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Non-Additivity of Subjective Expectations over Different Time Intervals*

Peter Haan† Chen Sun‡ Uwe Sunde§ Georg Weizsäcker¶

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Abstract

We examine the additivity of stock-market expectations over different time intervals. When asked about a ten-year interval, survey respondents expect a stock-price change that is not equal to, but closer to zero than, the sum of their expectations over two shorter time intervals that cover the same ten years. Such sub-additivity is irrational in that it cannot stem from aggregating short-term expectations. Model estimates show that the pattern is consistent with a time perception where shorter time intervals have a proportionally larger weight. We also find that the respondents' degree of additivity is correlated with making larger financial investments.

JEL-classification: D01, D14, D84, D9

Keywords: Expectation Formation, Time perception, Sub-additivity, Super-additivity

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1 Introduction

A large and deep literature asks about the consistency of choice over time: do people's choices over sooner-but-less consumption versus later-but-more consumption "add up" if the time horizon varies across choice tasks? The literature examines *preferences* in much detail (see, e.g., the survey by Ericson and Laibson, 2019). Economic choices over time, however, reflect not only preferences but also *expectations*. The growth rates of many relevant variables are unknown at the time of making a choice, e.g., when households face trade-offs in their consumption-savings choices, portfolio choices, or educational choices. The perceived time horizons that households consider when evaluating these growth rates can vary, quite strongly in some cases, and they may influence the perceived trade-offs and potentially lead to inconsistent choices. We thus ask about the consistency of expectations over time, i.e., whether people's expectations about growth rates "add up".

Concretely, suppose that a decision-maker considers the prices of an asset at two different points in time t and t', with $t < t'$, and let $\Lambda_{t,t'}$ denote the decision-maker's subjective expectation about the ratio of prices at these points in time. Our analysis asks about the consistency of $\Lambda_{t,t'}$ across different pairs (t, t'). For instance, for the three points in time $\{0, 1, 2\}$, is the decision-maker's long-term expectation $\Lambda_{0,2}$ consistent with a suitable aggregation of her short-term expectations $\Lambda_{0,1}$ and $\Lambda_{1,2}$? Essentially all dynamic models of economic decision-making have this property (i.e., expectations add up).[1] However, it is far from clear that the decision-maker, when asked about her expectations or when making her choice based on them, actually has such internally consistent expectations about future growth. Indeed, one strand of the literature on time preferences (following Read, 2001) suggests the opposite, namely that the *perception of time* may be non-constant, such that shorter time periods have larger (more than proportional) weight than longer ones. This can rationalize hyperbolic discounting of consumption utility and sub-additivity in discounting. Yet, the hypothesis of non-constant time perception should also concern expectations: it implies non-additivity of perceived time spells of different length, and therefore non-additive expectations.

This paper is the first to empirically examine non-additivity of expectations over dif-

[1] Abstracting from uncertainty, the consistency requirement is $\log(\Lambda_{0,1}) + \log(\Lambda_{1,2}) = \log(\Lambda_{0,2})$. With uncertainty, the same must hold in expectation.

ferent time horizons, to the best of our knowledge.[2] We design a representative survey among a large subsample of the Socio-Economic Panel (SOEP) and ask each respondent for his or her expectations about the future growth performance of the German stock market index over three time intervals: a horizon of ten years and two shorter sub-periods (either a horizon of one year and the subsequent nine years, or a horizon of five years and the subsequent five years). The questions are simple to understand and appear in immediate succession on the questionnaire, rendering it non-challenging to give an additive set of responses. Nevertheless, expectations violate additivity for the vast majority (>99%) of respondents. This finding is qualitatively robust to measurement/reporting errors and to the possibility that respondents ignore the compounding effects of growth: depending on how we allow for these deviations from rationality, the consistency rate of reported expectations increases considerably, but even if both types of error are allowed for, about half of the respondents show non-additive expectations.

Notably, giving consistent responses in our survey does not restrict the respondents' expectations about growth at any instant of time. Additivity should hold irrespective of the nature of one's expectations (it does not) but the direction of a possible bias may be different for respondents who expect negative versus positive growth at different points in time. When separating subsamples of respondents who report consistently positive expectations or consistently negative expectations, we find a pattern that is symmetric around zero: respondents with consistently positive expectations exhibit a too-small expectation for the entire period, whereas respondents with consistently negative expectations exhibit a too-large expectation for the entire period. For both of these groups, the typical pattern is, thus, that expectations are sub-additive with respect to absolute values. The null hypothesis of additivity can be rejected at high levels of statistical significance for both groups.

