Income-Dependent Equivalence Scales and Choice Theory
Implications for Poverty Measurement

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Income-Dependent Equivalence Scales and Choice Theory: Implications for Poverty Measurement

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Abstract

Equivalence Scales are a tool for removing the heterogeneity of household sizes in the measurement of inequality, and affect poverty assessments and poverty lines. We address the disadvantage that poor households may suffer due to their reduced ability to share goods within the household. This disadvantage is important to estimate and embed in standard analysis, as it seems to have a substantial quantitative impact on the measurement of poverty. We also suggest that future research on the role of subsistence incomes of different household types in utility functions may shed light on explanations for poverty and may guide anti-poverty policies.

Keywords: Equivalent incomes, household-size economies, inequality, demographics and poverty, child costs, Generalized Equivalence Scale Exactness

JEL Classification: I32, D14, D63, D15

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Introduction

The study of poverty is at the heart of economics, and the goal of overcoming it drives efforts of policy-makers worldwide. Meeting such goals requires confidence (a) in the tools we have to measure poverty, and (b) in our understanding of the determinants of poverty. Here, we focus on the role of household composition in the measurement and analysis of poverty. After presenting some core concepts, we suggest new directions for further research. We offer a survey of evidence showing that, compared to richer households, poorer multimember households may have a disadvantage in obtaining and sharing public within-household goods, such as housing, heating, and means of transportation. Therefore, commonly accepted tools for measuring poverty may underestimate its intensity and may hide some of its key determinants.

Equivalence scales: The core concept for linking household composition with income distribution and poverty

Equivalence scales (ESs) measure the extent to which multimember households benefit from sharing goods within the household, and measure differences in needs between household members (e.g., adults and children). As most income (and wealth) data are available at the household level, ESs play a crucial role in linking household size and inequality/poverty measurement: They make it possible to compare incomes of heterogeneous household types in terms of living standards, by converting household income into one-member-household (OMH)-equivalent incomes. The thought experiment that underlies the use of ESs in distributional analysis is:

- Fix a set of OMH incomes as “reference incomes”, and determine multimember-household “equivalent incomes (EIs)”, that is, multimember household incomes that match the living standards of these OMH reference incomes. The ratio between a multimember household EI and its corresponding OMH reference income gives this household type’s ES.

- Take adults and children from any multimember household, place them in fictitious OMHs, and assign each the observed multimember-household income divided by this household’s ES.

- The result is a distribution of OMH-equivalent incomes.

Because of household-size economies and lower needs of children compared to adults, ESs are typically smaller numbers than the number of household members. For example, the OMH EI for each member of a childless couple receiving 5,000 euros per month is typically more than 5,000/2=2,500 euros. Most ESs are assumed to be the same for the rich and the poor, yet ESs may vary by the level of material living standards, they may be income-dependent.

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1 For example, the first of the 17 Sustainable Development Goals (SDGs), is “No poverty”.
The core poverty-line concept linked with ESs

There are numerous different concepts of poverty lines and different poverty measurements are used in practice. Notably, to score human conditions, quantifications of multidimensional poverty focus on three broad dimensions to evaluate human living conditions: Health, Education, and Standard of living. Since ESs are closely linked to incomes, this chapter steps back from multidimensional poverty and focuses on one-dimensional poverty, a “breadline” that considers income as a “catch-all” measure of material living standards.

Standard practice: Using the same ESs for rich and poor

A vast body of research measures and explains ESs. The predominant practice among academics and institutions is to use “expert” ESs such as the OECD modified ES. The common assumption characterizing the OECD ES is that the scale and thus the benefits from sharing goods within multi-member households is the same for the rich and the poor: This assumption has been called “Equivalence Scale Exactness” (ESE) (Blackorby and Donaldson, 1991 and 1993) or “Independence of Base” (IB) (Lewbel, 1989). The IB/ESE assumption is central in the literature that derives ESs from estimations of demand systems.

