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Pio Baake Sebastian Schwenen Christian von Hirschhausen

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Abstract

In current power markets, the bulk of electricity is sold wholesale and transported to consumers via long-distance transmission lines. Recently, decentralized local power markets have evolved, often as isolated networks based on solar generation. We analyze strategic pricing, investment, and welfare in local power markets. We show that local power markets with peer-to-peer trading are competitive and provide efficient investment incentives, even for a small number of participating households. We identify positive network externalities that make larger markets more attractive but lead to inefficiencies where networks compete. Collectively, our results present a set of positive efficiency results for peer-to-peer electricity markets.

Keywords: Market Design, Networks, Peer-to-Peer Markets, Electricity.
JEL-Classification: D16, D26, D47, L94.

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1 Introduction

Electricity access for all has become a major topic for international energy, climate, and regulatory policy. Especially in developing economies, micro-grids based on solar generation increasingly provide rural areas with electricity. Local peer-to-peer markets, eventually separated from large scale power grids, have however also been field-tested in established electricity systems, that traditionally consisted of complex vertical and sequential market arrangements (Wilson, 2002).

In this article, we provide a model for studying electricity provision through peer-to-peer trading in local power markets. The model allows to analyze pricing behavior and investment incentives in local power markets, and to compare welfare implications that arise from network externalities and different market outcomes in the upstream market for generation and network technologies.

The results that we derive are substantially different from equilibrium models that seek to describe traditional power market architectures, composed of top-down vertical generation, transmission, and retail supply chains. For example, we show that local power markets yield competitive market prices. Importantly, this result holds for a small number of market participants (i.e., households) and despite strategic pricing behavior by all participating households. Furthermore, we find that positive network externalities arise and make larger markets relatively more attractive.

Next to the efficiency of decentralized planning and trading, paramount questions surrounding the design and regulation of local power markets pertain to the use of different and incompatible technology standards. Networks in rural areas have often started at low levels of power and quality, typically using simple and easily accessible direct current (DC) technology. With increasing demands and capital costs, alternating current (AC) network standards become feasible, allowing for higher voltage levels and higher peak and average consumption. Because we identify positive network effects in local power markets, different technology standards can separate markets and attenuate welfare.

In the environment that we study, households can decide to participate in a local market for electricity. Households participate by demanding electricity from the local network or by investing in generating plants, here solar plants, that add supply to the local network. Households that have invested in generation capacity can be net-selling or net-buying from the market, depending on whether their generation covers more or less than their consumption. Given aggregate household demand and aggregate solar supply, the market constitutes an equilibrium price for electricity. We model market clearing using a demand function approach. Having characterized pricing and investment equilibria in local power markets, we subsequently analyze implications that arise from two competing networks that operate with different technologies. Finally, as an extension, we model storage devices, that typically accompany local networks with the aim to arbitrage.

Our findings constitute a set of positive efficiency results for local power markets. We find that the equilibrium price for electricity in the local market is perfectly competitive. While both net-buying and net-selling households exercise strategic pricing, their strategies cancel out and the market price is competitive even with only a few participating households. In essence, households engage in demand reduction in a similar fashion as in Ausubel et al. (2014). Where households have invested in production plants and are net-sellers to the market, they however inflate their demand to push up market prices. Because we show that strategies are symmetric, demand reduction and demand inflation cancel out, insuring efficient pricing and allocation.

Furthermore, because equilibrium prices are competitive, households' investment incentives are efficient and yield aggregate market investment that maximizes welfare — if only one network exists. Conversely, if networks compete, network externalities can lead to inefficiencies. This is because households do not account for their positive externality on neighboring households, causing inefficient entry decisions into competing networks and deteriorating the efficiency of local power markets.

Our work contributes to several strands of research. First, our findings contribute to the large literature on electricity market architecture. Beginning with the deregulation of this sector, this literature has paid significant attention to the various levels of the traditional supply chain, such as on wholesale markets (Newbery, 1998; Wolfram, 1999; Borenstein et al., 2000; Fabra et al., 2006; Bushnell et al., 2008; Reguant, 2014; Schwenen, 2015; Holmberg and Wolak, 2018), retail markets and consumer behavior (Joskow and Tirole, 2006; Allcott, 2011; Allcott and Rogers, 2014; Giulietti et al., 2014), network regulation (Joskow, 2008; Tangerås, 2012), and, more recently, distributed generation (Brown and Sappington, 2017). We add to this literature by providing a model on the efficiency of local power markets, where strategic households simultaneously act as producers, consumers, and traders, and where we abstract from the canonical producer to consumer, wholesale to retail market architecture.

We also relate to the recent literature on the electrification of rural areas. Dinkelman (2011) finds positive effects on employment of a large grid connection plan in South Africa in the 1990's. Lee et al. (2020) provide experimental evidence on the effect of electrification in rural Kenya, and identify scale economies of connecting households to the power grid. Reporting evidence from rural India, Aklin et al. (2016) find that only a few hours of additional electricity supply increases household satisfaction substantially. We add with a theoretical study to this growing empirical literature on electrification. To our knowledge, no analytical model has so far been developed to understand efficiency and regulatory requirements for the electrification through local power markets.

Last, our model adds to the literature on bidding strategies in markets that follow double auction formats. Beginning with Wilson (1979) and Klemperer and Meyer (1989), this literature has started analyzing market interaction via demand and supply function equilibria. In many cases, the supply function framework has been used to study wholesale power markets (Green and Newbery, 1992; Baldick et al., 2004; Hortacsu and Puller, 2008; Hortaçsu et al., 2019). Recently, demand function equilibria have been used to model strategies of traders in sequential double auctions (Du and Zhu, 2017). While our model draws from the demand function equilibrium approach, we amend this framework to study peer-to-peer markets, which have previously been researched in the context of internet-based platforms such as Uber, eBay, and Airbnb (Einav et al., 2016).

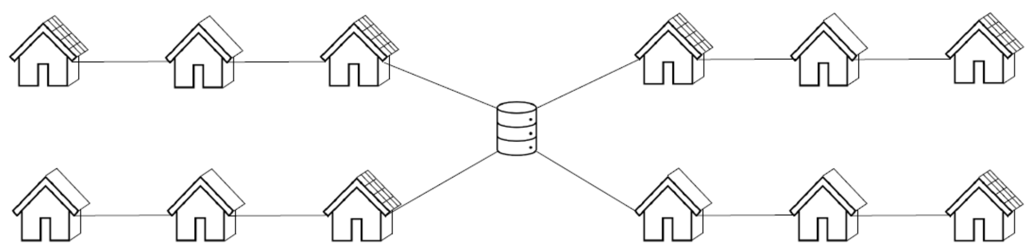
The remainder is organized as follows. Section two presents a model for local power markets. In section three, we study competition in the upstream market for solar modules and network technologies. Section four presents extensions. Section five concludes with policy implications.