We also investigate whether non-additivity of expectations over time is related to demographic characteristics and financial outcomes. Regarding demographic correlates, we find that the deviations from additivity are positively correlated with lower education and higher age. Regarding financial outcomes, we find that they are correlated with lower

[2] A sizable literature follows the decision-theoretic approach of allowing for non-additive subjective probability measures introduced by Gilboa (1987) and Schmeidler (1989). In contrast to our research question, the additivity properties in this literature concern the aggregation over different possible *events*, not over different time horizons.

investment propensity and lower investments/savings. This set of findings indicates that time perception may be a relevant component of financial literacy.

The observation that sub-additivity, not super-additivity, is the predominant pattern is also confirmed in further analyses. We estimate a nonparametric model of time perception where each of the time intervals that we use in the survey (year 1, years 1-5, years 1-10, years 2-10, years 5-10) has a potentially different weight. The estimates can replicate most patterns in the data and show that the interval containing only year 1 has a far larger weight than the other intervals, by a factor that ranges between 2.6 and 6.8 for the different comparisons with the longer intervals. More generally, shorter intervals have a proportionally larger weight than longer ones. Finally, we propose a parametric model that we estimate with our sample. Despite its simple functional form, the model has a good in-sample fit and replicates all main features of the data, which may be relevant for future analyses of expectations with variable time horizons.

The literature on time preferences includes mounting evidence that time perceptions may be non-additive for many individuals (see, e.g. Read, 2001; Scholten and Read, 2006; Ebert and Prelec, 2007; Kable and Glimcher, 2010; Bradford et al., 2019; Dohmen et al., 2022). Our findings add a piece of evidence from a new context, expectations. They also relate to the literature on the stock market participation puzzle, showing evidence that private households in many countries have a surprisingly low frequency of investing in stock (Haliassos and Bertaut, 1995). Studies that elicit expectations about stock-market growth find a positive correlation between expectations and stock market participation in most cases (Hurd et al., 2011; Dominitz and Manski, 2011; Arrondel et al., 2014) but not in all of them (Breunig et al., 2021b). We note that the strength of the statistical connection between expectations and investments should co-vary with the degree of congruence of the relevant time horizons – but not much evidence exists yet about how expectations change with different time horizons. Recent evidence on German households (Breunig et al., 2021a) documents that expectations about long-run stock returns are substantially lower than what one would expect from extrapolation of short-run expectations. This is consistent with expectations simply being more pessimistic for the far future than for the near future, but it is also consistent with non-constant time perceptions. Specifically, if the short run is perceived with a larger weight than the long run, then a set of positive growth expectations that cover increasingly long time horizons will show a natural tendency to be concave in the sense of diminishing per-period growth. Non-constant time perceptions may thus affect long-run expectations and induce a low propensity to invest in stock.[3]

Finally, we note that diminishing time weights may be viewed as a special case of a much wider phenomenon in the psychological literature: the effect of salient part-by-part presentations. For instance, research on partition dependence finds that people assign a greater total probability to an event if it is described as the union of sub-events rather than a single event (see, e.g., Fox and Rottenstreich, 2003). Analogously, increased time weights of short horizons may be driven by the salience of the near term. Salience effects and non-linear (diminishing) cognitive responses to stimuli have been shown to be powerful drivers of many choices (Bordalo et al., 2012; Köszegi and Szeidl, 2013) and economic valuations (Fischhoff et al., 1993).

The paper is organized as follows. The next two sections introduce definitions and describe the data collection. In Section 4 we demonstrate the prevalence of non-additivity and show how it correlates with observable characteristics and investment behavior. We then differentiate sub-additivity versus super-additivity and describe the data patterns in relation to the sign of the respondents' expectation reports. In Section 5 we estimate different models of time perception, yielding a concise description for the data patterns.

