Access Price Regulation and Price Discrimination in Intermediate Goods Markets

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Abstract

We consider a model of a monopolistic network operator who sequentially offers two-part access charges to symmetric downstream firms. We are particularly interested in analyzing an alternative to current regulatory practice of prescribing access. In particular, we look at the possibility of restraining the input monopolist’s market power by endowing downstream firms with a regulatory option: In case they disagree with the contracts proposed to them, downstream firms can claim a regulated access price. It turns out that this form of regulation may prevent foreclosure even though allowing for price discrimination in the intermediary market. It proves itself more beneficial to welfare than the current practice of prescribing access prices above marginal cost. Interestingly, even though one expects discrimination against the first mover, non-discriminatory input prices below cost can occur when the monopolist faces the alternative of a rather strictly cost-oriented regulated access price. Non-discrimination rules will either not become effective or result in less optimal price levels.

JEL Classification: D43, L13, L14, L42

Keywords: price discrimination, vertical contracting, exclusion, regulatory outside option

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1 Introduction

In telecommunications policy so-called service-based competition is regarded as the first step towards promoting competition. Coordinating different modes of market entry has been found necessary from this point of view. In recent times it gained importance, particularly, because for services like voice telephony an increasing number of service providers without their own network have come into existence. These require access to the underlying network in order to go into operation. In Germany as well as in other countries, a large part of this network is still owned by the formerly state-owned incumbent and monopolist. Therefore, requesting network access represents a typical bottleneck situation which is named one-way access problem in the telecommunications literature. Throughout Europe, but also in the U.S., this situation has been recognised and dealt with by obliging the network owner to grant access at a prescribed price. We question whether this measurement leads to a socially desired outcome compared to a situation which allows for a certain degree of price discrimination. Our particular idea is to examine the impact of levelling the network owner’s ability to price discriminate through potential regulation. This regulatory intervention would concede a higher flexibility in access pricing to the network provider. Nonetheless, it would not completely abandon regulatory influence. It, therefore, represents an alternative to present regulatory practice of prescribing access prices in telecommunications. In line with this, our interest lies in two-part tariffs as these as well as other non-linear pricing schemes are widely employed for network access charges.

Formally, we consider an intermediary market with a single upstream supplier and two downstream firms. Contracts specifying two-part input prices are agreed on sequentially. We presume observable contracts so as to induce identical results to a game of simultaneous agreements with unobservable contracts. Our aim is to analyze static implications for prices and output under price discrimination and under non-discriminatory rules given the suggested regulatory framework. This setup is close to McAfee and Schwartz (1984) and Marx and Shaffer (2004a, 2004b). Yet, McAfee and Schwartz (1984) and Marx and Shaffer (2004a, 2004b) solve their particular game assuming unobservability of contracts in order to investigate the effectiveness of non-discrimination obligations under different beliefs. According to them, the above described setup would induce a discriminatory contract favouring the later entrant and foreclosing the precedent firm from the market if the supplier was unconstrained provided perfect information. Qualitatively, we obtain the same strategic pricing behaviour as them: The second mover is offered more favourable terms when wholesale agreements

\[\text{1 An example of service providers would be providers of voice call services, generally known as call-by-call providers. Note that such service is realised by different means, e.g. there is traditional fixed line voice-service as well as Voice-over-IP. Yet, from a consumer’s perspective, this difference is oftentimes not perceived.}\]

\[\text{2 Strictly speaking, network operators are only asked to make reference cost-oriented offers which are subject to approval by the respective National Regulatory Authorities.}\]

\[\text{3 For Europe, see European Commission (2006)}\]
are reached successively. However, our final results differ from these observations as the supplier is not unconstrained albeit the ability to make take-it-or-leave-it offers: As in Katz (1987) the supplier’s market activities are restricted because downstream firms may choose an alternative outside option. While in Katz (1987) this option takes the form of vertical integration, in our model, downstream firms can always reject the proposed tariff and choose a mandated linear input price imposed through regulation. In light of this regulatory alternative, our model yields more particular results which shift the focus of the access price discussion to the role of cost and level playing field of the input supplier: Surprisingly, there is a chance that non-discriminatory prices in form of fixed charges only characterise the equilibrium outcome. It appears in case of a restricted ability to price discriminate in the upstream industry, strictly speaking, a regulatory outside option close to marginal cost combined with low marginal production cost. Moreover, foreclosure is more unlikely to arise with the regulatory constraint than without regulatory intervention. Yet, results appear to be ambiguous when conceding higher degrees of price discrimination and cost levels. In such a situation, either an almost monopolised level of downstream competition or the facilitation of exclusion might occur. As it comes to non-discrimination, McAfee and Schwartz (1984) find that a non-discrimination rule will not become effective when contracts are specified in two-part tariffs. This is confirmed by Marx and Shaffer (2004b) but for the case where the primary downstream firm is excluded from the market. They point out that non-discrimination claims are likely to come into effect then. In terms of welfare, opinions on the effect of price discrimination differ: While e.g. Katz (1987) and Yoshida (2000) demonstrate that price discrimination lowers welfare, O’Brien and Shaffer (1994) find that input price discrimination always raises welfare.4 Our findings concerning the effectiveness of non-discrimination rules comply with McAfee and Schwartz’s (1984) even with varying notions of non-discrimination: We claim that they do not become effective. Yet, price discrimination turns out to be detrimental to welfare as in Katz (1987) and Yoshida (2000). Nevertheless and most importantly, a flat ban on price discrimination as much as abandoning regulation completely, implies lower welfare than a situation with price discrimination as induced in our framework.

The paper proceeds as follows: Section 2 introduces the basic model. Section 3 studies the equilibrium outcome under constrained price discrimination. Section 4 analyses the impact of non-discrimination clauses. Section 5 briefly reflects on the assumptions made and Section

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4 Katz (1987) argues that price discrimination lowers welfare because it involves the distortion of at least one downstream firm being assigned a higher price. Yoshida (2000) comes to the same conclusion concerning welfare. Yet, his reasoning is that price discrimination leads to substantial losses for both downstream firms, but is beneficial to consumers and the discriminating upstream monopolist. In contrast, O’Brien and Shaffer (1994) find that input price discrimination always raises welfare as it results in lower retail prices implying significantly higher consumer surplus, even under different notions of non-discrimination.
2 The Model

We consider an intermediary market with a single upstream supplier \( N \) and two symmetric downstream firms \( S_i \) with \( i = 1, 2 \). The supplier provides an input which firms in the intermediary industry use to produce a homogeneous final good. Both \( S_1 \) and \( S_2 \) have identical production functions with constant returns to scale so that one unit of input transforms into one unit of output.

The supplier produces at constant marginal cost \( c \), restricting our attention to \( c \in (0;1) \) to ensure possible activity of the supplier and subsequent firms in the market.\(^5\) Further, we normalise the downstream firms’ production costs to zero. Therefore, their actual costs of producing the final good solely encompass their payments to the supplier. The two downstream firms sell their output in the same market and compete in quantities. These quantities are denoted \( x_1 \) for firm \( S_1 \) and \( x_2 \) for firm \( S_2 \). In aggregate, output adds up to \( X = x_1 + x_2 \) in the final goods market. The market price is characterised by the linear inverse demand \( P(X) = 1 - X \).\(^6\) The monopolist supplier and the downstream firms have to settle on contracts fixing intermediary prices. These take the form of individual two-part tariffs comprising a fixed fee \( F_i \) and a per-unit price \( a_i \) required to have a non-negative value.\(^7\)

The monopolist can make take-it-or-leave-it offers. Yet, the supplier’s contracting power is limited by a regulatory constraint: In case of disagreement between the supplier and downstream firms, the respective downstream firm can claim to acquire input at a prescribed per-unit-access price \( c^r = c + \Delta \) with \( 0 \leq \Delta \leq 1 - c \), so that \( c^r \geq c \). Requesting this regulatory alternative will not entail any additional cost for the downstream industry. Contracting takes place in sequential order and is observable. Service Provider \( S_1 \) is the first to settle its contract terms with the supplier.

In sum, we consider a three-stage game. In the first stage, supplier \( N \) offers firm \( S_1 \) a contract \( (a_1, F_1) \) for the purchase of the input factor. It is a take-it-or-leave-it option which \( S_1 \) can accept or reject. If \( S_1 \) rejects it can revert to the regulated contract \( (c^r, 0) \). In the second stage, after an agreement with the first downstream firm has been reached, the supplier likewise makes an offer \( (a_2, F_2) \) to firm \( S_2 \). In the third stage, downstream firms compete in the product market: Firms which have left the market earn zero. Firms which have stayed compete over the amount of service each of them can deliver to the final consumer and order

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\(^5\) One could allow for fixed costs of production, but as long as they are sunk they will have no effect on the upstream supplier’s behaviour. We assume that they are not that high as to tie up the upstream supplier’s market activity.