The direct link between ESs and poverty lines: Starting with measurement

For simplicity, let us assume that, under any methodology, we measure that $X$ is a vector of ESs, referring to all observed household constellations, and that each vector element is common for the rich and the poor within any household type (i.e., IB/ESE holds). Assume that $Y$ is the poverty line for an OMH, and let us look at the two main poverty-line concepts:

(A) Absolute poverty line, where poverty lines are defined through measures of human conditions such as calorie intake, heating, shelter, and sanitary conditions. In this case, values in vector $X$ influence how many converted OMHs fall under the poverty threshold $Y.$

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2 See, for example, the Global Multidimensional Poverty Index developed by the Oxford Poverty & Human Development Initiative: [https://ophi.org.uk/multidimensional-poverty-index/](https://ophi.org.uk/multidimensional-poverty-index/)

3 Generally, one can quantify distinct dimensions of poverty and map them to incomes. After this step, the link with ESs is straightforward. While there is little literature on linking distinct dimensions of poverty directly with ESs, two extensions are Pitt et al. (1990) and notably Kools and Knoef (2019), who propose a survey method and a model of choice based on utility functions in order to directly link health states with ESs.

4 For a literature review, see Schröder (2004, pp. 7-38).

5 These ESs are explained on the official website of the OECD: [https://www.oecd.org/economy/growth/OECD-Note-EquivalenceScales.pdf](https://www.oecd.org/economy/growth/OECD-Note-EquivalenceScales.pdf)

Three kinds of OECD ESs (see also Buhmann et al., 1988) that dominate the practice of converting household incomes into OMH incomes (EIs) are:

(a) the original “OECD ES”, assigning a value of 1 to the first household member, of 0.7 to each additional adult, and of 0.5 to each child,

(b) the “OECD-modified ES”, assigning a value of 1 to the first household member, of 0.5 to each additional adult, and of 0.3 to each child, and

(c) the “square-root ES”, ESs resulting from dividing household income by the square root of household size, with the resulting number assigned to both adults and children.
Relative poverty line, for example, with $Y$ being 60% of mean equivalent income. Relative poverty lines may change even if the rich get richer, while the poor live under the same conditions as before. Values in the vector $X$ influence the number of converted OMHs that fall under the poverty threshold $Y$, and different values of $X$ may lead to a substantial revision of $Y$.

ESs vary with the estimation method. A substantial literature analyzes the sensitivity of poverty to ESs. What unites these analyses is the assumption of IB/ESE. A later section provides an application showing how poverty estimates change when the IB/ESE assumption is dropped.

**First key extension: More general forms than IB/ESE**

It is reasonable to think that poorer multimember households have a disadvantage in sharing goods within the household. Poorer households spend a larger share of their financial resources on food, and food is difficult to share. Richer individuals in OMHs typically live in larger homes that do not become crowded when a partner or child joins the household. Poorer individuals in OMHs can only afford smaller homes and are forced to move if new members join the household.

If IB/ESE does not hold, the facility of having a single point estimate for the appropriate ES per multimember household type is lost: We may well have infinite ESs on the continuum of OMH reference incomes. In this case, research on the principles and parametric forms of ESs is needed to simplify this task.

**Second key extension: Explaining ES estimates by linking them to household choices**

Poor households are constrained to meeting their basic needs. However, even households under the poverty line, especially those with shelter (non-homeless), often have some discretionary income, allowing them to choose goods with higher sharing potential within the household. Freedom of choice in markets obliges us to employ some decision theory. This has been the approach of demand-system estimations of ESs. Existing demand systems are typically static models, and the challenge is to incorporate expectations, including the planning of cohabitation, parenthood, divorce, etc., in the analysis of choices across the life course. A later section focuses on recommending future research directions to specify utility functions.

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6 Buhmann et al. (1988) take household income data from the Luxembourg Income Study and find that the choice of the ES affects absolute and relative levels of poverty and country poverty, motivating a series of follow-up studies (e.g., Phipps, 1993; Banks and Johnson, 1994; Burkhauser et al., 1996; De Vos and Zaidi, 1997; Aaberge and Melby, 1998; Newhouse et al. 2017). Coulter et al. (1992) theoretically explain this sensitivity through the changing covariance between EIs and household size in micro data.
Estimating ESs and testing whether they depend on living standards

Brief historical overview of ES estimation methods

Estimating ESs has a long tradition. Engel (1857) argued that share of household financial resources spent on food could serve as a measure of households' material needs. According to his reasoning, if two household types have equal expenditure shares for food, they are equally well-off. A related approach remains to date, for defining the income thresholds of the US poverty line.