2 Definitions and Notation

Let P_t be a stock market index at time t. The price ratio between time t and t' is

$$\Lambda_{t,t'} = \frac{P_{t'}}{P_t}.$$

At time t_0, a decision-maker forms expectations about how the index changes over time. Specifically, she forms her expectations about the price changes from t_0 to t_1, from t_1 to t_2, and from t_0 to t_2, respectively. In the following, we adopt the convention of the finance literature and define growth expectations as logarithms of price ratios, which allows adding up over time. We describe the decision-maker's expectations as consistent

[3] The basic method of our elicitations is as in Breunig et al. (2021a) but the structure of questions follows Dohmen et al. (2012, 2022) enabling the possibility that reports can show non-additivity.

if they are additive over time, i.e.,

$$\mathbb{E}_{t_0}[\log(\Lambda_{t_0,t_2})] = \mathbb{E}_{t_0}[\log(\Lambda_{t_0,t_1})] + \mathbb{E}_{t_0}[\log(\Lambda_{t_1,t_2})] \,, \tag{1}$$

where $\mathbb{E}_{t_0}$ denotes expectations held at t_0. The property of additivity holds for any rational set of expectations, independent of instantaneous expectations and (auto-)correlation structure. But the agent's reported expectations may not be rational in this sense and deviate from additivity. Denote the agent's expectations about the log-price ratios over the three periods by x_{t_0,t_1}, x_{t_1,t_2} and x_{t_0,t_2}, respectively. Our main goal is to test whether

$$x_{t_0,t_2} = x_{t_0,t_1} + x_{t_1,t_2}. \tag{2}$$

Towards this goal, we use the agent's reported percentage changes in the index, denoted by q_{t_0,t_1}, q_{t_1,t_2} and q_{t_0,t_2}, respectively, and exploit the equivalence relation between the two set of variables:

$$x_{t_i,t_j} \equiv \log\bigl(1 + q_{t_i,t_j}\bigr), \quad (i,j) \in \{(0,1),(1,2),(0,2)\} \tag{3}$$

Two ways of violating additivity are possible: sub-additivity and super-additivity. *Sub-additivity* is the property that the expectation over a period is of smaller absolute magnitude if it is elicited directly over the entire period than if the period is divided into two sub-periods over which expectations are elicited separately. Formally, expectations are sub-additive if

$$0 \leq x_{t_0,t_2} < x_{t_0,t_1} + x_{t_1,t_2} \tag{4}$$

or

$$x_{t_0,t_1} + x_{t_1,t_2} < x_{t_0,t_2} \leq 0. \tag{5}$$

Similarly, expectations are *super-additive* if the expectation over a period is larger in magnitude if elicited directly as compared to separate elicitation over sub-periods, i.e.,

$$0 \leq x_{t_0,t_1} + x_{t_1,t_2} < x_{t_0,t_2} \tag{6}$$

or

$$x_{t_0,t_2} < x_{t_0,t_1} + x_{t_1,t_2} \leq 0. \tag{7}$$

When we focus on the time period (t_0, t_2), we call x_{t_0,t_2} the *direct* expectation and $x_{t_0,t_1} + x_{t_1,t_2}$ the *indirect* expectation over the period. Similarly, when we focus on the later of the two sub-periods, (t_1, t_2), we define x_{t_1,t_2} as the *direct* expectation and $x_{t_0,t_2} - x_{t_0,t_1}$ as the *indirect* expectation over this sub-period. In the empirical analysis we provide a formal test of whether direct and indirect expectations are equal.

3 Data

The empirical analysis is based on data from the SOEP-IS, the Innovation Sample of the Socio-Economic Panel (Goebel et al., 2019). The SOEP-IS is designed to be representative of Germany's population. In addition to standard socio-economic information, the SOEP-IS includes separate survey modules for specific research questions. To test for non-additivity we inserted questions about price expectation for the well-known German stock market index DAX over different time periods. The interviews were conducted between September and December 2019. During this time the DAX was mostly increasing.[4]

Throughout our module, we first ask whether individuals expect an increase or a decrease of the stock market index for a given period. Then, we elicit the expected magnitude of the change. For instance, the following questions refer to the time horizon of one year:

In the following we want to ask you some questions about financial issues. They refer to the German index of stocks (DAX), which summarizes the economic development of 30 major enterprises. We want to know what you expect about the future development of the DAX, expressed as profit or loss compared to the DAX's current value.