\(^6\) This demand can be derived from a variant of Bowley’s (1924) utility function considering two undifferentiated final goods: \( U(x_1, x_2) = (x_1 + x_2) - \frac{1}{2} (x_1 + x_2)^2 + 2x_1x_2 + (x_2)^2 \). Actually, our results should hold for all decreasing inverse demand functions with \( P' < 0 \) where \( P(X) \) is twice continuously differentiable considering functional properties, see Vives, X. (1999).

\(^7\) I.e. subsidies are not permitted as they induce inefficiencies.
inputs accordingly.

3 Equilibrium under Price Discrimination

We analyse the outcome of the previously described framework where the monopolistic supplier can price discriminate but his scope of action is limited due to potential price regulation. Without constraints the monopolistic supplier will have an incentive to foreclose one of the downstream firms from the market in order to maintain a vertical monopoly and its entailing profits.\(^8\) The paper is therefore related to the literature on foreclosure and we refer to Rey and Tirole (2005) as a recent survey in this field. In our framework, exclusion but also other equilibria are possible. The actual outcome varies with changing price regulation. In the following, we describe the characteristic features of the solutions distinguishing between non-exclusionary and exclusionary outcomes and then, order results by the prescribed access price level. The different equilibria are specified by solving the game backwards.

3.1 Strategic Interaction without Exclusion

**Downstream Competition:** In the non-exclusionary case, both downstream firms will be active in the market. Thus, there is a duopoly in the last stage of the game. As they compete in Cournot each downstream firm will choose its output so as to maximise its profits taking the contract with supplier \(N\) as given. Denoting profits from retail sales as \(\pi_i = (P - a_i) x_i\), downstream profits can be written as

\[
\Pi_i(x_i, x_j) = \pi_i - F_i,
\]

with \(i \neq j\), that is profits from retail sales minus the payable fixed fee. Firm \(S_i\) then optimally chooses quantity

\[
x_i^*(a_1, a_2) = \arg \max_{x_i} \Pi_i(x_i, x_j^*) \quad s.t. \quad x_i^* > 0.
\]

The equilibrium output quantities must satisfy the mutual best response property and constitute a Cournot Nash equilibrium. Therefore, reaction functions of the downstream firms can be characterised by the first-order condition

\[
\frac{\partial \Pi_i(x_i, x_j^*)}{\partial x_i} = \frac{\partial P}{\partial x_i} x_i + P - a_i^* = 0.
\]

This equation displays the standard optimality condition. It says that marginal revenue has to be equal to actual input cost and implies that both the optimal quantity chosen and individual downstream profit is strictly decreasing in the input price \(a_i\), but increasing with the rival’s price \(a_j\). One can see that \(P > a_i^*\) in equilibrium.

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\(^8\) There is no problem of double marginalisation due to intermediary prices in two-part tariffs.
**Contract with second downstream firm:** The supplier proposes a contract \((a_2, F_2)\) to downstream firm \(S_2\). Monopolist \(N\) will offer contract terms as to maximise its profits. Here, the fixed charge \(F_i\) serves to divide \(S_i\’s\) retail profits between the supplier and \(S_i\). Altogether, upstream profits consist of those from access provision and the additional retail profits which are extracted from the two downstream firms with help of the fixed fee.\(^9\) Hence, the profit function writes

\[
\Pi_N(a_1, F_1, a_2, F_2) = (a_1 - c)x_1^* + (a_2 - c)x_2^* + F_1 + F_2
\]

taking the agreement with the prior downstream firm as given in this stage of the game. An optimal per-unit price \(a_2^*(a_1, c')\) will be chosen according to

\[
a_2^*(a_1, c') = \arg \max_{a_2} \Pi_N \quad s.t. \quad a_2^* > 0.
\]

(2)

The distinctive feature of the model is that service providers can claim a regulated input price. As the value of the regulatory option equals \(V^r_2 = \pi_2(a_1, c')\), the supplier’s offer must satisfy the participation constraint

\[
\pi_2(a_1, a_2) - F_2 \geq V^r_2
\]

(3)

in order to reach an agreement with downstream firm \(S_2\). It becomes obvious that the supplier cannot entirely shift profits towards itself due to the regulatory option. Instead, it has to permit the downstream firm \(S_2\) to gain at least the amount it would obtain by choosing the regulatory option in order to implement discriminatory pricing. It will aim to do so because price discrimination raises the supplier’s profit. Condition (3), thus, becomes binding so that the fixed charge amounts to \(F_2 = \pi_2(a_1, a_2) - V^r_2\).

**Contract with first downstream firm:** The supplier likewise proposes a tariff \((a_1, F_1)\) to \(S_1\). Since the agreement is made before conditions for the rivals are set, the optimal reaction \(a_2^*(a_1)\) of the succeeding retailer is anticipated. Again the supplier will determine the per-unit-input-price by

\[
a_1^*(a_2^*(a_1), c') = \arg \max_{a_1} \Pi_N \quad s.t. \quad a_1^* \geq 0
\]

(4)

to maximise profits and the fixed charge \(F_1\) considering the regulatory outside option. As its value amounts to \(V^r_1 = \pi_1(c', a_2^*(a_1))\), the optimal contract must satisfy the participation constraint

\[
\Pi_1(a_1, a_2^*(a_1), F_1) \geq V^r_1,
\]

(5)

hence, the value of \(F_1 = \pi_1(a_1, a_2^*(a_1)) - V^r_1\). Given conditions (2) and (4) we show in the Appendix that both corner and interior solutions for the intermediary prices are possible. We find that the regulated price determines whether one or the other will arise. In principal, we can say:

\(^9\) That is a standard finding for two-part tariffs. It is also known that \(\frac{\partial a_i}{\partial F_i} < 0\), i.e. variable and fixed charge move the opposite directions.
Lemma 1.

Equilibrium per-unit wholesale prices for non-exclusionary cases will always be below the regulatory option, i.e. \( a^*_i < c^r \). (The regulatory outside option is never invoked.)

Proof. See Appendix

The reason for intermediary prices below the regulated level is to be found in marginal profits of the downstream firms \( \frac{\partial \Pi_i}{\partial a_i} < 0 \), and the fact that the regulated price \( c^r \) amounts to \( c \) or above. In other words, the downstream firms must be given an incentive to choose the discriminatory contracts which requires the discriminatory price to be below the mandatory option.

Let us now examine the interdependency of access prices. For that, we look at the case of interior solutions first.\(^{10}\) With \( F_2 \) from (3) and the supplier’s maximisation problem in (2), the optimal access price \( a_2 \) will be chosen based on the reaction function

\[
\frac{\partial \Pi_N}{\partial a_2} = (a_1 - c) \frac{\partial x^*_1}{\partial a_2} + x^*_2 + (a_2 - c) \frac{\partial x^*_2}{\partial a_2} + \frac{\partial F_2}{\partial a_1} = 0. \tag{6}
\]

For its rival \( S_1 \)

\[
\frac{\partial \Pi_N}{\partial a_1} = (a_1 - c) \frac{\partial x^*_1}{\partial a_1} + x^*_1 + (a_2 - c) \frac{\partial x^*_2}{\partial a_1} + \frac{\partial F_1}{\partial a_2} \frac{\partial a_2}{\partial a_1} + \frac{\partial F_2}{\partial a_1} = 0 \tag{7}
\]

represents the first-order condition for the access price considering \( F^*_1 \) from (5) and condition (4). Note that conditions (1) and (6) have been inserted into the result of \( \frac{\partial \Pi_N}{\partial a_1} \), hence, the envelope theorem has been applied. As we compare first order conditions (6) and (7) and, furthermore, consider the monotonicity of reaction functions as well as lower bounds on \( a_i \), we find:

Lemma 2. Asymmetric per-unit prices for the service providers can arise with firm \( S_2 \)’s price lower than \( S_1 \)’s, i.e. \( a_2 \leq a_1 \). Per-unit access charges are weak strategic complements in the non-exclusionary case.