2 A model of local power markets

Market participants and technology. Figure 1 shows the basic market environment: We study a local power market with n households. Each household has demand for electricity and can invest in generation assets (here solar plants) to generate and consume electricity. In case of excess electricity, each household can sell to all $n-1$ neighbors via a common micro-grid. Vice versa, when own generation capabilities are exhausted, each household can buy electricity from its neighbors. Depending on the amount of installed solar plants relative to aggregate demand, local power trade establishes an equilibrium price for electricity. Finally, one or several storage units can operate in the local market. Storage units buy and sell electricity with the aim to profit from arbitrage. In the following, we first focus on the interaction between households in this peer-to-peer market and introduce storage further below.

In expectation of the equilibrium price and resulting consumption utility and costs (profit) from buying (selling) electricity, households decide on their optimal amount of plants to be installed and connected to the local grid. We model the outside option from not buying plants as the costs of purchasing electricity from the local market. Hence, the outside option is endogenous to the investment and consumption choices of all neighbors.

Figure 1: *Schematic representation of local power market with solar plants and storage.*



The utility of each household i is denoted as $U_i(x_i, \varepsilon_i)$ and increases in power consumption x_i. In addition, utility depends on the realization of an idiosyncratic error term, ε_i, known only to household i. For this reason, the demand of neighboring households is unknown and the equilibrium power price is uncertain.

Local consumption and trade. We consider a market where households announce their demand for electricity, while supply is determined by all generating resources that are connected to the local grid. Hence, all production units are pooled, produce at full output, and cannot be used strategically. Finally, aggregate demand and supply determine the price for electricity, equilibrium consumption, and net-selling (buying) positions of each household.[1]

Households simultaneously decide on their consumption schedule $X_i(p, \varepsilon_i)$ that specifies their demand from the local grid at each price p. In equilibrium, each household consumes $X_i(p^*, \varepsilon_i)$ where p^* is the equilibrium price that equates supply and demand. Formally, the

[1]Detailed parameters for announcing demand may vary in practice. Households may submit time-variant demand functions or condition demand on other variables than price. We abstract from multi-dimensional demand schedules and focus on the standard case of price-quantity demand functions.

equilibrium price is given by

$$p^* : \sum X_i(p, \varepsilon_i) = \sum q_i, \tag{1}$$

with q_i being the installed generation capacity of household i. A household's profit from trading electricity becomes $p(q_i - X_i(p, \varepsilon_i))$. Consequently, the quasi-linear utility from consumption and trade is

$$U_i\left(X_i(p, \varepsilon_i), \varepsilon_i\right) + p(q_i - X_i(p, \varepsilon_i)). \tag{2}$$

Suppose that the idiosyncratic consumption shock ε_i is drawn from a distribution F prior to submitting demand schedules. The support of ε_i is equal for all households, finite with $\varepsilon_i \in [-\varepsilon_o, \varepsilon_o]$, and symmetric around $\mathbb{E}[\varepsilon_i] = 0$. Because the type of each neighboring household (i.e., their realized demand) and therefore the clearing price is unknown prior to announcing demand, strategies must be Bayesian-Nash optimal. Households must maximize expected utility and, before deciding on $X_i(p, \varepsilon_i)$, form an expectation on aggregate demand and the equilibrium price.

To capture the uncertainty in price, conditional on household i's demand function, we draw from the auction literature (Wilson, 1979; Hortacsu and Puller, 2008) and use the market clearing condition in equation (1) to map randomness from demand to price. Specifically, the distribution function of the equilibrium electricity price p^*, given household i's demand X_i at price p, becomes

$$\begin{aligned} H_i(p, X_i(p, \varepsilon_i)) &= Pr\left(p^* \leq p \mid X_i\right) \\ &= Pr\left(\sum_{j \neq i} X_j(p, \varepsilon_j) + X_i \leq \sum q_i \;\middle|\; X_i\right), \end{aligned} \tag{3}$$

where j indexes all households except household i. The distribution function H_i states the probability that $p^* \leq p$, this is, the probability that supply is larger than demand at this price. The support of H_i on $[\underline{p}, \bar{p}]$ depends on the support of all idiosyncratic demand shocks.

Using this probability measure, the expected utility of household i can be written as

$$EU_i = \int_{\underline{p}}^{\bar{p}} \left[U_i\left(X_i(p,\varepsilon_i),\varepsilon_i\right) + p(q_i - X_i(p,\varepsilon_i))\right] dH_i(p, X_i(p,\varepsilon_i)). \tag{4}$$

Households maximize expected utility by specifying optimal demand over the range of possible prices. In optimum, a household is indifferent between shifting demand from one possible price level to another. Formally, optimality is given by the Euler-Lagrange first order condition, which after rearranging yields:

$$\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p = (q_i - X_i(p,\varepsilon_i))\,\frac{H_{X_i}(p, X_i(p,\varepsilon_i))}{H_p(p, X_i(p,\varepsilon_i))}, \tag{5}$$

where H_{X_i} and H_p are the derivatives of H_i with respect to X_i and p.[2] To interpret the optimality condition in equation (5), note that H_p is the probability density function of price and must be positive. In contrast, H_{X_i} must be negative, because additional demand decreases the likelihood that the price is below any given value. Consequently, $\frac{H_{X_i}}{H_p} < 0$, and equation (5) shows that in equilibrium households that are net-sellers to the local market must have marginal utility from consumption below the market price. We summarize this finding in the following Proposition.

Proposition 1. *Households that are net-sellers to the local market mark-up sales above their marginal utility of consumption. Households that are net-buyers from the local market mark-down demand below their marginal utility of consumption, i.e.,*

$$\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} < p \quad \Longleftrightarrow \quad q_i > X_i(p,\varepsilon_i)$$

[2] We implicitly assume that for all q_i and ε_i we have $X_i > 0$ and $p \geq 0$.

$$\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} > p \iff q_i < X_i(p,\varepsilon_i).$$

Proof. The result follows from equation (5). A detailed proof is shown in Appendix A. □

The optimality condition suffices for computing the equilibrium demand strategies, given model primitives for household utility on the left hand side of equation (5). Notice that in addition the derivatives of H_i on the right hand side of equation (5) depend on the functional form of utility and its corresponding demand function. For this reason, equation (5) can only in few cases be evaluated analytically (Hortaçsu, 2011).

In proceeding, we assume utility that exhibits saturation and results in linear demand. We view linear demand schedules to be a good approximation for consumers that stick to simple rather than more complex, nonlinear demand heuristics when submitting their willingness to pay to the market.[3] Specifically, we assume utility of

$$U_i(x_i,\varepsilon_i) = (\theta_i+\varepsilon_i)x_i - \frac{1}{2}x_i^2, \tag{6}$$

with $\frac{\partial U_i(x_i,\varepsilon_i)}{\partial x_i} = \theta_i + \varepsilon_i - x_i$. The parameter θ_i represents a household's maximal willingness to pay (for $\varepsilon_i = \mathbb{E}[\varepsilon_i] = 0$) and can be viewed as a parameter that specifies household size. We rank households sizes as $\theta_1 > \theta_2 > ... > \theta_n$.