Demand-system analysis uses principles of revealed preferences and choice theory, arguing that ESs can be identified through estimates of household cost functions. Various factors make this approach challenging. Some within-household information is difficult to observe, for instance, the quantity/quality of domestic production (Gronau, 1973, 1977) and the intra-family allocation of these commodities (Browning, 1992, p. 1470). As a result, estimates of ESs from revealed preferences depend on (a priori untestable) exogeneity assumptions and identification restrictions as well as assumptions about within-household sharing rules and within-household production.

Particularly in the context of poverty measurement, the IB/ESE assumption serves as a starting point toward new research directions. Survey approaches have been suggested that derive ESs directly from people’s assessments of the relationship between their income, household type, and material living standards, based on the assumption that people are well informed about the material living standards that a given income confers on household members. These survey estimates can be used to test the validity of identification restrictions that demand systems impose (see, e.g., Koulovatianos et al., 2005a,b) or to validate demand-based estimates (see Kapteyn, 1994).

Equivalent income functions

In an estimated demand system, the estimated indirect utility of multimember households leads to equivalent income (EI) functions, that is, incomes that equate the living standards of households for different OMH income levels. Specifically, using an OMH as a reference household, for a given reference income, $y^r$, an EI function is given by,

$$y^h = \Phi(V(y^r)),$$

Equivalent income functions

7 The expert approach to defining ESs is to have an expert define the needs of children and adults in a household. It first appears in the early empirical works of Engel (1857, 1895). For a literature review of expert approaches see Schröder (2004, pp. 7-8). Further identification issues are summarized in Lewbel (1997) and Slesnick (1998), encompassing simultaneity of demand and supply equations; estimation of ESs without price variation; interpersonal comparability, ordinality, and cardinality of household utilities.
where $y^h$ is the EI of household type $h$, $V(y^r)$ is the indirect utility (value) function of the single childless adult, and $\Phi$ is the inverse of the value function of $h$.

ESs are derived from EI functions, such as that given in equation (1). Specifically, let $ES^h(y^r)$ denote the ES for household type $h$ that corresponds to OMH reference income $y^r$. Then equation (1) implies that,

$$ES^h(y^r) = \frac{y^h}{y^r} = \frac{\Phi(V(y^r))}{y^r}.$$

(2)

Even if we assume that we have a way to confidently estimate ESs, equation (2) implies that we need to estimate potentially infinite ESs on the whole span of reference incomes, $y^r$. For simplifying this practical problem, we can assume utility functions that imply a simple parametric form for the composite function $\Phi(V(\cdot))$ in equation (1), as implied by IB/ESE.

**EI functions under IB/ESE**

The EI function consistent with IB/ESE is,

$$y^h = \Phi(V(y^r)) = R^h y^r,$$

and, based on equation (2),

$$ES^h(y^r) = R^h.$$

(3)

Equation (3) conveys the main concept of IB/ESE: ESs do not depend on reference incomes, $y^r$. From a practical viewpoint, only one parameter, $R^h$, gives the ES for each household type.

**A generalization of IB/ESE: Fixed costs of consumption**

IB/ESE as a convenient identification restriction in demand systems. Donaldson and Pendakur (2004 and 2006) generalized this assumption by considering fixed costs in household consumption. Specifically, Donaldson and Pendakur (2006) suggest a formulation for EI functions with a fixed component and a variable component that is proportional to the reference income. This formulation, “Generalized Absolute Equivalence Scale Exactness” (GAESE), is characterized by the EI function,

$$y^h = A^h + R^h y^r,$$

and, accordingly, ESs are given by,

$$ES^h(y^r) = \frac{A^h}{y^r} + R^h.$$

(4)

Equation (4) still guarantees convenience, because for each household type, $h$, ESs are fully characterized by a pair of parameters $(A^h, R^h)$. The key concept behind GAESE is fixed costs of consumption, such as housing rent, maintenance flows of durables, etc., affecting the discretionary income of a household, with the latter being lower for poorer households.
Demand-system estimation of ESs is self-referential

The ESE or GAEESE property of ESs is instilled in a demand system by assumptions on the utility functions. Utility functions analyze free choice and provide utility comparisons among different types of households. Yet their structure always retains the ESE or GAEESE property once it is assumed in the utility function structure. Therefore, testing whether a demand system that is consistent with GAEESE fits the data, as in Donaldson and Pendakur (2006), is self-referential: It does not constitute an independent test. Some survey-based approaches seek external validation of the GAEESE property.