- *This question asks about the next year, i.e., the next twelve months. Do you expect the DAX to rather gain or lose during the next year, compared to the current value?*

[4]The DAX is a blue chip stock market index that summarizes economic development of (then) 30 major German companies trading on the Frankfurt Stock Exchange. It started at a base value of 1000 index points on December 31, 1987. Its performance is covered widely and frequently in many German news services.

- *Expressed in numbers: What [Gain/Loss] do you expect for the next year overall, in percent?*

Respondents are randomly allocated in two groups, relating to different time periods. Those in one treatment group are asked to report their expectations for the next year and their expectations for the nine years that start after the next year. We refer to this group as Treatment group 19 (T19). Respondents in the other group – Treatment group 55 (T55) – are asked about expectations for the next five years and for the subsequent period of five years. In addition to their reports about the sub-intervals, individuals in both treatment groups report their expectations for the full period of ten years. For each group, we randomly allocate the order of the questions. Our SOEP-IS sample includes information of about 1,990 individuals, of whom 1,345 individuals report all three expectations. For the statistical analysis we exclude individuals with missing information and extreme values at the top and the bottom (1%) as we preregistered.[5] This leaves us with 662 individuals for group T19 and 652 individuals for group T55.

The following results sections will include evidence on the heterogeneity of expectations and correlations between expectations and financial decisions. Among other variables, the SOEP-IS provides detailed information about education, income, employment status and whether individuals hold financial assets.[6]

4 Non-additivity of Subjective Expectations

4.1 Distributions of expectations

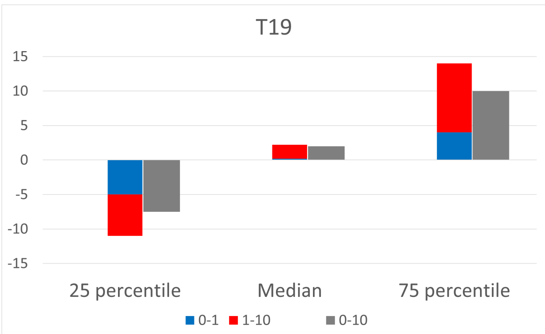
We illustrate the main data pattern by considering the quartiles of reported expectations. Figure 1 plots the quartiles of the log-price ratios for the two treatment groups, juxtapos-

[5] Among the 639 respondents who do not report all the three expectations, 81% do not respond to any of the expectation questions. The expectations reported by respondents who do respond to some expectations questions are not significantly different from those of respondents with responses to all three expectations (p-values > 0.1 in Wilcoxon rank-sum tests for all the five time horizons).

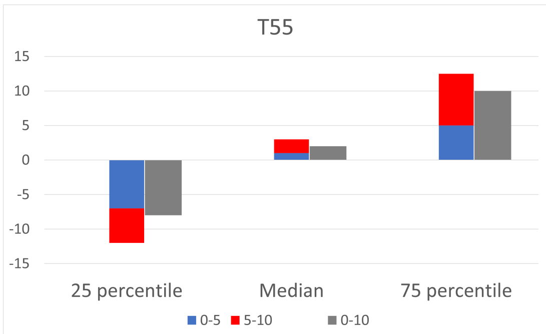
[6] We preregistered our study on AsPredicted (#67104) before we started to analyze the data. We execute the plan faithfully, including the exclusion of the top and bottom 1% observations, the main dependent variable and test, as well as the regressions between our main measure of non-additivity and background variables. The only exception is that the test results in Table 1 exclude expectation reports with inconsistent signs. However, including theses observations would not affect the significance of our findings (see Section 4.2).

ing the expectations that were obtained directly over the 10-year horizon (shown in grey bars) versus indirectly, i.e., when considering the partitions into subperiods of one and nine years, or five and five years, respectively. The figure's indirect expectations (stacked red and blue bars) have a visibly larger absolute magnitude than the corresponding direct expectations. Table A.2 shows that this pattern generalizes across the entire distribution of expectations. It is also consistent with the very low count of perfect additivity: the variable Diff $= x_{0,10} - x_{0,k} - x_{k,10}$ takes on the value of zero in only 6 out of 1,314 observations (0.5%); all remaining reports of expectations are non-additive (see Table A.4).[7]

Figure 1 also shows that a considerable share of respondents expect the stock market to decrease in value and that, correspondingly, the median respondent expects only very modestly positive gains. Table A.1 summarizes the respondents' distribution of expectations and shows their wide heterogeneity.