Given true willingness to pay of $\theta_i + \varepsilon_i - x_i$ as indicated above, we allow each household to shade its demand and announce a linear demand function of the form

$$X_i(p,\varepsilon_i) = \alpha_i + \beta_i\varepsilon_i - \gamma_i p, \tag{7}$$

where α_i, β_i, and γ_i are choice variables for each household. Put differently, households can hide their reservation value by announcing α_i instead of θ_i and can shade their sensitivity

[3] Linear bid functions are in line with solutions proposed by Baldick et al. (2004) on supply function equilibria. Also Du and Zhu (2017) focus on linear demand equilibria when analyzing traders who operate in sequential double auctions.

to the error term and price by choosing β_i and γ_i.

Recalling equation (5), the probability of affecting the market price by announcing higher or lower demand can now be written as $H_{X_i} = -1$. In addition, the density function of price yields $H_p = \sum \gamma_{-i}$ with $\sum \gamma_{-i} = \sum_{j \neq i} \gamma_j$. We provide detailed derivations in Appendix A. Substituting $\frac{H_{X_i}}{H_p}$ and equation (7) into the first order condition in (5) yields

$$\theta_i + \varepsilon_i - (\alpha_i + \beta_i \varepsilon_i - \gamma_i p) - p = (q_i - (\alpha_i + \beta_i \varepsilon_i - \gamma_i p)) \frac{-1}{\sum \gamma_{-i}}. \tag{8}$$

As shown in Appendix B, for $n \geq 3$ coefficient matching yields equilibrium demand functions $X_i^*(p, \varepsilon_i)$ with

$$\alpha_i = \frac{q_i + \theta_i \sum \gamma_{-i}}{1 + \sum \gamma_{-i}} \quad \text{and} \quad \beta_i = \gamma_i = \frac{\sum \gamma_{-i}}{1 + \sum \gamma_{-i}}. \tag{9}$$

This equilibrium is in stark contrast to truthful demand of $\alpha_i = \theta_i$ and $\beta_i = \gamma_i = 1$. As can be seen, bid shading for the reservation value, α_i, depends on the household's amount of solar plants. Moreover, the steepness of household i's demand function, γ_i, depends on the slope of other households' demand functions. This is intuitive because in equilibrium each household optimizes its demand schedule vis-à-vis the slope of its residual supply, which in turn is determined by the demand functions of its neighbors. This equilibrium feature reveals the complementarity in demand strategies: The more price-sensitive the demand of household i's neighbors becomes (the more $\sum \gamma_{-i}$ in equation (8) increases), the less can household i impact the market price and thus has little incentives to deviate from announcing true marginal utility. As stated in the following Proposition, this complementarity results in symmetric equilibrium demand.

Proposition 2. *In equilibrium, households submit symmetric demand functions, conditional on their size θ_i and installed generation capacity q_i. In markets with $n \geq 3$, the equilibrium demand function parameters are*

(1) $\gamma_i^* = \frac{n-2}{n-1}$

(2) $\beta_i^* = \frac{n-2}{n-1}$

(3) $\alpha_i^* = \frac{q_i+\theta_i(n-2)}{n-1}$.

With $n < 3$, no trade occurs in the local market and each household consumes its own electricity.

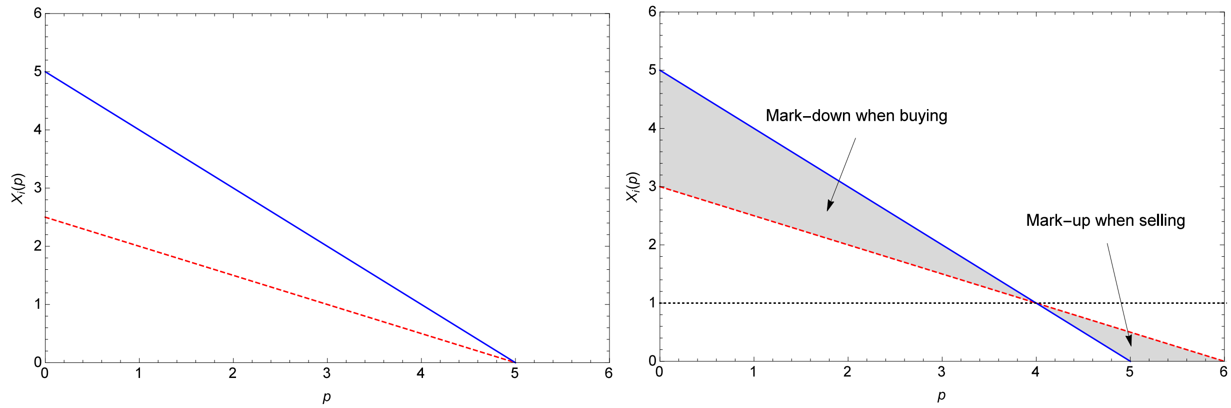
Proof. First, we show in Appendix B that trade ceases for $n < 3$. For $n \geq 3$ and given symmetry, rearrange γ_i from equation (9) to $\gamma_i = \frac{\sum \gamma_{-i}}{1+\sum \gamma_{-i}} = \frac{(n-1)\gamma_i}{1+(n-1)\gamma_i}$ $\forall$ i. Solving for γ_i yields $\gamma_i^* = \frac{n-2}{n-1}$. The equilibrium parameters α_i^* and β_i^* follow immediately. In Appendix C, we show that asymmetric strategies cannot exist if $\gamma_i > 0$. □

How does the magnitude of demand reduction change in market size, this is, as the number of households increases? From Proposition 2 it follows that $\lim_{n\to\infty} \alpha_i^* = \theta_i$ and $\lim_{n\to\infty} \beta_i^* = \lim_{n\to\infty} \gamma_i^* = 1$. Hence strategic demand shading ceases for a large number of neighbors. Notice that β_i and γ_i approach 1 from below. In addition, for any $q_i \in [0, \theta_i]$ (as reasonable to assume), α_i approaches θ_i from below.

Figure 2 illustrates equilibrium demand functions for three households so that $\beta_i = \gamma_i = \frac{1}{2}$. Panel (a) depicts true demand (solid line) and strategic demand (dashed line) for a representative household with $\theta_i = 4, \varepsilon_i = 1$, and $q_i = 0$. Panel (b) shows true demand (solid line), strategic demand (dashed line), and solar capacity (dotted horizontal line) for a household with $\theta_i = 4, \varepsilon_i = 1$, and $q_i = 1$.