Survey methods and tests of GAEESE without choice models

A tradition of survey approaches to the estimation of ESs, was pioneered by Kapteyn and van Praag (1976) and explained by Bradbury (1989). This literature uses Likert scales, asking respondents to rate how happy or satisfied they are with their income or life. Then, using a utility theory, the rating is mapped to incomes. Yet, the key objective of externally validating whether ESs satisfy IB/ESE or if they are income-dependent is to avoid using a utility theory.

In order to avoid the use of utility theories, Koulovatianos, Schröder, and Schmidt (2005a and 2005b) proposed a survey method in which respondents state EIs directly. Specifically, they ask questions of the form: “What net household income level can make a household with two adults and one child as well off as a one-person household with a net income of 1,000 dollars?” In this example, the amount of 1,000 dollars is the reference income, \( y^r \). By repeating the same type of questions for many household types and for many reference income levels, \( y^r \), they construct EI functions that enable tests of the IB/ESE and GAEESE formulation.

The main result in Koulovatianos, Schröder, and Schmidt (2005a and 2005b) is that ESs are income-dependent in six countries, and that the poor have a disadvantage in benefitting from sharing goods within the household. Koulovatianos, Schröder, and Schmidt (2007) explicitly test for GAEESE using a specification test of the form,

\[
\frac{y_i^h}{y^r} = R^h + A^h \frac{1}{y^r} + \alpha_0^h \text{Ref. Income Dummies} + a^h_{PERSONAL_i} + \epsilon_i^h, \tag{5}
\]

where \( y_i^h \) is the EI that was stated by survey respondent \( i \) about a household of type \( h \), for a given reference income, \( y^r \), (the ratio \( y_i^h / y^r \) is the ES for \( h \), stated by respondent \( i \)). Personal characteristics of respondents are a set of variables “\( PERSONAL_i \)”. “Ref. Income Dummies” is a set of dummy variables that assigns 1 whenever reference income is equal to the corresponding reference income given in a question, and 0 otherwise. Therefore, if the functional form given by (5) is not sufficient to explain the variation in survey data, any additional variation is captured by these reference-income dummies, and a test for their inclusion is a direct test of the GAEESE formulation of equation (4). A total of 42 tests in six

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8 Nevertheless, this approach manages to test GAEESE against IB/ESE, providing some useful information.

9 Seidl (1994) subjected the utility theory used in this line of happiness research to critical scrutiny. An alternative and more recent study using subjective ESs is Bishop et al. (2014).
countries do not reject GAESE. Yet, Koulovatianos, Schröder, and Schmidt (2007, pp. 19-22) criticize their survey, explaining that the GAESE pattern may be the result of a survey-framing effect called “anchoring”.

In order to deal with this potential problem of anchoring, Koulovatianos, Schröder and Schmidt (2019, Figure 1, p. 177, and Table 1, p. 180) show that all responses of EIs, by different, independent respondent groups that evaluate different reference incomes, $y^r$, lie on straight lines, consistently with equation (4), and perform further tests of the validity of their survey method.

In a nutshell, the main results are:

(a) the poor have a disadvantage in sharing goods within the household due to fixed costs of consumption

(b) this income dependence of ESs may not pose a concern for researchers, because GAESE holds and ESs are fully characterized by a pair of parameters $(A^h, R^h)$.

New directions for choice theory and modeling of poverty

Models can guide us in prescribing policies to combat poverty. To the extent that these GAESE findings reflect a ubiquitous regularity, poverty modeling can be carried out in a relatively straightforward manner.

Two crucial principles of choice theory are (i) opportunity cost, captured by convex-programming techniques, and (ii) forward-lookingness and expectations, captured by dynamic-programming techniques. In Koulovatianos, Schröder, and Schmidt (2019, Proposition 1, p. 173, and Theorem 1, p. 172), it is explained that GAESE is broadly consistent with dynamic dynasties that have utility functions of the form,

$$\int_0^\infty e^{-\rho t} \frac{[c - \beta_h(t)]^{1-\frac{1}{\alpha}}}{1 - \frac{1}{\alpha}} dt$$

where $\rho, \alpha > 0$ are constant parameters, and $\beta_h(t) > 0$ is a family-type subsistence income (for family type $h$ at time $t$) that can vary over time across the individual life course and depending on health and educational needs in different phases of life.