Proof. See Appendix

The possibility of asymmetric input prices becomes obvious recognising the additional factors \( \frac{\partial F_1}{\partial a_2} \frac{\partial a_2}{\partial a_1} + \frac{\partial F_2}{\partial a_1} \) in (7). Observing that these terms have a positive sign and the concavity of supplier \( N \)’s profit function in \( a_i \) shows \( a_2 < a_1 \), see the Appendix for details. Generally, such pricing behaviour comes with the sequential settlement of contracts representing a typical commitment problem: Successive contracting enables the supplier to offer \( S_2 \) a per-unit-charge \( a_2 \) lower than \( a_1 \). Maintaining a relatively high intermediary per-unit-price for firm \( S_1 \) and a rather low for \( S_2 \) expands the latter’s profits and reduces the former’s. More precisely, the \( S_2 \)’s profit increase outweighs \( S_1 \)’s loss. Accordingly, the supplier raises the

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\(^{10}\) Alternatively, see Vives, X. (1999). Conclusions for corner solutions can be drawn by considering the monotonicity of reaction functions (input prices) at the transition from corner to interior solutions.
joint amount of profits it can transfer to itself.\textsuperscript{11} As for the complementarity of prices it is essential to understand that by settling on access charges the degree of retail competition is determined. With an increased price for one of the downstream firms the supplier is enabled to augment the rival’s price, too.

3.2 Strategic Interaction with Exclusion

The regulatory charge and the ensuing degree of price discrimination can reach a level which incurs one downstream firm’s exclusion, see the Appendix for detailed results. It shows that exclusion will target $S_1$ as it is the firm which receives the more unfavourable contract according to Lemma 2. Both the up- and the downstream industry are monopolised in this case. Therefore, in the retail stage, the amount of service placed in the product market by the remaining downstream firm $S_2$ is characterised by

$$x^E_2(a^E_2) = \arg\max_{x_2} \Pi_2(0, x_2) \quad \text{s.t.} \quad x^E_2 \geq 0.$$  

Prior to this stage, the upstream supplier’s profit consideration adheres to $\Pi_N(a_2, F_2) = (a_2 - c)x^E_2 + F_2$ and determines the level of

$$a^E_2 = \arg\max_{a_2} \Pi_N \quad \text{s.t.} \quad a^E_2 \geq 0$$

given that $a_1$ is set so high that $S_1$ withdraws from market activities. Again, the supplier’s power to extract downstream rents is limited due to the potential regulatory option. Accordingly, the fixed charge imposed depends on the value of the regulatory option $V^E_{2} = \pi_2(0, c^r)$ so that it amounts to

$$F^E_2 = \Pi_2(0, a_2) - V^E_{2}.$$  

Even though at first sight a demur, exclusionary price discrimination cannot be undermined by requesting the regulated access price $c^r$. It is not worthwhile for $S_1$ to claim it as the level of $a^E_1$ at which exclusion occurs is below the regulatory price. Details for this result are given in the next section and the Appendix.

3.3 The Regulated Access Price and Equilibrium Prices

We illustrate the various equilibrium pricing strategies in Table 1. Contracts and ensuing retail competition change with the production cost level $c$ and associated regulated access price $c + \Delta$. Therefore, one can distinguish between the outcomes by determining threshold levels $\Delta(c)$ of the regulatory price dependent on the actual production cost. We will use following notation and order for all $0 < \Delta(c) < 1 - c$ to specify the equilibrium constellation: $\Delta' < \Delta'' < \Delta''' < \Delta''''$. The same notation applies referring to access prices $a_i$.\textsuperscript{12}

\begin{itemize}
  \item \textsuperscript{11} This occurrence builds on imperfect downstream competition. It is helpful to look at the case of identical prices $\hat{a}_2 = \hat{a}_1$ and the changes due to deviation from there.
  \item \textsuperscript{12} $\Delta'''$ is determined due to $\Delta(c) \in (0; 1 - c)$ and $\Delta(c) \geq 0$.
\end{itemize}
Table 1: Equilibrium Wholesale Prices

<table>
<thead>
<tr>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0     if $0 \leq \Delta \leq \Delta'(c)$</td>
</tr>
<tr>
<td>$a_1''$</td>
<td>0     if $\Delta'(c) &lt; \Delta \leq \Delta''(c)$</td>
</tr>
<tr>
<td>$a_1'''$</td>
<td>$a_2''$ if $\Delta''(c) &lt; \Delta \leq \Delta'''(c)$</td>
</tr>
<tr>
<td>$\geq a_1''''$</td>
<td>$a_2''''$ if $\Delta'''(c) &lt; \Delta &lt; \Delta''''(c)$</td>
</tr>
</tbody>
</table>

Considering the concavity of supplier $N$’s profit function, corner solution $a_i = 0$ becomes an option in case regulation proposes thresholds below $\Delta'(c)$ as an alternative pricing scheme. Note that both $a_1$ and $a_2$ are monotone increasing functions of $\Delta(c)$ limited by zero from below and by $c^r$ from above. Proposition 1 summarises our results:

**Proposition 1.** The following types of equilibria emerge when price discrimination is feasible:

i. **Non-Discriminatory Inclusion:** No variable access prices, precisely, $a_1 = a_2 = 0$ will be charged for $0 < \Delta < \Delta'$. It means that prices are non-discriminatory in this region.

ii. **Price discrimination against firm $S_1$** is the equilibrium strategy for the supplier if $\Delta' < \Delta < \Delta''$. It takes the form of
   a. **Discriminatory Accommodation:** For $\Delta' < \Delta < \Delta''$ the second downstream firm’s input price $a_2 = 0$ while the first firm’s input price $a_1 > 0$.
   b. **Exclusionary Accommodation:** Both firms are charged a variable access price with $a_1 > a_2 > 0$ for $\Delta'' < \Delta < \Delta'''$. $S_2$’s contract provokes an output quantity $x_2 = \frac{1-c_2}{c}$ which would usually arise in monopoly.

iii. **Exclusion:** Exclusion of $S_1$ occurs in equilibrium if $\Delta''' < \Delta < \Delta''''$. Access charges amount to $a_2 = c$ while for $a_1$ all $a_1 \geq a_1'''$ constitute Nash equilibria.

The markup on actual cost inducing the above described pricing behaviour shrinks with higher production cost, so that $\frac{d\Delta(c)}{dc} < 0$ holds.

**Proof.** See Appendix

Figure 1 illustrates the regions where the different contracts arise.
The lower the permitted markup $\Delta$ on production cost the lower is a downstream firm’s opportunity cost to revert to the regulated access price. We name such a situation stricter regulation which characterises region I where the tolerated markup range is $\Delta < \Delta'$. The variable access prices here result to zero and facilitate strong retail competition without tariff induced advantages.\(^{13}\) We call this outcome "non-discriminatory inclusion". Regulation is attenuated a little in regions IIa and IIb with $\Delta' < \Delta < \Delta''$. Price discrimination takes place but does not induce a downstream firm to exit the market. We find that:

**Corollary 1.** A higher regulated access price leads to stronger price discrimination.

**Proof.** See Appendix

In other words, the higher the upper limit on intermediary prices, the harsher is the adopted price discrimination. Formally, an increasing price difference $a_2 - a_1$ turns out to be profitable for the supplier due to $\frac{\partial^2 \Pi_2}{\partial a_2 \partial a_1} < 0$. With this, the marginal loss $\frac{\partial \Pi_1}{\partial a_1} < 0$ from a higher $a_1$ is not affected as much the lower the rival’s input price $a_2$. Likewise, the gain $\frac{\partial \Pi_2}{\partial a_2} < 0$ from a lower $a_2$ is higher the higher $a_1$.\(^{14}\) Therefore, as soon as a higher regulatory price $c + \Delta$ permits stronger price discrimination, it is carried out. Even though price discrimination on the access level gives downstream firm $S_2$ a comparative advantage over firm $S_1$ it does not benefit from it in the end: The resulting higher revenues are seized by the supplier via the fixed fee $F_2$.

\(^{13}\) Contracts would be subsidised if permitted by assumptions.

\(^{14}\) It is helpful to look at the case of identical prices $\hat{a}_2 = \hat{a}_1$ and the changes due to deviation from there.
Beneath $\Delta''$ the variable access price $a_2$ of $S_2$ amounts to zero while $S_1$ pays a positive per-unit-price $a_1$. Thus, both downstream firms are accommodated whereby $S_2$'s variable price reaches its lower boundary of zero and $S_1$'s is limited by the upper boundary $c + \Delta''$. We therefore named this area "discriminatory accommodation". Above $\Delta''$ not only the price gap but also the level of input prices rises. Quasi-monopolistic behaviour of the second downstream firm is induced by the maintained price level: It places as much on the final market as it were a monopolist and its rival is left to the quantity which has to be admitted due to the regulatory outside option. We name this particular situation "exclusionary accommodation". In region III the permitted markup on cost is so high that it allows exclusion. As a consequence, the initial retailer is forced to exit the market and no competition takes place at any stage of the vertical chain. The outcome is therefore the same as if the supplier acted without any regulatory constraints. In contrast to the other results, this area is not defined by unique equilibrium values but a threshold value $a''''_1$ where all $a_1 \geq a''''_1$ constitute possible Nash equilibria. Interestingly, the threshold is not determined by looking at the border when activity in the final market turns to be unprofitable for the first downstream firm given marginal cost pricing of its rival and the usage of a regulatory price, i.e. condition $\Pi_1(c^r, c) = 0$ as one would assume at first hand. Conversely, the boundary beyond which exclusion occurs is characterized by $\Pi_1(a_1, a_2) = 0$ which yields a lower threshold than the aforementioned would require.\footnote{See Lemma 1.}

**Corollary 2.** Regulation may prevent exclusion. But if rather weak it won’t alter the incentive to exclude. Then, $a_2 = c$.