As can be seen in panel (a), households that do not produce electricity strategically announce lower demand at any price. This strategy is similar to standard demand reduction equilibria (e.g., Ausubel et al., 2014), where bids for the first unit are equal to marginal utility and demand is more understated for each additional unit. As apparent in panel (b), also the household with $q_i = 1$ strategically reduces demand. However, as compared to the

Figure 2: *Demand functions for a local market with three households.*



(a) $\theta_i = 4, \varepsilon_i = 1, q_i = 0$ (b) $\theta_i = 4, \varepsilon_i = 1, q_i = 1$

demand curve of its neighbors with no solar output, this household shifts demand upward in a parallel fashion, thereby increasing the market price to its favor. For all $X_i(p, \varepsilon_i) > q_i = 1$ this household demands electricity at prices lower than marginal utility, while for $X_i(p, \varepsilon_i) < q_i = 1$ it is willing to sell electricity at a mark-up on marginal utility.[4]

Using Proposition 2 and equation (7), the equilibrium demand schedule becomes

$$X_i^*(p, \varepsilon_i) = \frac{n-2}{n-1}(\theta_i + \varepsilon_i - p) + \frac{1}{n-1} q_i, \tag{10}$$

and the market clearing condition in (1) yields

$$\sum_i^n \left(\frac{n-2}{n-1}(\theta_i + \varepsilon_i - p) + \frac{1}{n-1} q_i \right) = \sum_i^n q_i. \tag{11}$$

The equilibrium price is

$$p^* = \frac{1}{n} \sum_{i=1}^n (\theta_i + \varepsilon_i - q_i). \tag{12}$$

[4]Figure 2 in addition shows that for trade to take place, the market price must be above the price at which at least for one household $X_i(p, \varepsilon_i) < q_i$ holds. In turn, buying households must have marginal utility higher than this price.

and depends only on market fundamentals. We summarize this finding in the next corollary.

Corollary 1. *The equilibrium market price is independent of demand reduction strategies and only depends on market fundamentals for demand, θ_i and ε_i, and supply, q_i of all households $i = 1, .., n$. Demand functions determine market shares in consumption at the competitive market price.*

The equilibrium price is independent of bid shading, because strategies of net-buying and net-selling households cancel out. To see this, consider the case where $n-1$ households have zero supply and only one household i generates electricity. In equilibrium, the market supply from household i of $q_i - X_i(p, \varepsilon_i)$ must equal demand of the $n-1$ buying households, $\sum X_{-i}(p, \varepsilon_{-i})$. The demand reduction of the buying households will be exactly offset by the selling household. This requires that the selling household reduces its supply by announcing higher demand and consuming more electricity. The household consumes more electricity, because the cost of consumption declines when at the same time the price of its supply increases.

Finally, the expected power price follows from equation (12) and when recalling that $\mathbb{E}[\varepsilon_i] = 0$. Whereas the expected power price is always competitive, investment still matters for households, because the distribution of generation assets among households determines their consumption shares.

Investment incentives and network effects. Next, we derive investment incentives and investigate whether network externalities arise in local power markets. We consider investment to take place prior to market clearing and prior to announcing demand. At the investment stage, households consequently do not know their realized demand shock ε_i. In addition, households have to form a prior on the demand shock of their neighbors.

To separate out the different demand shocks, define $\sum_{j \neq i} \varepsilon_j := \Psi_i$ with $g_i(\Psi_i)$ being the density function of Ψ_i. Recalling that $\varepsilon_i \in [-\varepsilon_o, \varepsilon_o]$, g must be distributed in $[-(n-$

$1)\varepsilon_o, (n-1)\varepsilon_o]$. Using this definition, the equilibrium price in (12) can be rewritten as $p^* = \frac{1}{n}[\Psi_i + \varepsilon_i + \sum_i (\theta_i - q_i)]$. Household i finds its optimal investment by maximizing expected utility in (4) net of investment costs, weighted over all possible demand shocks:

$$\mathbb{E}[EU_i] = \int_{-\varepsilon_o}^{\varepsilon_o} \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} \left[U_i(X_i(p^*, \varepsilon_i), \varepsilon_i) + p^*(q_i - X_i(p^*, \varepsilon_i))\right] g(\Psi_i) d\Psi_i f(\varepsilon_i) d\varepsilon_i - p_s q_i \quad (13)$$

where p_s is the market price for solar units. The first order condition for household i's optimal investment choice becomes[5]

$$q_i = \theta_i - (n-1)p_s - \frac{n-2}{n} \sum_i (q_i - \theta_i)\,. \quad (14)$$

Equation (14) shows that individual investment in solar plants is a strategic substitute to aggregate market investment. Summing up the optimality condition over all n households, we obtain $\sum q_i = \sum \theta_i - n\left((n-1)p_s - \frac{n-2}{n}\sum(q_i - \theta_i)\right)$. Solving this expression for $\sum q_i$ yields aggregate equilibrium investment of

$$\sum_i q_i^* = \sum_i \theta_i - np_s. \quad (15)$$

Last, when substituting the aggregate equilibrium investment in (15) into the expression for optimal investment of household i in (14), rearranging yields $q_i^* = \theta_i - p_s$. This solution is straightforward: each household only invests in solar plants as long as its maximum valuation for electricity, θ_i, is above the price of solar plants. We summarize this finding in the next Proposition.

Proposition 3. *Household i's equilibrium investment in generation assets is* $q_i = \theta_i - p_s$.

Proof. The result follows from equation (15) and is derived in full in Appendix D. □

[5] We focus on interior solutions.

It follows that larger households contribute with relatively more supply to the local market.

Corollary 2. *Individually optimal investment maximizes welfare.*

Corollary 2 follows from evaluating $\frac{\sum_i \mathbb{E}[EU_i]}{\partial q_i} = 0$, which parallels the solution in equation (15). We formally proof this result in Appendix D.

Using Proposition 3 and equation (12), the equilibrium power price —given optimal investment— eventually becomes $p^* = \frac{1}{n}\sum_i(\varepsilon_i + p^s)$. With $E[\varepsilon_i] = 0$, the expected power price of the local market simply is

$$\mathbb{E}[p^*] = p_s, \tag{16}$$

implying that, for optimal investment levels, the expected electricity price in local power markets equals the costs of generation assets.

Finally, substituting the equilibrium investment into equation (13), we obtain the expected utility of household i as

$$\mathbb{E}[EU_i^*] = \frac{1}{2}\left((\theta_i - p_s)^2 + \frac{n-2}{n-1}\sigma\right), \tag{17}$$

where σ denotes the variance of the demand shock, $\mathbb{E}[\varepsilon_i^2]$. The first expression represents consumer surplus, while the second expression signals positive network effects.[6] Network effects exist, because the expected utility depends positively on the total number of market participants.

Corollary 3. *Positive network externalities exist in local power markets. A household's utility increases the more households participate in the local market.*

As a result, positive network externalities make larger networks relatively more attractive to consumers, and competing networks can harm welfare if technologies are not compatible.

[6]For instance, with ε_i being uniformly distributed and Ψ_i following an Irwin-Hall distribution, expected utility equals $\frac{1}{2}\left(\theta_i - p_s\right)^2 + \frac{n-2}{6(n-1)}\varepsilon_0^2$.