(a) Group T19



(b) Group T55

Note: Quartiles of expectations over the ten-year period (grey) and the two sub-periods (blue and red), respectively.

Figure 1: Quartiles of direct and indirect expectations over the ten-year period

The mere count of inconsistent responses overstates the relevance of non-additivity, however, if individuals report expectations with error or imprecision. We therefore also allow for rounding or calculation inaccuracy (see Table A.5). With a 1-percentage-point tolerance of error in each of the three reported expectations, the rate of potentially additive responses increases substantially, to 38% (504 out of 1,314 cases). Nevertheless, the tendency towards sub-additivity remains the same.

[7]This is in line with the findings of Breunig et al. (2021a) whose survey covered up to 30 years of investment horizon and who find a strongly concave pattern of expectations, with modest average increases over later time periods after an early steep increase.

Another explanation for non-additive responses may be that respondents have difficulties in calculating compounded growth rates, i.e., suffer from exponential-growth bias (Stango and Zinman, 2009; Levy and Tasoff, 2016; Ensthaler et al., 2018). That is, respondents may calculate the growth rate over the ten-year period by simply adding up the expected percentage changes over the two sub-periods, ignoring cumulative effects. If we allow for this possibility and tolerate any degree of exponential-growth bias in the 10-year expectations, 14% of responses are counted as potentially additive whereas 86% reveal non-additive expectations (Table A.6). When combining the previous two approaches, i.e., accounting for the possibility of rounding errors or compounding errors, or a mixture of both, the share of responses that are counted as potentially additive increases to 43%, still leaving the majority (at least 57%) of responses non-additive (Table A.7).

4.2 Tests of sub-additivity versus super-additivity

To investigate the bias's direction, it is useful to differentiate between respondents with positive versus negative expectations. Confirming Figure 1's tendency, the data lean toward sub-additivity: direct expectations are lower than indirect expectations in 68% of the cases where a respondent's directly elicited 10-year expectation is weakly positive (497 out of 734). Conversely, among the remaining respondents who expect negative growth, 63% (364 out of 575) report direct expectations that are larger (i.e., also closer to zero) than their indirect expectations.[8]

For a statistical test of the directed hypotheses given in expressions (4), (5), (6) and (7), we restrict the sample to responses where these hypotheses can be checked, i.e., responses with consistent signs of expectations.[9] We assign respondents to four groups, depending on the sign of their expectations (consistently non-negative vs. non-positive) and the treatment (Treatment 19 vs. Treatment 55). In two of the four groups, all respondents have non-negative expectations over the 10-year period, and we perform Wilcoxon signed-rank tests on the differences between the direct and the indirect expectations, with (4) or (6) as the alternative hypothesis. In the other two groups, respondents have non-positive

[8]Table A.4 reports the exact numbers of respondents with additive versus non-additive sets of responses, according to the different classifications.

[9]We thereby exclude a total of 139 respondents (10.7%) who exhibit combinations of direct and indirect 10-year expectations that have opposite signs. Including these observations would not change any results regarding the prevalence of sub-additivity.

expectations, and we perform the tests with (5) or (7) as the alternative hypothesis.

Table 1 displays the test results. The first two rows show that the alternative hypothesis (4) is favored for the groups with non-negative expectations, while the last two rows show that the alternative hypothesis (5) is favored for the non-positive groups. Consistently, the direct expectation over the 10-year period is smaller in magnitude than the indirect one. The pattern of sub-additivity thus appears for respondents with optimistic and pessimistic expectations, and in a statistically significant way in both cases.

Table 1: Tests of additivity

Sign of $x_{0,10}$	Treatment	$x_{0,10} - x_{0,k} - x_{k,10}$			All	z stat	p-value
		> 0	$= 0$	< 0			
Non-negative	T19	91	5	227	323	-6.930	0.0000
	T55	44	1	272	317	-12.101	0.0000
Non-positive	T19	183	1	85	269	6.985	0.0000
	T55	183	0	83	266	7.785	0.0000

Note: Wilcoxon signed-rank tests on the differences between the direct and the indirect expectations over the ten-year period. Observations where the direct and the indirect expectations are of opposite signs are excluded.