**Proof.** See Appendix

For the overall level of access prices, we additionally conclude:

**Corollary 3.** For non-exclusionary outcomes $a_2 < c$ for all $0 < \Delta(c) < \Delta''$, i.e. the variable price $a_2$ will always be below cost. In contrast, for variable price $a_1$, there is some threshold $\tilde{\Delta} \in [0; \Delta'']$ so that $a_1 < c$ for all $\Delta < \tilde{\Delta}$ and $a_1 > c$ for all $\Delta > \tilde{\Delta}$.

**Proof.** See Appendix

As price discrimination in the given setup leads to predatory pricing by means of the intermediary price $a_2$, this price attains a value below cost. The competitor’s price $a_1$, on the contrary, is determined by the degree of price discrimination still allowed by regulation. Its value changes with the intensity of regulation and leads to an access price $a_1 > c$ for higher thresholds of $\Delta(c)$. 

\footnote{See Lemma 1.}
3.4 Equilibrium Welfare in view of Access Price Regulation

Let us now turn to the welfare effects when the announced regulatory markup changes for a given input supply cost level. Welfare is given by the sum of profits of active parties and consumer surplus $CS$. For the non-exclusionary cases it amounts to

$$W = \Pi_N + \Pi_1 + \Pi_2 + CS,$$

while for exclusion it is

$$W = \Pi_N + \Pi_2 + CS.$$

As the pricing behaviour changes with the regulatory announcement welfare outcomes vary. The detailed results are stated in the Appendix. Looking specifically at the effect of the optional regulated price, we find:

**Proposition 2. Welfare and Access Price Regulation**

i. Welfare is maximised for potential regulation close to marginal cost: If $\Delta' > 0$ that occurs for $0 < \Delta \leq \Delta'$, otherwise this holds for $\Delta = 0$.

ii. A higher mandatory access price leads to lower welfare if price discrimination occurs and both downstream firms are active, i.e. $\frac{\partial W}{\partial \Delta} < 0$ for $\Delta' < \Delta < \Delta''$.

iii. Exclusion may be most detrimental to welfare.$^{16}$

**Proof.** See Appendix

Up to a certain extent of production cost and regulatory markup $\Delta$ welfare tendencies build on consumer surplus. Consumer surplus, in turn, is higher, the lower retail and intertwined access prices. This is the case for non-discriminatory inclusion or otherwise equilibria with no markup on marginal cost. It enforces intermediary prices of zero or at least relatively low access prices furthering competition downstream and maximising welfare in our framework. For a higher potential regulated access price stronger price discrimination is executed and access prices are raised simultaneously. This leads to higher retail prices and weakens retail competition. Hence, discriminatory prices will serve the monopolist’s interests, but decreases downstream firms’ profits and consumer welfare compared to the outcome for non-discriminatory inclusion. This is the reason why the welfare function is strictly decreasing in $\Delta(c)$ for the areas where both service providers stay active. For certain cost and regulation levels exclusion is most detrimental to welfare. However, note that the welfare function displays discontinuity at threshold $\Delta''$ due to $\Pi_1 > 0$ in (8). Consequently, for certain mandatory prices in the region of exclusionary accomodation welfare is equivalent to the one in case of exclusion. It is that under lowered retail competition, as is the case for equilibria in the areas of exclusionary accomodation and exclusion, the gains of both the

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$^{16}$ For some cases exclusionary accomodation and exclusion yields the same level of welfare (but distribution of benefits is different).
downstream and the upstream industry can contribute significantly to welfare. Therefore, the amount of welfare achieved can be identical in these regions, yet, distribution of surplus is different.\footnote{The ability to price discriminate will directly have an impact on welfare provided that asymmetric prices prevail. Then, in line with Corollary 2 and Proposition 3, a wholesale pricing scheme granting wholesale prices below marginal cost to both downstream firms leads to higher welfare than a scheme with prices above marginal cost. Indeed, at least the more competitive downstream firm can increase its sold quantity in case of price discrimination to a certain extent as long as its own variable price is set to zero. Nevertheless, it does not gain from it because the supplier can seize all of the additional profit through the fixed charge. So, interestingly, the downstream firm with the higher variable access charge gains even though its market activities are deterred.}

Comparing welfare of the discriminatory setup to the case of an unconstrained monopolist, we find that welfare in the two setups correspond in the exclusionary case.\footnote{Yet, due to the constraint of } Given our previous findings, we conclude that a certain extent of price discrimination levelled by the regulatory option is socially more desirable than the case of an unconstrained monopolist.

4 Non-Discrimination Clauses

We analyse whether non-discrimination rules alter market outcomes which occur under price discrimination. As the term "non-discriminatory pricing" has different legal and economic interpretations we consider two different notions of it. First, we look at a ban of price discrimination regardless of an optional regulated access price. Second, we examine the effect of non-discriminatory offers given the above described option of regulation.

4.1 Ban on Price Discrimination

Consider first the strictest interpretation which enforces the same tariffs for the downstream industry by completely prohibiting discrimination. That implies both firm $S_1$ and $S_2$ are charged the same tariffs $(a, F)$ for requesting network access which corresponds to a simultaneous setting of contracts. Downstream, this yields the same quantities being offered by the two downstream firms in the final market complying with condition (1). We denote, hence, output $x_1 = x_2 = x_{ND}$. As network access is concerned, profits of the monopolistic supplier can be written as $\Pi_N = 2 \cdot (a - c) x_{ND} + 2F$ which are optimised considering (2) and (3). Optimisation leads to a variable access charges $a(c) > c$ independent of the regulated price. The actual access prices will maintain a retail price as if the downstream industry was monopolised. Comparing this outcome to the ones in case of potential regulation, stated in Proposition 1 and 2, we conclude:

**Proposition 3.** A ban on price discrimination is socially less desirable than price discrim-
ination facing potential regulation.

**Proof.** See Appendix

When the monopolistic supplier can only charge the same price components to the downstream firms, it cannot engage in rent shifting via predatory pricing and price discrimination. It will therefore maximise the joint downstream profits it can shift to itself on the intermediary level with help of the two-part tariff. The equilibrium in this situation provokes aggregate final output which would occur for a monopolised downstream industry by setting intermediary variable prices above marginal cost $c$. It induces the same welfare outcome as in case of exclusion. Therefore, a ban on price discrimination cannot be more efficient than the proposed regulated price discrimination.

### 4.2 Non-Discriminatory Offers

Giving the disadvantaged downstream firm the possibility to claim the same and seemingly more favourable contract of its rival complies with the legal understanding of non-discrimination. To investigate the effect of such a rule, we add a recontracting stage to the previously used setup. Thus, the principal contracting process remains the same as before, but prior to downstream competition, there is now the possibility of revising contracts: After the first round of tariff agreements, downstream firms are now able to claim the same conditions as their rival or alternatively, switch to the regulatory option.\(^\text{19}\) Once recontracting is completed, downstream competition takes place. It remains to check whether such a rule might come into effect. For that, the disadvantaged firm’s profit under price discrimination has to be compared to the one it could obtain from claiming a non-discriminatory-price or alternatively, the regulated option. Therefore, we check whether

$$\pi_1(a^*_2, a^*_2) - F^*_2 \geq \pi_1(a^*_1, a^*_2) - F^*_1$$

and also whether $\pi_1(e^*, a^*_2) \geq \pi_1(a^*_1, a^*_2) - F^*_1$. In case these conditions hold it would be worthwhile for the underprivileged firm to claim the non-discriminatory terms. As for the regulatory option, commitment to existent contracts is given due to condition (5). Additionally, we can show that $S_1$ would not claim the seemingly more favourable contract terms of its rival, for a detailed proof see the Appendix: The supplier can commit both parties to their contracts by employing two-part tariffs. This occurs because the higher payable fixed fee more than offsets the potential gains from a lower variable fee. It exceeds the amount of profits a retail firm can make given uniform contracts. This finding builds on the dependency of downstream marginal profits on the rival’s access price: The level of the fixed fee was determined taking into account $S_2$’s low variable price and the comparative advantage over $S_1$’s tariff. Therefore, the reduction in wholesale charge $a^*_1$ is worth less to $S_1$ as this tariff is

\(^{19}\) This sequence of stages builds on McAfee and Schwartz (1994).
already at its rival’s disposal. This can be analytically derived by looking at cross-partials of the respective profits.