We explore the welfare implications of suppliers with different network technologies in the next section.

3 Competition among network suppliers

We use this section to show how differentiated products in the upstream market for solar modules can deteriorate the positive efficiency results we have obtained so far. As argued, technologies can follow alternating current (AC) or direct current (DC) standards. On the one hand, the provision of solar plants connected to high-quality AC grids can increase welfare. On the other hand, a separated market where some households remain in DC networks while others opt into separate AC networks can downsize positive network externalities. In what follows, we investigate this welfare trade-off and explore conditions that yield optimal welfare when networks with different technologies compete.

To fix ideas, assume that there exists a high-quality technology (e.g. solar plants and grid equipment that use AC technology) and a less expensive but inferior technology (e.g. solar plants and grid equipment that use DC technology). The two technologies are not compatible so that the market may potentially separate. We introduce the difference in quality by stating

$$\theta_i \equiv \begin{cases} \theta_i & \text{if using high-quality technology} \\ \omega\theta_i & \text{if using low-quality technology} \end{cases} \tag{18}$$

with $0 < \omega < 1$. The high-quality technology is available at price p_s^H while the low-quality technology costs p_s^L per module. Prices are competitive and each local market is a price-taker. Furthermore, we assume that the choice between either technology in a local market does not change the "global" relative prices for each technology.

Participation incentives. Denote the number of households in the high-quality network as m. From equation (17), the expected utility of household i in the high-quality network becomes

$$\mathbb{E}[EU_i^H] = \frac{1}{2}\left(\left(\theta_i - p_s^H\right)^2 + \frac{m-2}{m-1}\sigma\right). \tag{19}$$

The competing low-quality network hence consists of $n - m$ households and the expected utility of household i when being part of the low-quality network becomes

$$\mathbb{E}[EU_i^L] = \frac{1}{2}\left((\omega\theta_i - p_s^L)^2 + \frac{n-m-2}{n-m-1}\sigma\right). \tag{20}$$

To identify market shares and resulting network externalities, we first characterize the participation constraint of the pivotal household m. The ranking of household sizes $\theta_1 > \theta_2 > ... > \theta_n$ implies that if household m is the smallest household in the high-quality network, then all larger households with $\theta_i \geq \theta_m$ must prefer the high-quality network, while all smaller households with $\theta_i < \theta_m$ prefer the less expensive low-quality supply.

If household m joins the high-quality network, it has expected utility of

$$\mathbb{E}[EU^H(\theta_m)] = \frac{1}{2}\left(\left(\theta_m - p_s^H\right)^2 + \frac{m-2}{m-1}\sigma\right). \tag{21}$$

Conversely, if the same household m switches and joins the competing network with inferior technology, it expects utility of

$$\mathbb{E}[EU^L(\theta_m)] = \frac{1}{2}\left((\omega\theta_m - p_s^L)^2 + \frac{n-(m-1)-2}{n-(m-1)-1}\sigma\right). \tag{22}$$

Notice that the above accounts for household m switching networks as, upon the change of household m, the low-cost network has $n - (m - 1)$ participants.

The participation constraint (PC) for household m to choose the high-quality network must therefore satisfy $\mathbb{E}[EU^H(\theta_m)] - \mathbb{E}[EU^L(\theta_m)] \geq 0$ or written in full notation

$$PC(m) \equiv \frac{1}{2}\left(\left(\theta_m - p_s^H\right)^2 - (\omega\theta_m - p_s^L)^2 + \frac{n+1-2m}{(m-1)(m-n)}\sigma\right) \geq 0. \tag{23}$$

Welfare. To characterize efficient and inefficient market outcomes, we compare the participation incentives of each household with aggregate welfare across all households. Welfare can be written as the sum of utilities, including network effects, for all households in each network. In defining welfare we again use that household m is the household that joins the high-quality network "at the margin". This is, welfare equals

$$W(m) = \frac{1}{2}\left(\sum_{i=1}^{m}\left[(\theta_i - p_s^H)^2 + \frac{m-2}{m-1}\sigma\right] + \sum_{i=m+1}^{n}\left[(\omega\theta_i - p_s^L)^2 + \frac{n-m-2}{n-m-1}\sigma\right]\right). \tag{24}$$

Because the number of households in each network is discrete, we compute the optimality condition for welfare by taking first differences: Welfare when household $m-1$ instead of household m is marginal becomes

$$W(m-1) = \frac{1}{2}\left(\sum_{i=1}^{m-1}\left[(\theta_i - p_s^H)^2 + \frac{m-3}{m-2}\sigma\right] + \sum_{i=m}^{n}\left[(\omega\theta_i - p_s^L)^2 + \frac{n-m-1}{n-m}\sigma\right]\right) \tag{25}$$

and subtracting (25) from (24) yields the discrete welfare derivative

$$\frac{\Delta W}{\Delta m} = \frac{1}{2}\left((\theta_m - p_s^H)^2 - (\omega\theta_m - p_s^L)^2 + \frac{(n-2)(1-2m+n)}{(m-2)(m-1)(m-m)(1+m-n)}\sigma\right). \tag{26}$$

The first two terms represent the change in utility for household m. The last term represents the change in relative network effects caused by household m, i.e., the difference between network externalities that the high-quality network gains and the low-quality network loses as m switches sides.

Note that the above two welfare expressions in (24) and (25) consider cases where both markets always co-exist. Recalling Proposition 2, this is true where each market consists of at least three participants so that trade never ceases, neither for $W(m)$ nor for $W(m-1)$, and network externalities always realize. We provide welfare analyses for the extreme cases where the change of household m to another network ends trade in household m's former network in Appendix E.

Finally, we illustrate the inefficient misalignment between market outcomes and welfare by subtracting the participation constraint in (23) from the welfare derivative above in equation (26) and arrive at

$$\frac{\Delta W}{\Delta m} - PC(m) = \frac{2m-n-1}{2(m-2)(1+m-n)}\sigma \quad \begin{cases} < 0 & \text{if } m > \frac{n+1}{2} \wedge \sigma \neq 0 \\ = 0 & \text{if } m = \frac{n+1}{2} \vee \sigma = 0 \\ > 0 & \text{if } m < \frac{n+1}{2} \wedge \sigma \neq 0. \end{cases} \tag{27}$$

As apparent, if $\frac{\Delta W}{\Delta m} - PC(m) < 0$, the incentives for household m to switch to the high-quality network are larger than the resulting welfare gains, because household m does not account for the foregone positive network effects when departing from the low-quality network. This is the case when $m > \frac{n+1}{2}$, so when the high-quality network is relatively larger.[7] In contrast, if $\frac{\Delta W}{\Delta m} - PC(m) > 0$, household m's incentives to switch to the high-quality network are less than the overall welfare gain. Taken together, equation (27) implies that larger networks are too large and smaller network are too small. Finally, if networks are of equal size and household m is the median household, losses and gains from externalities cancel out at the margin for $m = \frac{n+1}{2}$. We summarize these results in the next Proposition.