As a robustness check, we repeat the tests while allowing for compounding errors in the assignment into groups (Table A.8), which confirms the results in Table 1.

The possibility of noisy responses leads to another potential concern about our particular data pattern: the direct expectation being closer to zero than the indirect expectation might merely reflect that the indirect expectation is noisier. Indeed, the indirect expectation stems from two directly reported expectations, each of which may be subject to error. If all expectation reports are subject to unrelated noise, this would widen the distribution of indirect expectations more than that of direct expectations, consistent with the main data pattern. To address this concern, we repeat the tests but for expectations about the second sub-period ($(1, 10)$ and $(5, 10)$, respectively) rather than the 10-year period. Here, too, a large level of reporting noise would tend to push the distribution of indirect expectation to be more spread out than for direct expectations – but this is not the case in the data. Instead, the direct expectation is greater in absolute magnitude than the indirect one, consistent with sub-additive expectations.[10]

[10] We compare $x_{k,10}$ with $x_{0,10} - x_{0,k}$ and drop observations from respondents whose direct and indirect expectations over the second sub-period have opposite signs, analogous to what we do for the comparison between $x_{0,10}$ and $x_{0,k} + x_{k,10}$ (Panel A of Table A.9). The results show that direct expectations over the

Another concern with our tests of additivity is that survey responses might not reflect the aggregation of deterministic expectations over time, but the aggregation of expectations about two random variables. As long as respondents compute geometric means of expected price ratios (as in (1)), the test of additivity remains valid also in this case. If respondents report arithmetic means and correctly aggregate, the difference between the direct and the indirect expectations is not equal to zero under the null hypothesis, but equal to the covariance between the expectations over the two sub-periods.[11] In this case, an alternative explanation for the long-run expectation being smaller than the sum of the short-run expectations, as observed in our non-negative sub-sample, would be mean reversion in growth rates, i.e., a negative covariance, consistent with evidence from field experiments in which the majority of respondents exhibit mean reversion in expectations (see, e.g., Laudenbach et al., 2021). At the same time, however, the predominant pattern in our non-positive sub-sample is that the long-run expectation is *greater* than the sum of the short-run expectations. To account for the observed patterns for this subsample, the covariance would have to be positive. This suggests that mean reversion in expectations does not provide a unified explanation to the results in our non-negative and non-positive sub-samples. Moreover, the results of calibration exercises indicate that this explanation is unlikely to account for the quantitative deviations observed in our data.[12]

time period $(k, 10)$ are greater in magnitude than indirect ones. The results from a similar robustness check that allows for compounding error also reveal that additivity is rejected for all groups, while sub-additivity is supported (see Panel B of Table A.9).

[11]Concretely, respondents might not report geometric means of possible price ratios, i.e., $\mathbb{E}[\log(\Lambda_{t_1,t_2})]$, but report arithmetic means, i.e., $\mathbb{E}[\Lambda_{t_1,t_2} - 1]$, where $(t_1, t_2) = (0, k), (k, 10), (0, 10)$. Then the reported long-run expectation is

$$\mathbb{E}[\Lambda_{0,10} - 1] = \mathbb{E}[\Lambda_{0,k} - 1] + \mathbb{E}[\Lambda_{k,10} - 1] + \mathrm{Cov}[\Lambda_{0,k}, \Lambda_{k,10}]$$

where $\mathrm{Cov}[\Lambda_{0,k}, \Lambda_{k,10}]$ is the covariance of the growth rates over the two sub-periods.

[12]In our calibration, we assume that the returns in two consecutive years follow a bi-variate normal distribution with identical means and standard deviations, and calibrate the perceived means, standard deviations and the covariance to the expectations reported by Laudenbach et al. (2021). The resulting mean is 7.5%, the standard deviation is 26.6%, and the correlation coefficient between two consecutive years is -0.14. The predicted covariance is thus -0.01. Among the 499 respondents who present a negative Diff in our non-negative sub-sample, 422 (85%) exhibit a Diff with a larger magnitude than the above value of covariance, suggesting that even for our non-negative sub-sample, mean reversion has limited explanatory power for the observed patterns.