Regarding the question of what numbers to assign to $\beta_h(t)$, a conventional answer is proposed by Koulovatianos, Schröder, and Schmidt (2007, Figure 1): Subsistence is the income level at which nutrition and health goods can no longer be shared, that is, when the ES is equal to the number of household members.

The monthly subsistence incomes per person appearing in Koulovatianos, Schröder, and Schmidt (2007, Table 4), adjusted in 2019 euros, range between 312-383 euros in Germany, between 271-354 in France, 246-286 in Cyprus, 201-340 in China, 130-184 in India, and 187-249 in Botswana. Such values have been used in savings and portfolio analysis by Achury et
al. (2012), and Hubar et al. (2020), showing that cross-sections of risk-taking and savings rates can be better matched using parsimonious models.\(^{10}\)

**Application: Why income-dependent ESs matter quantitatively for the measurement of poverty**

Using the German Socio-Economic Panel (SOEP, Goebel et al., 2019), we assess the implications of a constant and an income-dependent ES for measured poverty. Our application considers three ESs:

(a) the OECD modified ES;
(b) the income-dependent ES, consistent with GAESE, as explained above;
(c) the household-type-specific mean values of the income-dependent scale.

We consider only those household types for which values of the ESs for (b) and (c) are available (single adults and couples with up to three children). We use 60 percent of the weighted median EI as the poverty line using the respective ES.

Panel A of Figure 1 provides the poverty lines together with the distributions of EIs in the low-income segment for all three ESs. The poverty lines are close, falling in the interval between 897 euros (averaged income-dependent scale) and 963 euros (OECD modified scale). The EI distributions for the OECD and the averaged income-dependent scale are also close, but the distribution for the income-dependent scale, as a result of low within-household size economies in low-income households, has a much fatter left tail.

Beyond the visual representation in Panel A, we look at poverty indices corresponding to each ES concept. As an incidence measure, we use the head count ratio, that is, the share of the population below the poverty line. To measure the intensity of poverty, we use the poverty gap ratio, that is, the mean shortfall of the total population from the poverty line (counting the non-poor as having zero shortfall), expressed as a percentage of the poverty line.

\(^{10}\) For a detailed analysis of dynamic models of poverty and subsistence, see, e.g., Maialeh (2020, Ch. 5).

Figure 1 – Poverty lines and densities of equivalent disposable household income, and population composition below the poverty line for alternative ESs

Comparing results using the standard OECD ESs, with income-dependent ESs, the poverty line shifts from 963 to 918 euros per month, but poverty indicators change substantially:

(1) The poverty gap ratio (incidence of poverty) increases from 2.47 to 4.10
(2) The head count ratio (intensity of poverty) increases from 9.78 to 14.21

In order to validate that these poverty increases in (1) and (2) above are not due to some methodological bias, we compare the standard OECD ESs, with the average income-
dependent ESs, a case where the poverty line shifts from 963 to 897 euros per month. Comparing results using the standard OECD ESs with income-dependent ESs:

(A) The poverty gap ratio (incidence of poverty) increases slightly from 2.47 to 2.53
(B) The head count ratio (intensity of poverty) increases marginally from 9.78 to 9.82

The increases in (1) and (2) demonstrate that poverty is substantially underestimated when failing to use income-dependent ESs to account for the full disadvantage of the poor in sharing goods within the household.

The use of an income-dependent scale has implications not only for the incidence and intensity of poverty, but also for the composition of the poor population. This can be seen from Panels B through D in Figure 1. By means of pie charts, they depict, for each ES, the share of the poor population in each of the eight household types considered. Using the OECD scale, about 52 percent of the poor population are living in single-adult households, about 24 percent in two-adult households without children, and about 24 percent in households with children. For the averaged income-dependent scale, these percentages are 43, 29, and 28 percent, respectively. In contrast, when using the income-dependent scale, the share of the poor living in single adult households is much lower, at 31 percent, and markedly higher for two-adult households without children (37 percent), and for households with children (32 percent) as a result of high fixed costs of consumption.

Conclusion

ESs are a tool for understanding how household sizes affect poverty assessments and poverty lines. Future research could focus on the disadvantage that poor households suffer due to their reduced ability to share goods within the household. This disadvantage is important to address as it may have a substantial quantitative impact on the measurement of poverty. Future research on subsistence incomes may shed light on explanations for poverty and may guide anti-poverty policies.
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