i)) \times (\text{trimmean(horzcat(C0(1,i) +... DD(1,i),C0j+DCj(2,:)),n/N*100) - L0) - a*(DD(1,i)^2) - b*(DD(2,i)^2)) \right); \]

DStrategy = fminunc(fOwnProfit,zeros(2,N),options);
PDev(i,i) = fpayoffc(V0,C0,N,n,L0,a,b,CStrategy,DStrategy,i);
end

%%% Step 3: Check whether constraints hold and produce output
if PDev - PCol <= psi

msgbox(sprintf(['Break-up: No.',...
'\n\nAdvised positions:',...
'\nBank 1: Adjustment = %.4f, New position = %.4f',...
'\nBank 2: Adjustment = %.4f, New position = %.4f',...
'\nBank 3: Adjustment = %.4f, New position = %.4f',...
'\nBank 4: Adjustment = %.4f, New position = %.4f',...
'\n\nAdvised submission targets:',...
'\nBank 1: Adjustment = %.4f, New submission = %.4f',...
'\nBank 2: Adjustment = %.4f, New submission = %.4f',...
'\nBank 3: Adjustment = %.4f, New submission = %.4f',...
'\nBank 4: Adjustment = %.4f, New submission = %.4f'],...
CStrategy(1,1), V0(1)+CStrategy(1,1), ...
CStrategy(1,2), V0(2)+CStrategy(1,2), ...
CStrategy(1,3), V0(3)+CStrategy(1,3), ...
CStrategy(1,4), V0(4)+CStrategy(1,4), ...
CStrategy(2,1), C0(1)+CStrategy(2,1), ...
CStrategy(2,2), C0(2)+CStrategy(2,2), ...
CStrategy(2,3), C0(3)+CStrategy(2,3), ...
CStrategy(2,4), C0(4)+CStrategy(2,4)))
else
    msgbox('Break-up: Yes.')
end

Given the parameters of the rate setting process, the cut-off value \( \Psi \) is found in routine \texttt{fpsi} that simulates a sufficient amount of daily payoff values in case of Bayesian-Nash, Collusion, Defection and Nash (100,000 times in the results reported here), such that the fixed point \( \rho \) can be identified with sufficient precision. The Bayesian-Nash strategies are found by calculating the expected baseline values of the other \( N-1 \) banks and for each bank separately using the \texttt{fminunc} function in MATLAB\textsuperscript{\textregistered} under the assumption that the calculated expected baseline values hold for the other banks. Nash strategies following break-up are found by each bank consecutively maximizing its own payoff function, repeated for a sufficient number of
rounds. Banks respond to each other for up to 24 rounds, after which either a non-cooperative equilibrium in pure strategies is reached, or none is concluded to exist, in which case the outcome of round 24 is taken as the mixed-strategy equilibrium drawing.

Step 1, at 0_t in the morning, all cartel members report their baseline drawings, exposures v_{0i} and eligible transactions rate c_{0i}, which are entered as inputs in the prompt as shown in Figure 9.

![Figure 10: Baseline exposure and eligible transaction rate prompts (for N = 4).](image)

In Step 2, the script subsequently derives the optimal cartel strategies, using the \texttt{fminunc} function. Taking V_0 as the 1 \times N vector of baseline exposures and C_0 as the 1 \times N vector of baseline eligible transaction rates, the code minimizes the objective function ObjFunc along the 2 \times N choice matrix DC, which represents the front running choice variables (first row) and eligible transactions rigging choice variables (second row). Note that ObjFunc is specified as the negative of the sum of the individual payoff functions, which is subsequently minimized. Output CStrategies are the optimal cartel strategies. Plugging these into the individual payoff functions provides each bank’s cartel profits. This is done in the routine \texttt{fpayoff}. Similarly, the defection payoffs are found by maximizing own payoffs given that other banks adhere to cartel strategy.

Note that all banks optimally adjust their positions by the same amount, independent of their initial portfolio position, because manipulation costs are quadratic and profits linear, the same for all banks.

Finally, in Step 3 it is checked whether the difference between the defection payoff and collusion payoff of each bank is below the critical cut-off value \Psi. The cartel
Figure 11: Exchange of information and cartel instructions (for $N = 4$).

Instructions of all members are given to each, together with all shared information, as illustrated in Figure 10. If none of the banks has a payoff differential above $\Psi$, a collusive quote is scripted (‘Break-up: No.’), including which strategies each bank should implement. If at least one bank has a payoff differential above $\Psi$, all banks receive the notification that collusive optimization is not stable (‘Break-up: Yes.’), instructing them to revert to one-period static Nash.