Proposition 4. *Local power markets with competing networks*

(1) maximize welfare if no network externalities exist or networks are of equal size

(2) fail to internalize network externalities if networks are of different size; the larger

[7] This result is due to declining marginal network externalities, $\frac{\partial^2 \frac{m-2}{m-1}\sigma}{\partial m^2} = -\frac{2}{(m-1)^3}\sigma < 0$, so that incremental losses from a small network are larger than incremental gains to a bigger network.

network is too large

(3) maximize positive externalities if only one network type exists.

Proof. The first two statements follow from equation (27). The third statement follows immediately when markets do not separate. Results for the extreme cases where networks cease to exist are presented in Appendix E. □

Another implication of Proposition 4 is that while positive externalities are maximized for $m = 0$ and $m = n$, welfare can decline temporarily as networks grow towards either corner solution, because households in shrinking networks suffer relatively large losses in utility. Furthermore, for statement (3) of Proposition 4, notice that the type of network that prevails in equilibrium, i.e., $m = 0$ or $m = n$, will generally depend on the expectations of households (Katz and Shapiro, 1985). However, it is straightforward to show that if households are all relatively large, the high-quality network must prevail and expectations are irrelevant. Too see this, consider the case of $\sigma = 0$, where household i's decision between the networks reduces to comparing utilities of $\frac{1}{2}(\theta_i - p_s^H)^2 \gtrless \frac{1}{2}(\omega\theta_i - p_s^L)^2$. Rearranging yields $(1-\omega)\theta_i \gtrless p_s^H - p_s^L$, so that for sufficiently high θ_i the gain in utility outweighs the price difference, and household i prefers the high-quality network. A similar condition is easily obtained for $\sigma > 0$, where critically large households prefer the high-quality network, while households that are sufficiently small opt into the low-quality network.

Corollary 4. *Local power markets that consist of sufficiently small (large) households are efficient as only one low-quality (high-quality) network exists.*

In sum, the results of this section illustrate that the efficiency of local power markets can deteriorate when market participants are too heterogeneous and networks with different technology standards co-exist.

4 Storage

In closing, we extend the model and introduce storage capacities. Storage capacities allow for buying production during low price realizations and selling stored generation in times of high demand. Especially where generation stems from fluctuating solar plants, storage typically accompanies the architecture of local power markets.

Our main result of this section is that storage does not change our positive efficiency results. Instead, storing devices cushion demand shocks and reduce price volatility. To incorporate storage activity, we model market participants whose sole intention is arbitrage. A storage participant bids into the market at the same time as households do, demanding power whenever the clearing price is below the expected market price, and supplying power to the grid whenever the realized price is above its expectation. Formally, the supply function of storage can be written as

$$X_s(p) = s(p - \mathbb{E}[p]). \tag{28}$$

Notice that storage as additional buyer (seller) in the market only impacts the expected utility of households via the equilibrium market price p^* and its distribution function H. In detail, the distribution of the clearing price with storage is

$$\begin{aligned} H(p, X_i(p, \varepsilon_i)) &= Pr\left(p^* \leq p \mid X_i\right) \\ &= Pr\left(\sum_{j \neq i} X_j(p, \varepsilon_j) + X_i(p, \varepsilon_i) - X_s(p) \leq \sum q_i \;\middle|\; X_i\right). \end{aligned} \tag{29}$$

Using this price distribution, the equilibrium strategy of household i for buying or selling on the local market (see Proposition 2) changes to

$$\alpha_i = \frac{q_i + \theta_i \sum(s + \gamma_{-i})}{1 + s + \sum \gamma_{-i}} \quad \text{and} \quad \beta_i = \gamma_i = \frac{s + \sum \gamma_{-i}}{1 + s + \sum \gamma_{-i}}. \tag{30}$$

Importantly, strategies remain symmetric and the resulting equilibrium price remains a function of market fundamentals, which now relate to demand, supply, and storage capacity. However, storage changes the distribution of realized equilibrium prices. In Appendix F, we show that the realized equilibrium power price with storage becomes

$$p^* = \sum_{i=1}^{n} \left(\frac{1}{n}(\theta_i - q_i) + \tau \varepsilon_i \right) \tag{31}$$

with

$$\tau = \frac{2 + n + s - \sqrt{(n-2)^2 + 2ns + s^2}}{2(2n + s)} < \frac{1}{n}. \tag{32}$$

Because $\tau < \frac{1}{n}$, price shocks in either direction caused by net positive or net negative aggregate demand shocks are cushioned as compared to the no-storage equilibrium price of $\sum_{i=1}^{n} \frac{1}{n} (\theta_i - q_i + \varepsilon_i)$. The expected equilibrium price does not change with storage and remains $p^* = \sum_{i=1}^{n} \frac{1}{n}(\theta_i - q_i)$. This is because in expectation storage does not operate, and only buys (sells) if realized prices are below (above) expectation. As a result, with unchanged expected power prices it follows that storage does not change optimal investment incentives.

5 Conclusion

The provision of electricity is increasingly organized in local power markets. In this article, we have provided a model to study pricing behavior, investment incentives, and welfare in local power markets.

We have derived a set of positive efficiency results. First, local power markets can provide electricity at competitive prices. Importantly, this result holds for a small number of participating households and despite strategic demand reduction. Furthermore, combining generation and trading possibilities in local peer-to-peer markets yields efficient investment incentives. Each household's optimal investment guarantees that total welfare is maximized.

However, we show that these positive efficiency results can deteriorate due to network externalities that, with heterogeneous households, can lead to inefficient entry into competing networks.

The results of this article point to the efficiency of local power markets, and underscore the importance of monitoring the upstream market for generation and network technologies to make households benefit from the shift towards local generation and peer-to-peer trading in electricity markets.

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Appendix

A Equilibrium demand functions

Integrating (4) by parts yields

$$\begin{aligned} EU_i &= \int_{\underline{p}}^{\overline{p}} \left[U_i\left(X_i(p,\varepsilon_i),\varepsilon_i\right) + p(q_i - X_i(p,\varepsilon_i))\right] dH(p, X_i(p,\varepsilon_i))dp \\ &= \left[U_i\left(X_i(p,\varepsilon_i),\varepsilon_i\right) + p(q_i - X_i(p,\varepsilon_i))\right] H(p, X_i(p,\varepsilon_i))\Big|_{\underline{p}}^{\overline{p}} \\ &- \int_{\underline{p}}^{\overline{p}} \frac{d}{dp}\left[U_i\left(X_i(p,\varepsilon_i),\varepsilon_i\right) + p(q_i - X_i(p,\varepsilon_i))\right] H(p, X_i(p,\varepsilon_i))dp. \end{aligned}$$