Alternatively, one could examine a variation to the case just examined recognising practical information constraints: Courts and retailers encounter difficulties observing discriminatory behaviour with help of subsidies because they can appear in numerous forms. Offering the same variable price without the necessity to check on fixed payments therefore denotes a practical alternative to the stricter legal interpretation of non-discrimination allegiances. Here also, we compare profits of $S_1$ under price discrimination with profits obtainable when invoking non-discriminatory terms as foreseen, i.e.

$$\pi_1(a_1^*, a_2^*) - F_1 \geq \pi_1(a_1^*, a_2^*) - F_1^*.$$  

We find:

**Proposition 4.** Non-discrimination rules which oblige the monopolistic supplier to offer downstream firms the same per-unit price will not become effective in a setting of discriminatory two-part tariffs. Potential price regulation does not change this result.

**Proof.** See Appendix

The reason for that and the crucial role two-part tariffs play here has already been explained in 4.2: It does not pay off for $S_1$ to choose $S_2$’s preferrable marginal price because it comes with a fixed fee set too high: Claiming it results in a loss for $S_1$. Thus, both for non-discrimination rules related to the per-unit charge and for rules considering the entire tariff, the fixed part of the contract is used to maintain the discriminatory agreement. We conclude that non-discriminatory offers cannot serve to alleviate possible anticompetitive effects of price discrimination when two-part tariffs are employed. Neither will welfare improve by enforcing equivalent uniform contracts in the downstream industry. Potential regulation does not affect this result.

5 Discussion

Apart from the regulated access price as an outside option, our results build on the restriction to two-part tariffs, the specification of linear demand and constant marginal cost of input supply. We discuss these underlying assumptions to our model to test the robustness of obtained results. This, actually, could indicate further areas of research.

**Two-Part Tariffs**

When employing two-part tariffs on the intermediary level the supplier uses the fixed fee to extract as much rent as possible from the downstream firms. In an environment of

\[\text{as a consequence, the two firms compete more aggressively in the retail market.}\]

\[\text{E.g. under-the-table-payments, rebates or other allowances.}\]
sequential contracting this leads to a second-mover advantage. As the supplier’s price discrimination ability is, additionally, restrained by the regulatory option this might induce below cost pricing towards both downstream firms. If we assumed linear pricing instead, the fixed charge as an instrument to shift profits would cease to exist. Then, a ”regular” Stackelberg constellation on the intermediary level would be present with Nash equilibria \((a_i, a_j) \in \{a_i < c, a_j > c\}\) with \(i \neq j\) and \(a_i \neq a_j\).\(^{22}\) As we would not achieve below cost pricing for both firms on the access level, this would feed back into higher retail prices than in two-part tariffs which, ultimately, could decrease welfare. Therefore, the assumption of two-part tariffs is crucial to our results. It maintains the possibility of stronger retail competition and no variable input price.\(^{23}\)

**Product Differentiation**

In our framework we consider homogenous products. We motivated this assumption observing an increasing convergence of Telecommunications and Internet services. Such a viewpoint is congruent with the attitude of European policy to assess demand-side-substitutability when defining market segments.\(^{24}\) The perspective of differentiated services could be justified the same way since there is a great variety of tariff packages and related services in the telecommunications, nowadays. We would obtain similar results to what we found if we changed not only the degree of differentiation but also the competitive surroundings in the downstream markets: With differentiated products and a Bertrand duopoly results should equal the ones obtained in a Cournot duopoly of homogenous products due to standard duality findings. In the most extreme case, differentiation can ”separate” retail markets which is corresponding to two vertically monopolised market segments removing the interdependency effects on which our results build.

**Constant Marginal Cost**

Results were demonstrated by the specific use of a linear demand example and assuming constant marginal cost upstream. One could imagine a different shape of the upstream cost function considering the requirements of Telecommunications or Internet applications inducing capacity constraints, like online gaming or video streaming. In these circumstances, convex cost representing growing cost with more requested capacity or likewise a degradation of quality seem a legitimate concern possibly affecting the shape of the supplier’s profit function.\(^{25}\) The conditions we used to illustrate our results would indicate ambiguous results in this case, so that this question is left open for further research.\(^{26}\)

\(^{22}\) Calculating the linear demand example, this yields \((a_1, a_2) \in [c^\tau, \frac{1}{2}(1 + c + 2c^\tau)), \left(\frac{1}{2}(1 + c + 2c^\tau), c^\tau\right]\).

\(^{23}\) These considerations, evidently, relate to an environment presuming sequential moves and discriminatory pricing. With simultaneous moves, an equivalent result, particularly concerning retail competition, could be reproduced by imposing uniform pricing below marginal cost.

\(^{24}\) See Article 7 procedures to monitor the implementation of the EU Regulatory framework, available at: [http://ec.europa.eu/information_society/policy/ecomm/article_7/index_en.htm](http://ec.europa.eu/information_society/policy/ecomm/article_7/index_en.htm)

\(^{25}\) For convex downstream cost, see e.g. Baake, P., Kamecke, U. and Normann, T. (2002).

\(^{26}\) Furthermore, it indicates a possible extension of the framework considering congested networks.
Innovation Incentives

We paid attention to the relationship between pricing and welfare with exogeneous input cost examining the implications of price discrimination in the short-run. Yet, policy discussion nowadays is often concerned with appropriate incentives for investment and a balance between short- and long-run incentives. Therefore, it is worthwhile, broadening the scope of our setup as to permit possible investments and the question whether innovation and innovation spillovers are induced by it. This suggests an additional investment stage to explore the link between pricing and potential regulation. Moreover, the innovative effects caused by stronger market competition have to be re-assessed and specified.

6 Conclusion

In this paper, we examined how the regulatory threat of prescribing access charges affects discriminatory price setting of a network operator. We showed that for a potentially regulated access price close to marginal cost, exclusion is omitted and retail competition furthered, leading to a socially desirable outcome. We found that the possibility of a regulated input price affects the usage and strength of price discrimination provided that discriminatory tariffs are in two parts. In fact, prescribing access prices above marginal cost proves itself socially more undesirable than most discriminatory outcomes facing a potential statutory price. Additionally, we saw that uniform pricing cannot be induced by offering all respective parties the same menu of tariffs by means of a non-discrimination rule. These results imply that banning price discrimination cannot serve to raise welfare or install stronger competition. Also, it suggests to re-examine the effects of prescribing price levels for network access. Most importantly, the results suggest that threatening regulation may be a useful role for government intervention inducing a socially more desirable outcome than the present regulatory approach.

Observing that outcome changes with varying cost brings the cost aspect of access provision to mind. From this perspective it is noteworthy, that our results not only depend on the overall cost level but also on the markup on costs conceded by potential regulation. As recent policy articulates the interest to stimulate investments, this aspect could be further analysed extending our framework by an additional innovation stage. With that, our short-run perspective could be complemented by long-run dynamic implications of discrimination and regulatory intervention. Endogenising cost as well as the question whether increasing competition raises both investment activities and innovation might be aspects to look into. It is left for further research.
7 Appendix

Proof of Lemma 1:
Let us first show $a_i < 0$. We will make use of a proof by contradiction:
In contrast to Lemma 1, we suppose that there exists at least one $a_i^* > c$. Considering this relationship and the fact that $\frac{\partial \pi_i}{\partial a_i} < 0$, we conclude that
\[
\pi_i(a_i^*, a_j^*) < \pi_i(c^*, a_j^*)
\]
for $i \neq j$ and $i, j = \{1, 2\}$. Yet, from this it follows that
\[
\pi_i(a_i^*, a_j^*) - F_i < \pi_i(c^*, a_j^*)
\]
which contradicts the participation constraints given in (3) and (5). Therefore, $a_i^* > c$ cannot be an equilibrium outcome.
To see that $a_i = c$ cannot be a solution, we consider the supplier’s incentive to offer discriminatory contracts
\[
\Pi_N(a_i, a_j) > \Pi_N(c, a_j).
\]
Then, for a solution $a_i = c$ in equilibrium
\[
\Pi_N(c, a_j) > \Pi_N(c, a_j)
\]
must hold. It does not. Then, $a_i = c$ cannot be a solution.
Therefore, no $a_i^* \geq c$ exists as a solution to the given problem.
q.e.d