4.3 Correlation with background variables

We now examine how violations of additivity are related to demographics and financial outcomes. We use a continuous measure of non-additivity: the absolute deviation of reported expectations from the additive benchmark, $M_{\text{nonadd}} = |x_{0,10} - x_{0,k} - x_{k,10}|$.[13]

The regressions of M_{nonadd} on various demographic variables results in almost no significant coefficients, the exceptions being that older respondents and respondents with low education (no vocational or tertiary education degree) show greater deviations from additivity.[14] This is reminiscent of previous findings of larger rationality violations for similar sub-groups (e.g. Choi et al., 2014) and the coefficient estimates are sizable but arguably not surprisingly large: respondents above 65 years of age and respondents without educational degree each show higher non-additivity scores than other respondents by about seven percentile ranks on average.

The correlations between non-additivity and financial behavior are economically more relevant. Financial investments require forming expectations about asset price change in the short run and in the long run. We therefore ask whether an individual's degree of additivity is related to her financial behavior. We consider three outcome variables: i) an indicator of having any financial investment (including savings, bonds, stocks, etc.), ii) the amount of investment conditional on having some investment, and iii) and an indicator for saving regularly. Table 2 contains the regression results. Columns (1) and (2) show that respondents who exhibit greater deviations from additivity are less likely to report any financial investments. The coefficient is barely affected by including socio-economic control variables (household composition, economic status and educational attainment), suggesting that the deviation from additivity is a behavioral trait whose relation with financial behavior does not simply reflect one's background. The association between non-additivity and having financial investments is remarkably strong: the coefficient of non-additivity is of similar size as the corresponding coefficient of college education (relative to the control of no educational degree). The distance between the lower and upper boundaries of the interquartile range corresponds to an increased likelihood of having investments by almost nine percentage points, which is remarkably high relative to a population average of 44% who have any investments. Columns (3) and (4) show that

[13]Figure A.2 plots the distribution of our measure for the two treatment groups (T19 and T55).
[14]Table A.10 shows the regression results.

Table 2: Correlation with Financial Outcomes

	Having investment probit		Amount of investment OLS		Saving regularly probit	
VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
Non-additivity: quantile rank	-0.195***	-0.174***	-0.262	-0.266	-0.085*	-0.060
	(0.050)	(0.047)	(0.233)	(0.222)	(0.046)	(0.041)
Vocational education		0.109**		0.259		0.096**
		(0.052)		(0.257)		(0.040)
College education		0.162***		0.848***		0.106**
		(0.055)		(0.266)		(0.046)
Controls A	Yes	Yes	Yes	Yes	Yes	Yes
Controls B	No	Yes	No	Yes	No	Yes
Observations	1,293	1,283	532	530	1,279	1,271

Note: Marginal effects displayed for Probit estimation, and coefficients for linear regressions. Controls A include gender, age groups, and being German. Controls B include marital status, number of minor children, unemployment, and log net income. Standard errors clustered at the household level in parentheses.*/**/*** indicate significance at 10%, 5%, and 1% level.

people who deviate more from additivity on average also have financial assets of lower value, although this association is insignificant. Once again, controlling for household composition, economic status and educational attainment does not affect the coefficient estimate. Columns (5) and (6) show the regression results for saving regularly. The results show that people who deviate more from additivity are less likely to save regularly, but here the correlation becomes insignificant once household composition and economic status are added as control variables.[15]

5 A Model of Non-additive Expectations

5.1 A generic compression model

In this section we develop a conceptual framework that operationalizes non-additivity of expectations and can account for the data patterns. We start from a set of stylized

[15]Extended specifications that differentiate whether a respondent exhibits sub-additive or super-additive expectations reveal that the association between non-additive expectations and having financial investment is stronger for respondents with sub-additive expectations, but significant for both groups (Table A.12). We also estimate an extended specification that differentiates whether a respondent holds a positive or negative expectations. The association between non-additive expectations and financial behavior is slightly weaker for respondents with negative expectations, but also significant for both groups (Table A.13).

empirical facts that a model should match and then formulate and estimate models of compressed time perception, where different time intervals obtain different weights.