Because $H(\underline{p}) = 0$ and $H(\bar{p}) = 1$ we obtain

$$\begin{aligned} EU_i = &\, U_i\left(X_i(\overline{p},\varepsilon_i),\varepsilon_i\right) + \overline{p}(q_i - X_i(\overline{p},\varepsilon_i)) \\ &- \int_{\underline{p}}^{\overline{p}} \left[\left(\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p\right) X_i'(p,\varepsilon_i) + q_i - X_i(p,\varepsilon_i)\right] H(p, X_i(p,\varepsilon_i))dp. \end{aligned}$$

Therefore households maximize

$$\max_{X_i(p,\varepsilon_i)} \left[U_i\left(X_i(\overline{p},\varepsilon_i),\varepsilon_i\right) + \overline{p}(q_i - X_i(\overline{p},\varepsilon_i)) - \int_{\underline{p}}^{\overline{p}} \mathcal{L}(p, X_i(p,\varepsilon_i), X_i'(p))dp\right]$$

with

$$\mathcal{L}(p, X(p), X_i'(p)) := \left[\left(\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p\right) X_i'(p) + q_i - X_i(p,\varepsilon_i)\right] H(p, X_i(p,\varepsilon_i)).$$

With free salvage value (Kamien and Schwartz, 2012), the first order condition for an unspecified $X_i(\overline{p},\varepsilon_i)$ becomes

$$\frac{d}{dp}\mathcal{L}_{X'} = \mathcal{L}_X$$

with

$$\left(\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p\right) - \mathcal{L}_{X'} = 0 \text{ for } p = \overline{p}.$$

Computing the derivatives yields

$$\mathcal{L}_{X'} = \left(\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p\right) H(p, X_i(p,\varepsilon_i))$$

$$\mathcal{L}_X = H_{X_i}\left[\left(\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p\right)X_i'(p,\varepsilon_i) + q_i - X_i(p,\varepsilon_i)\right] + \left[\frac{\partial^2 U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i^2}X_i'(p,\varepsilon_i) - 1\right]H(p, X_i(p,\varepsilon_i))$$

and

$$\frac{d}{dp}\mathcal{L}_{X'} = (H_p(p, X_i(p,\varepsilon_i)) + H_{X_i}(p, X_i(p,\varepsilon_i))X_i'(p,\varepsilon_i))\left(\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p\right) + \left(\frac{\partial^2 U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i^2}X_i'(p) - 1\right)H(p, X_i(p,\varepsilon_i)).$$

Using the above and rearranging $\frac{d}{dp}\mathcal{L}_{X'} = \mathcal{L}_X$ yields equation (5) in the main text

$$\frac{\partial U_i(X_i(p,\varepsilon_i),\varepsilon_i)}{\partial X_i} - p = (q_i - X_i(p,\varepsilon_i))\frac{H_{X_i}(p, X_i(p,\varepsilon_i))}{H_p(p, X_i(p,\varepsilon_i))}.$$

For $p = \overline{p}$ it must hold that

$$\left(\frac{\partial U_i(X_i(\overline{p},\varepsilon_i),\varepsilon_i)}{\partial X_i} - \overline{p}\right) - \mathcal{L}_{X'} = \left(\frac{\partial U_i(X_i(\bar{p},\varepsilon_i),\varepsilon_i)}{\partial X_i} - \overline{p}\right) - \left(\frac{\partial U_i(X_i(\overline{p},\varepsilon_i),\varepsilon_i)}{\partial X_i} - \overline{p}\right) = 0.$$

Finally, to obtain the derivatives of the price distribution H_i, use the demand function $X_i(p,\varepsilon_i) = \alpha_i + \beta_i\varepsilon_i - \gamma_i p$ and rearrange equation (3) in the main text as

$$\begin{aligned} H_i(p, X_i(p)) &= Pr\left(p^* \le p \mid X_i\right) \\ &= Pr\left(\sum_{j\neq i} X_j(p,\varepsilon_j) + X_i \le \sum q_i \;\middle|\; X_i\right) \\ &= Pr\left(\sum_{j\neq i}(\alpha_j + \beta_j\varepsilon_j - \gamma_j p) + X_i \le \sum q_i \;\middle|\; X_i\right) \\ &= Pr\left(\sum_{j\neq i}\beta_j\varepsilon_j \le \sum q_i - \sum_{j\neq i}(\alpha_j - \gamma_j p) - X_i \;\middle|\; X_i\right). \end{aligned}$$

Differentiating with respect to X_i yields -1 and differentiating with respect to p gives $\sum_{j\neq i}\gamma_j = \sum \gamma_{-i}$. When using these derivatives for equation (5) we obtain equation (8) in the main text

$$\theta_i + \varepsilon_i - (\alpha_i + \beta_i\varepsilon_i - \gamma_i p) - p = (q_i - (\alpha_i + \beta_i\varepsilon_i - \gamma_i p))\frac{-1}{\sum \gamma_{-i}}.$$

B Coefficient matching

Applying coefficient matching to solve

$$\theta_i + \varepsilon_i - (\alpha_i + \beta_i \varepsilon_i - \gamma_i p) - p = (q_i - (\alpha_i + \beta_i \varepsilon_i - \gamma_i p)) \frac{-1}{\sum \gamma_{-i}}$$

yields the conditions

$$0 = \varepsilon_j - \varepsilon_j \beta_j - \frac{\varepsilon_j \beta_j}{\sum \gamma_{-i}}$$

$$0 = -p + p\gamma_j + \frac{p\gamma_j}{\sum \gamma_{-i}}$$

$$0 = -\alpha_j + \theta_j + \frac{q_j - \alpha_j}{\sum \gamma_{-i}}.$$

with solutions as in Proposition 2. The solution for $n < 3$ is trivial with $\alpha_i = q_i$ and $\beta_i = \gamma_i = 0$. To see this solution, multiply both sides of the optimality condition by $\sum \gamma_{-i}$ and plug in $(\alpha_i, \beta_i, \gamma_i) = (q_i, 0, 0)$. For $n < 3$, this is the only equilibrium candidate that survives. In this equilibrium, all trade breaks down and each household consumes its own production. In this case the stand-alone utility of household i is straightforward to calculate as

$$EU_i = \int_{-\varepsilon_o}^{\varepsilon_o} \left[(\theta_i + \varepsilon_i) q_i - \frac{1}{2} q_i^2 \right] f(\varepsilon_i) d\varepsilon_i - p_s q_i = \theta_i q_i - \frac{1}{2} q_i^2 - p_s q_i$$

which is maximized at $q_i^* = \theta_i - p_s$, providing utility of $\frac{1}{2}(\theta_i - p_s)^2$.