Proof of Lemma 2:
Existence of unique interior solutions: By assuming linear demand and constant returns to scale, the profit function $\Pi_N$ becomes continuous and twice differentiable. Looking at its second derivative, we find that $\frac{\partial^2 \Pi_N}{\partial a_i} < 0$ holds. As the set $(a_1, a_2)$ is compact, we can conclude on the existence of unique interior solutions with help of Brouwer’s fixed point theorem.
Asymmetry and strategic complementarity of access prices: To prove Lemma 2 we have to show $a_2 < a_1$. We start out by comparing the reaction functions given in (6) and (7) characterising interior solutions to the supplier’s maximisation problem. We find that they are symmetric except for additional arguments $\frac{\partial F_1}{\partial a_2} \frac{\partial a_2}{\partial a_1} + \frac{\partial F_2}{\partial a_1}$ in downstream firm $S_1$’s reaction function. Given the concavity of the supplier’s profit function $\Pi_N$ and $\frac{\partial^2 \Pi_N}{\partial a_i} > 0$ we have to show that
\[
\frac{\partial F_1}{\partial a_2} \frac{\partial a_2}{\partial a_1} + \frac{\partial F_2}{\partial a_1} > 0
\]
to confirm $a_2 < a_1$. We look at the two components separately and make use of the binding participation constraints given in (3) and (5) determining $F_2 = \pi(a_1, a_2) - \pi(a_1, c)$ and $F_1 = \pi(a_1, a_2^*(a_1)) - \pi(c, a_2^*(a_1))$.
For the second component we, then, get
\[ \frac{\partial F_2}{\partial a_1} = \frac{\partial \pi_2(a_1, a_2)}{\partial a_1} - \frac{\partial \pi(a_1, c')}{\partial a_1} = -\frac{4}{9}a_2 + \frac{4}{9}(c + \Delta) \]

using the explicit functions \( \pi_2(a_1, a_2) \) inserting equilibrium values from (14) stated further below on page 20. Referring back to Lemma 1 this yields \( \frac{\partial F_2}{\partial a_1} > 0 \). Alternatively, one can check \( \frac{\partial \pi_2}{\partial a_2} < 0 \) and \( \frac{\partial^2 \pi_2}{\partial a_2 \partial a_1} < 0 \) characterised by inserting equilibria given in (1) and likewise determine the sign of \( \frac{\partial F_2}{\partial a_1} \) with help of Lemma 1.

Referring to the first component \( \frac{\partial F_1}{\partial a_2} \) the same reasoning applies for factor \( \frac{\partial F_1}{\partial a_2} \), so that we can assign a positive value. It remains to show that \( \frac{\partial a_2}{\partial a_1} > 0 \). Given the explicit interior solution \( a^*_2 = \frac{1}{4}(-1 + 2a_1 + 3c) > 0 \) by (13) on page 20, we can also confirm a positive sign for this factor.\(^{27}\)

The weaker condition \( a_2 \leq a_1 \) considers the possibility of corner solutions at the lower boundary of \( a_i \).

q.e.d

**Proof of Proposition 1:**

To obtain the results stated we have to solve the linear demand example as stated in sections 3.1 and 3.2. We start by restating the downstream firms’ and the supplier’s profit functions,

\[ \Pi_i(x_i, x_j) = (P(x_i, x_j) - a_i)x_i - F_i \]

and

\[ \Pi_N(a_1, a_2, F_1, F_2) = (a_1 - c)x_1 + (a_2 - c)x_2 + F_1 + F_2. \]

Provided that there is no exclusion, the explicit solution to (1) characterising equilibrium quantities in the retail stage is

\[ \frac{\partial \Pi_i(x_i, x_j^*)}{\partial x_i} = 1 - 2x_i - x_j^* - a_i = 0. \quad (10) \]

Considering mutual best responses \( x_1^*(x_2^*) \) and \( x_2^*(x_1^*) \) yield the reduced forms

\[ x_i^*(a_i, a_j) = \frac{1}{3}(1 - 2a_i + a_j) \quad \text{and} \]

\[ p^*(a_1, a_2) = \frac{1}{3}(1 + a_1 + a_2), \quad (11) \]

depicting the dependency of final output and prices on wholesale prices. Note that one can already state the boundaries of the different types of equilibria using (11) and referring to the assumptions on \( c \) and \( \Delta \) given on page 4 as

1. \( 2a_2 - 1 < a_1 \) for no exclusion of \( S_2 \),
2. \( a_1 < \frac{1 + a_2}{2} \) for no exclusion of \( S_1 \),
3. \( 0 < c < 1 \land 0 < \Delta < 1 - c \) for market activity of the supplier and regulated prices above marginal cost.

\(^{27}\) Alternatively, one could check the sign of \( \frac{\partial a_2}{\partial a_1} \) by making use of the FOC given in (6) and by recognising

\[ \frac{\partial a_2}{\partial a_1} = \frac{\partial^2 \Pi_N}{\partial a_1 \partial a_2} / \frac{\partial^2 \Pi_N}{\partial a_2^2} \]

when assuming strategic complementarity of the access charges.
Condition (3) on page 6 determines the value of \( F_2 \) affected by the regulatory constraint \( c + \Delta \). We get explicit solutions by reinserting equilibrium values \( a_2^*(a_1) \) characterised by (2). The first-order condition for interior solutions given in (6) writes
\[
\frac{\partial \Pi_N}{\partial a_2} = \frac{1}{9}(-1) + 2a_1 + 3c
\]
and yields
\[
a_2^*(a_1) = \begin{cases} 
\frac{1}{4}(-1 + 2a_1 + 3c) & \text{if } a_1 > \frac{1}{2} - \frac{3}{2}c \\
0 & \text{if } a_1 \leq \frac{1}{2} - \frac{3}{2}c
\end{cases} \tag{13}
\]
as possible variable wholesale prices. Plugging these results into \( F_2 \), we obtain
\[
F_2^* = \begin{cases} 
\frac{1}{3}(1 - c)^2 - \frac{1}{6}(1 + a_1 - 2c - 2\Delta)^2 & \text{if } a_1 > \frac{1}{2} - \frac{3}{2}c \\
\frac{1}{9}(1 + a_1)^2 - \frac{1}{6}(1 + a_1 - 2c - 2\Delta)^2 & \text{if } a_1 \leq \frac{1}{2} - \frac{3}{2}c
\end{cases} \tag{14}
\]
We use these results to solve (6) in order to obtain values for the variable price \( a_1 \) with profit function
\[
\Pi_N = \begin{cases} 
-\frac{1}{8}(1 - 2a_1 + c)^2 + \frac{1}{4}(1 - c)^2 - \frac{1}{6}(1 + a_1 - 2c - 2\Delta)^2 + F_1 & \text{if } a_1 > \frac{1}{2} - \frac{3}{2}c \\
\frac{1}{9}(a_1(-2a_1 + 1 + c) - 2c) + \frac{1}{6}(1 + a_1)^2 - \frac{1}{6}(1 + a_1 - 2c - 2\Delta)^2 + F_1 & \text{if } a_1 \leq \frac{1}{2} - \frac{3}{2}c
\end{cases}
\]
Again, the regulatory constraint as given in (5) determines the fixed charge \( F_1 \) requiring equilibrium variable prices of \( a_1^* \). The first-order condition as described in (7) results in
\[
\frac{\partial \Pi_N}{\partial a_1} = \begin{cases} 
\frac{1}{9}(1 + 15c + 12\Delta - 14a_1) & \text{if } a_2 > 0 \\
\frac{1}{9}(-1 + 7c + 4\Delta - 4a_1) & \text{if } a_2 = 0
\end{cases} \tag{15}
\]
and yields equilibrium values
\[
a_1^* = \begin{cases} 
\frac{1}{4}(-1 + 2a_1 + 3c) & \text{if } a_1 > \frac{1}{2} - \frac{3}{2}c \\
0 & \text{if } a_1 \leq \frac{1}{2} - \frac{3}{2}c
\end{cases} \tag{16}
\]
By considering the possibility of both interior and corner solutions in the non-exclusionary case and the boundaries given in 1., 2. and 3. on page 19 we can define following threshold levels (upper bounds):

<table>
<thead>
<tr>
<th>Threshold ( \Delta )</th>
<th>Equilibrium Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta' ) = ( \frac{1}{2} - \frac{3}{2}c ) if ( c &lt; \frac{1}{7} )</td>
<td>Type I: Non-Discriminatory Inclusion</td>
</tr>
<tr>
<td>( \Delta'' ) = ( \frac{3}{7} - 3c ) if ( c &lt; \frac{2}{7} )</td>
<td>Type Ia: Discriminatory Accomodation</td>
</tr>
<tr>
<td>( \Delta''' ) = ( \frac{2}{3} - \frac{1}{3}c )</td>
<td>Type Ib: Exclusionary Accomodation</td>
</tr>
</tbody>
</table>

Following equilibrium results occur for the equilibrium types within these thresholds: Input contracts for the case of Non-Discriminatory Inclusion (i.) will take values
\[
a_1' = a_2' = 0 \text{ and }
\]

\[28\] Threshold level \( \Delta''' = 1 - c \) for the exclusionary case is given by assumption, see footnote 11.
\[ F'_1 = F'_2 = \frac{4}{9}(a - c - \Delta)(c + \Delta). \]

Downstream quantities and retail price amount to
\[
x'_1 = x'_2 = \frac{1}{3} \quad \text{and} \quad p' = \frac{1}{3}.
\]

For these values profits sum up to
\[
\Pi_N = -\frac{2}{9}(3c + 4(c^2 + \Delta^2 + 2c\Delta - c - \Delta),
\Pi_1 = \Pi_2 = \frac{1}{9}(1 - 2c - 2\Delta)^2.
\]

For the case of Discriminatory Accommodation (iia.) equilibrium values amount to following access prices:
\[
a''_1 = \frac{1}{4}(-1 + 7c + 4\Delta) \quad \text{and} \quad F''_1 = \frac{1}{36}(-1 + 3c)(-5 + 11c + 3\Delta)
\]
\[
a''_2 = 0 \quad \text{and} \quad F''_2 = \frac{1}{3}(1 + c)(c + \Delta)
\]

These results produce
\[
x''_1 = \frac{1}{9}(3 - 7c - 4\Delta) \quad \text{and}
\]
\[
x''_2 = \frac{1}{12}(3 + 7c + 4\Delta) \quad \text{and}
\]
\[
p'' = \frac{1}{12}(3 + 7c + 4\Delta)
\]
downstream. Then, profits write
\[
\Pi_N = \frac{1}{72}(1 - 15c^2 - 48\Delta^2 + 2c - 16\Delta) \quad \text{and}
\Pi_1 = \Pi_2 = \frac{1}{17}(1 - 2c - 2\Delta)^2 \quad \text{and}
\]
\[
\Pi_2 = \frac{1}{174}(3 + 7c - 4\Delta)^2.
\]

In case of Exclusionary Accommodation (iib.) equilibrium prices add up to
\[
a'''_1 = \frac{1}{17}(-1 + 15c + 12\Delta) \quad \text{and} \quad F'''_1 = -\frac{11}{17}(11 - 11c - 20\Delta)(-1 + c - 2\Delta)
\]
\[
a'''_2 = \frac{1}{17}(-2 + 9c + 3\Delta) \quad \text{and} \quad F'''_2 = \frac{11}{17}(-1 + 3c - 2\Delta)(-17 + 17c + 8\Delta).
\]

This yields
\[
x'''_1 = \frac{1}{17}(2 - 2c - 3\Delta) \quad \text{and}
\]
\[
x'''_2 = \frac{1 - c}{2} \quad \text{and}
\]
\[
p''' = \frac{1}{17}(3 + 11c + 6\Delta)
\]
in the product market and results in profits
\[
\Pi_N = \frac{1}{67}(1 + c^2 - 38\Delta^2 - 2c + 46\Delta + 106c\Delta) \quad \text{and}
\Pi_1 = \pi_1 = \frac{1}{17}(5 - 5c - 11\Delta)^2 \quad \text{and}
\Pi_2 = \pi_2 = \frac{(13 - 13c - 16\Delta)^2}{1764}.
\]
Given conditions on page 8 in case of *Exclusion*, the first order condition in the retail stage writes

\[
\frac{\partial \Pi_2(x_2)}{\partial x_2} = 1 - 2x_2 - a_2 = 0
\]

so that the retail segment’s dependency on actual wholesale outcome is characterised by

\[
x_2(a_2) = \frac{1 - a_2}{2},
\]

\[
p(a_2) = \frac{1 + a_2}{2}.
\]

With this optimal reaction, Network supplier \( N \) optimises its profit \( \Pi_N = (a_2 - c)x_2 + F_2 \) according to the first-order condition on page 8 by fixing a contract

\[
a_2^E = c, \quad F_2^E = \frac{1}{4}(1 - c)^2 - \frac{1}{4}(1 - c - \Delta)^2,
\]

where again the regulatory outside option again determines the fixed fee and the actual profits of the supplier. As the supplier solely obtains profits from the fixed charge, the related participation constraint and the supplier’s profit are identical given by

\[
\Pi_E^N = \frac{1}{4}(1 - c)^2 - \frac{1}{4}(1 - c - \Delta)^2
\]

while for firm \( S_2 \)

\[
\Pi_E^S = \frac{1}{4}(1 - c - \Delta)^2.
\]

To verify \( \frac{d\Delta(c)}{dc} < 0 \) we look at the first derivative of relevant thresholds given on page 20:

\[
\frac{d\Delta'}{dc} = -3 < 0
\]

\[
\frac{d\Delta''}{dc} = \begin{cases} 
-3 & \text{if } c < \frac{2}{3} < 0 \\
0 & \text{if } c \geq \frac{2}{3} = 0 \quad \text{(already at the lower bound)}
\end{cases}
\]

\[
\frac{d\Delta'''}{dc} = -\frac{1}{3} < 0
\]

\[
\frac{d\Delta''''}{dc} = -1 < 0
\]

This verifies that thresholds \( \Delta \) are decreasing in \( c \) within the given value range.

\[ q.e.d \]

**Proof of Corollary 2:**

From Lemma 1 and 2 we know that \( a_2 \leq a_1 < c + \Delta \). From the concavity of \( \Pi_N \) or convexity of \( \Pi_i \) in \( a_i \) we can then directly see that \( \frac{da_1}{d\Delta(c)} \geq \frac{da_2}{d\Delta(c)} \). From standard derivation rule for sums we can conclude that

\[
\frac{d(a_1 - a_2)}{d\Delta(c)} \geq 0.
\]

\[ q.e.d \]
Proof of Corollary 3
To verify the level of per-unit-access prices with respect to \( c \), we look at upper and lower boundaries of the prices. The proof builds on the fact that both variable prices are strictly monotone increasing in \( \Delta(c) \), i.e. \( \frac{da}{d\Delta(c)} > 0 \), in case of interior solutions.
Calculating yields the lower bound
\[
\overline{a}_2 = \lim_{\Delta \to \Delta''} a_2 = 0 \quad \text{and}
\]
the upper bound
\[
\overline{a}_2 = \lim_{\Delta \to \Delta''} a_2 = c.
\]
Considering all non-exclusionary outcomes this shows that \( a_2 < c \) for all \( 0 < \Delta(c) < \Delta''' \).
For variable \( a_1 \) the lower bound is given by
\[
\overline{a}_1 = \lim_{\Delta \to \Delta'} a_1 = 0 \quad \text{and}
\]
the upper bound by
\[
\overline{a}_2 = \lim_{\Delta \to \Delta''} a_1 = \frac{1}{2} + \frac{11}{14} c > c.  \quad \text{29}
\]
As \( \frac{da}{d\Delta(c)} > 0 \) and there are solutions for all \( \Delta(c) \) in the interval, there exists some threshold \( \tilde{\Delta} \in [\Delta'; \Delta'''] \) s.t.
\[
\lim_{\Delta \to \tilde{\Delta}} a_1 \leq c \quad \text{if} \quad \Delta < \tilde{\Delta}
\]
and
\[
\lim_{\Delta \to \tilde{\Delta}} a_1 \geq c \quad \text{if} \quad \Delta > \tilde{\Delta}.
\]
q.e.d.

Proof of Corollary 4
See Proof of Proposition 1.

Proof of Proposition 2
In order to draw conclusions on welfare, we calculate the explicit solutions of the different equilibrium types.\(^{30}\) For Non-Discriminatory Inclusion we get
\[
W' = \frac{4}{9} \cdot \frac{2c}{3}.
\]
In case of Discriminatory Accomodation welfare amounts to
\[
W'' = \frac{1}{288} \left(119c^2 - 16\Delta^2 - 258c - 24\Delta + 40c\Delta + 135\right)
\]
whereas in case of Exclusionary Accomodation it is
\[
W''' = \frac{1}{392} \left(187c^2 - 36\Delta^2 - 374c - 36\Delta + 36c\Delta + 187\right).
\]

\(^{29}\) Instead of calculating an explicit result for \( \overline{a}_1 \) one could as well make use of Lemma 2 and the fact that \( \overline{a}_2 = c \) to show that \( \overline{a}_1 > c \).