C Symmetry

We show that only symmetric strategies exist for all positive γ_i. We proceed by induction arguments. Consider two households i and j, plus the remaining $(n-2)$ households with (ex-ante possibly distinct) γ_k. From the derivation of bidding strategies we know that $\gamma_i = \frac{\sum \gamma_{-i}}{1 + \sum \gamma_{-i}}$. Hence, we can write the optimality conditions for each household in equation (9) as

$$0 = \gamma_i - \frac{\gamma_j + \sum_{-i,-j}^{n-2} \gamma_k}{1 + \gamma_j + \sum_{-i,-j}^{n-2} \gamma_k}$$

$$0 = \gamma_j - \frac{\gamma_i + \sum_{-i,-j}^{n-2} \gamma_k}{1 + \gamma_i + \sum_{-i,-j}^{n-2} \gamma_k}$$

and solve for γ_i and γ_j. The only solution with positive γ_i is for $\gamma_i = \gamma_j$. Hence $s = 2$ households (i and j) exist with symmetric strategies $\gamma_i = \gamma_j$. Now taking one additional household l out of $\sum_{-i,-j}^{n-2} \gamma_k$ we can write

$$0 = \gamma_i - \frac{(s-1)\gamma_i + \gamma_l + \sum_{-i,-l}^{n-3} \gamma_k}{1 + (s-1)\gamma_i + \gamma_l + \sum_{-i,-l}^{n-3} \gamma_k}$$

$$0 = \gamma_l - \frac{s\gamma_i + \sum_{-i,-l}^{n-3} \gamma_k}{1 + s\gamma_i + \sum_{-i,-l}^{n-3} \gamma_k}$$

with $s = 2$. We again solve for γ_i and γ_l and again find only one solution with positive parameters, which is the symmetric solution. Continuing with $\gamma_i = \gamma_j = \gamma_l$ and $s = 3$ yields one additional symmetric household. Continue until $s = n - 1$ for which all households' γ_i are symmetric.

D Equilibrium investment

Starting from expected utility

$$\mathbb{E}[EU_i] = \int_{-\varepsilon_o}^{\varepsilon_o} \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} \left[U_i(X_i(p^*), \varepsilon_i) + p^*(q_i - X_i(p^*))\right] g(\Psi_i) d\Psi_i f(\varepsilon_i) d\varepsilon_i - p_s q_i$$

we take the first order condition and arrive at

$$\int_{-\varepsilon_o}^{\varepsilon_o} \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} \left[\frac{2(n-1)(\theta_i + \varepsilon_i - q_i) + (n-2)\left(\sum \theta_{-i} + \Psi_i - \sum q_{-i}\right)}{n(n-1)}\right] g(\Psi_i) d\Psi_i f(\varepsilon_i) d\varepsilon_i = p_s$$

We then solve the inner integral by parts using that $G(-(n-1)\varepsilon_o) = 0$ and $G((n-1)\varepsilon_o) = 1$. Further, we use that for a symmetric PDF g with a mean of zero, the anti-derivative of its CDF, $\overline{G}(\Psi_i) = \int G(\Psi_i) d\Psi_i$, evaluated at the bound of the support yields $\overline{G}((n-1)\varepsilon_o) = \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} G(\Psi_i) d\Psi_i = \Psi_i G(\Psi_i)|_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} - \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} \Psi_i g(\Psi_i) d\Psi_i = (n-1)\varepsilon_o$ and $\overline{G}(-(n-1)\varepsilon_o) = 0$. Using a corresponding procedure for the outer integral yields equation (14) in the main text.

To find investment that maximizes welfare we evaluate $\frac{\sum_i \mathbb{E}[EU_i]}{\partial q_i} = 0$ and obtain

$$q_i = \theta_i - (n-1)^2 p_s - (n-2) \sum_i (q_i - \theta_i).$$

Note that this expression differs from the optimal investment that maximizes only household i's expected utility. Summing over all households we obtain the aggregate investment

$$\sum q_i = n\left(\theta_i - (n-1)^2 p_s - (n-2)\sum_i (q_i - \theta_i)\right).$$

When solving for $\sum q_i$ we again obtain

$$\sum_i q_i = \sum_i \theta_i - np_s,$$

as for the case when household i instead maximizes its own expected utility rather than welfare.

E Welfare with separated networks

Welfare results in Proposition 4 are for markets with at least three participants in either network. Below, we show that the result that larger markets are inefficiently large extends to the case where large markets entirely crowd out trade in smaller markets. To begin, consider the case where the high-quality market grows from $n-m=3$ to $n-m=2$, ending trade in the low-quality network. Welfare for $n-m=2$ is

$$W(n-m=2) = \frac{1}{2}\left(\sum_{i=1}^{m}\left[(\theta_i - p_s^H)^2 + \frac{m-2}{m-1}\sigma\right] + \sum_{i=m+1}^{n}(\omega\theta_i - p_s^L)^2\right).$$

For $n-(m-1)=3$ instead the two networks still co-exist and welfare becomes

$$W(n-(m-1)=3) = \frac{1}{2}\left(\sum_{i=1}^{m-1}\left[(\theta_i - p_s^H)^2 + \frac{m-3}{m-2}\sigma\right] + \sum_{i=m}^{n}\left[(\omega\theta_i - p_s^L)^2 + \frac{n-m-1}{n-m}\sigma\right]\right).$$

Differencing yields the welfare gain for moving from $n-m=3$ to $n-m=2$ of

$$\frac{\Delta W}{\Delta m} = \frac{1}{2}\left((\theta_m - p_s^H)^2 - (\omega\theta_m - p_s^L)^2 + \left(1+m-n+\frac{1}{1-m}+\frac{1}{m-2}+\frac{1}{n-m}\right)\sigma\right).$$

The participation constraint for household m at $n-m=3$ to join the high-quality network instead of remaining the the low-quality network is the same as in the main text. Finally,

$\frac{\Delta W}{\Delta m} - PC(m=3) = \frac{2n-1+m^2-m(1+n)}{2(m-2)}\sigma < 0 \; \forall \; m \in [3, n-2]$, implying that the larger network entirely crowds out the smaller network, where all trade ends. The marginal household does not internalize this effect when moving to the larger network. The same argument applies when the low-quality network entirely crowds out the high-quality network, so for moving from $m = 3$ to $m = 2$.

F Demand functions with storage

Starting from the equilibrium strategies in equation (30), one easily obtains the equilibrium aggregate market demand $\sum_i X_i(p, s, \varepsilon_i)$ analogous to the left hand side of equation (11) for the case without storage. First note that the expected market demand is $\sum_i X_i(p, s, 0)$. When solving the market clearing condition $\sum_i X_i(p, s, 0) - X_s(p) = \sum_i q_i$ for the equilibrium price, we obtain the expected market price with storage as $\mathbb{E}[p^*] = \sum_i \frac{1}{n}(\theta_i - q_i)$, i.e., storage does not operate in expectation. Using the expected price in the storage supply function in equation (28) in the main text yields $X_s(p) = s(p - \mathbb{E}[p]) = s(p - \sum_i \frac{1}{n}(\theta_i - q_i))$. In turn, this allows to solve for the realized price $\sum_i X_i(p, s, \varepsilon_i) - X_s(p) = \sum_i q_i$ which yields equation (31) in the main text.