\(^{30}\) We make use of the same notation as for the access prices.
If Exclusion occurs it sums up to

\[ W''' = \frac{3}{8}(1 - c)^2. \]

To show that \( \frac{\partial W}{\partial \Delta} < 0 \) for \( \Delta' < \Delta < \Delta'' \) as stated in Proposition 2ii. we look at the first derivative of the respective welfare functions. This yields

\[ \frac{\partial W''}{\partial \Delta(c)} = \frac{1}{36}(-3 + 5c - 4\Delta) < 0 \]

and

\[ \frac{\partial W'''}{\partial \Delta(c)} = \frac{9}{98}(-1 + c - 2\Delta) < 0 \]

in the defined range of values \( \Delta(c) \). Then, to prove Proposition 2i. we check whether

\[ W'(c) > W''(\Delta', c) \land W'(c) > W'''(\Delta'', c) \land W'(c) > W'''(c). \]

For \( W''(\Delta', c) \) we obtain

\[ W''(\Delta', c) = \frac{4}{9} - \frac{2}{3}c < W' \]

and for welfare in case of exclusionary accomodation we get

\[ W'''(\Delta'', c) = \frac{1}{8}(-5c^2 - 2c + 3) < W' \]

in the defined range of values with \( c < \frac{2}{\sqrt{5}} \). For \( c > \frac{2}{\sqrt{5}} \) calculation yields

\[ W''' = \frac{187}{392}(1 - c^2) < W'. \]

Also in the exclusionary case, the above-made assumption holds.

We can verify Proposition 2iii. by calculating two examples. For the case of \( c = 0.2 \) and \( \Delta = 0.6 \) welfare sums up to \( W''' \approx 0.228 < 0.24 \approx W''' \) which shows that exclusion is not the least socially optimal outcome. On the contrary, assigning \( \Delta = 0.3 \) in this case yields \( W''' = 0.275 > 0.24 = W''' \). Therefore, for a given cost level \( c \) exclusion may be most detrimental, but is not necessarily.

q.e.d.

Proof of Proposition 3:
As before, the game is solved backwards. According to condition (1) or (A1) and considering uniform variable charges \( a_1 = a_2 = a \) reduced forms of quantites and retail price dependent on the access charge \( a \) write

\[ p^{ND}(a) = \frac{1}{3}(1 + 2a) \] and

\[ x^{ND}(a) = \frac{1}{3}(1 - a). \]
As the only participation constraint given is market activity, the monopolist, successively, sets the fixed charge \( F = \pi_i = x^2 \) and the variable charge with respect to the profit function stated on page 13. The first order condition, hence, writes

\[
\frac{\partial \Pi_N}{\partial a} = 2 - \frac{4}{3}a + \frac{2}{3}c.
\]

This yields the equilibrium prices

\[
p^{ND} = \frac{1}{2}(1 + c) \quad \text{and} \quad a = \frac{1}{4}(1 + 3c).
\]

Product quantities amount to

\[
x^{ND} = \frac{1}{4}(1 - c)
\]

so that the supplier sells the amount of access which would arise for a vertical monopoly in aggregate. It remains to show that welfare for this equilibrium outcome is not higher than welfare for equilibrium outcomes under price discrimination. To do so, we refer to the welfare function on page 12 and look at welfare changes caused by equilibrium prices given non-discrimination compared to discrimination in our setup, see page 23. We find that welfare in the case of non-discrimination and in the case of exclusion from the discriminatory setup correspond, i.e.

\[
W^{ND} = W'''.
\]

Yet, distribution of benefits differs. This becomes obvious by looking at industry profits: As there is no further constraint than non-discrimination on the input monopolist’s activities, the fixed charge \( F \) serves to extract downstream profits completely. Hence, \( \Pi_1^{ND} = 0 \leq \Pi_1 \), i.e. profits under the discriminatory regime are at least as high as profits under a ban on price discrimination. Supplier \( N \), on the contrary, benefits from the rule obtaining monopolistic profits \( \Pi_N^{ND} = \frac{1}{4}(1 - c)^2 \). These profits match total industry profits in the discriminatory regime, i.e. \( \Pi_N^{ND} = \Pi_N + \Pi_2 \), which further implies \( \Pi_N^{ND} > \Pi_N \) for positive \( \Pi_2 \). Consumer surplus stays the same in the non-discriminatory and in the discriminatory regime as retail prices and aggregate quantity coincide.

q.e.d.

Proof of Proposition 4:

To find out whether a non-discriminatory rule would be invoked the prospective profit of disadvantaged firm \( S_1 \) under non-discriminatory conditions are compared to the profit under price discrimination. In general, a non-discrimination claim would be made iff

\[
\Pi_1(a_1^*, a_2^*) > \Pi_1(a_1^*, a_2^*).
\]

corresponding to the conditions given on page 14 and 15. Proving this builds on the following three preliminaries:
1. Downstream firms' profits are symmetric, i.e. \( \Pi_1(a_1, a_2) = \Pi_2(a_2, a_1) \).

2. As we know that \( \frac{\partial^2 \pi_i}{\partial a_i \partial a_j} < 0 \), a decrease in the own wholesale price is less valuable the lower the rival’s wholesale price. It is because lower wholesale terms make the rival more aggressive on the retail level.

3. The prescribed regulatory tariff is higher than the equilibrium discriminatory fees, i.e. \( a_i < c + \Delta c \) as found in Lemma 3.1

Rewriting (15) in terms of flow profits and fixed fees and plugging in participation constraints (3) and (5), we get

\[
\begin{align*}
\pi_1(a_2^*, a_1^*) - F_2 & > \pi_1(a_1^*, a_2^*) - F_1 \\
\pi_1(a_2^*, a_1^*) - \pi_2(a_1^*, a_2^*) + \pi_2(c^*, a_1^*) & > \pi_1(a_1^*, a_2^*) - \pi_1(a_1^*, a_2^*) + \pi_1(c^*, a_2^*) \\
\pi_1(a_2^*, a_1^*) - \pi_1(c^*, a_2^*) & > \pi_2(a_1^*, a_2^*) - \pi_1(a_1^*, c^*)
\end{align*}
\]

Using symmetry this can be rewritten

\[
\begin{align*}
\pi_1(a_2^*, a_1^*) - \pi_1(c^*, a_2^*) & > \pi_1(a_2^*, a_1^*) - \pi_1(c^*, a_1^*) \\
\pi_1(a_2^*, a_1^*) - \pi_1(a_2^*, a_1^*) & > \pi_1(c^*, a_2^*) - \pi_1(c^*, a_1^*)
\end{align*}
\]

or

\[
\begin{align*}
\pi_1(a_2^*, a_1^*) - \pi_1(c^*, a_1^*) & > \pi_1(c^*, a_2^*) - \pi_1(c^*, a_1^*)
\end{align*}
\]

Rewriting in form of integrals yields

\[
\int_{c^*}^{a_2} \frac{\partial \pi_1}{\partial a_1} da_1(\cdot, a_2) > \int_{c^*}^{a_2} \frac{\partial \pi_1}{\partial a_1} da_1(\cdot, a_1).
\]

The last three expressions contradict the assumption of negative cross-partial. Thus, we can conclude that neither the other party’s wholesale price nor the regulatory option represent effective options for \( S_1 \) in case of competition downstream. The same reasoning applies in case there is no competition downstream.

q.e.d.
References


Errata:

Proof of Lemma 1, p.18:
Let us first show $a_i \leq c^r$. We will make use of a proof by contradiction:
In contrast to Lemma 1, we suppose that there exists at least one $a_i^* > c^r$. Considering this relationship and the fact that $\frac{\partial \Pi_i}{\partial a_i} < 0$, we conclude that
$$\pi_i(a_i^*, a_j^*) < \pi_i(c^r, a_j^*)$$
for $i \neq j$ and $i, j = \{1, 2\}$. Yet, from this it follows that
$$\pi_i(a_i^*, a_j^*) - F_i < \pi_i(c^r, a_j^*)$$
which contradicts the participation constraints given in (3) and (5). Therefore, $a_i^* > c^r$ cannot be an equilibrium outcome.
To see that $a_i = c^r$ cannot be a solution, we consider the fact that $\frac{\partial \pi_i}{\partial a_i} < 0$. Therefore, a profit maximising tariff $(a_i, F_i)$ will entail components $a_i < c^r$ and $F_i > 0$.
We conclude that no $a_i^* \geq c^r$ exists as a solution to the given problem.
q.e.d

Proof of Lemma 2, bottom of p.18:
(...Referring back to Lemma 1 this yields $\frac{\partial F_2}{\partial a_1} > 0$. Referring to the first component $\frac{\partial F_1}{\partial a_2} \frac{\partial a_2}{\partial a_1}$ the same reasoning applies for factor $\frac{\partial F_1}{\partial a_2}$, so that we can assign a positive value. (...)

First-order condition above (13), p.20:
$$\frac{\partial \Pi_N}{\partial a_2} = \frac{1}{9} (2a_1 - 4a_2 + 3c - 1).$$