As empirical targets, we consider moments of additivity and their relation to the signs of expectations, using data from the entire sample – without the sample restrictions that we made in Section 4.2. Panel A of Table 3 presents the means and the medians of the deviations from additivity for different constellations of the signs of reported expectations, including variations in the sign of the sum of expectations over the two shorter intervals. The entries in the first three columns show the response patterns in treatment group T19, whereas the second set of three columns shows responses in group T55. Several patterns are noteworthy. First, the mean and the median of the deviations in group T19 are of opposite sign than the expectation over the first sub-period ($x_{0,1}$). This suggests that the deviation from additivity in group T19 is strongly driven by the underlying expectation for year 1 being underweighted in the formation of $x_{0,10}$ relative to when $x_{0,1}$ is elicited directly. Second, the mean and median of the deviations in group T55 have a different sign than the indirect expectation over the ten-year period ($x_{0,5} + x_{5,10}$), with only one exception. This suggests that the deviations from additivity for group T55 are driven by a moderate underweighting of both $x_{0,5}$ and $x_{5,10}$. Taken together, this suggests the possibility that underlying "true" expectations are compressed in the reports, and the longer the period is, the greater is the compression.

A general but simple model of time perception can capture this pattern. Suppose that log-price ratios are reported as

$$x_{t,t+\tau} = \kappa(t, \tau)\xi_{t,t+\tau} \tag{8}$$

where $x_{t,t+\tau}$ is the reported expectation over the period from t to $t + \tau$, $\xi_{t,t+\tau}$ represents the "true" underlying expectation of the log price ratio over the period from t to $t + \tau$, and $\kappa(t, \tau)$ is a compression factor (or weighting factor). That is, we suppose that the decision-maker has an underlying set of expectations as the basis of her reports, but distorts them as captured by the compression factors (weights) $\kappa(t, \tau)$. Econometric discipline is imposed by assuming that all respondents have identical weights, while the underlying expectations are fully flexible for each respondent. We only assume that the underlying expectations are additive. From this assumption follows a simple relation

Table 3: Statistics of differences between the direct and the indirect expectations over the ten-year period

Panel A: Actual data

$x_{0,10} - (x_{0,k} + x_{k,10})$	Group T19 #Obs	Mean [S.D.]	Median	Group T55 #Obs	Mean [S.D.]	Median
$x_{0,k} \geq 0, x_{k,10} \geq 0$	269	-0.048 [0.078]	-0.030	300	-0.062 [0.106]	-0.039
$x_{0,k} \leq 0, x_{k,10} \leq 0$	219	0.084 [0.140]	0.051	238	0.098 [0.183]	0.051
$x_{0,k} \geq 0, x_{k,10} \leq 0,$ $x_{0,k} + x_{k,10} \geq 0$	11	-0.071 [0.056]	-0.095	15	-0.064 [0.101]	-0.040
$x_{0,k} \geq 0, x_{k,10} \leq 0,$ $x_{0,k} + x_{k,10} \leq 0$	63	-0.013 [0.064]	-0.019	27	0.027 [0.148]	0.000
$x_{0,k} \leq 0, x_{k,10} \geq 0,$ $x_{0,k} + x_{k,10} \geq 0$	70	0.041 [0.053]	0.028	26	-0.019 [0.084]	0.002
$x_{0,k} \leq 0, x_{k,10} \geq 0,$ $x_{0,k} + x_{k,10} \leq 0$	41	0.034 [0.151]	0.051	47	0.090 [0.248]	0.021

Panel B: Predicted by the general compression model

$\hat{x}_{0,10} - (x_{0,k} + x_{k,10})$	Group T19 #Obs	Mean [S.D.]	Median	Group T55 #Obs	Mean [S.D.]	Median
$x_{0,k} \geq 0, x_{k,10} \geq 0$	269	-0.057 [0.048]	-0.044	300	-0.076 [0.061]	-0.051
$x_{0,k} \leq 0, x_{k,10} \leq 0$	219	0.086 [0.082]	0.058	238	0.110 [0.107]	0.080
$x_{0,k} \geq 0, x_{k,10} \leq 0,$ $x_{0,k} + x_{k,10} \geq 0$	11	-0.053 [0.031]	-0.072	15	-0.038 [0.027]	-0.030
$x_{0,k} \geq 0, x_{k,10} \leq 0,$ $x_{0,k} + x_{k,10} \leq 0$	63	-0.022 [0.038]	-0.012	27	0.000 [0.018]	0.000
$x_{0,k} \leq 0, x_{k,10} \geq 0,$ $x_{0,k} + x_{k,10} \geq 0$	70	0.026 [0.029]	0.019	26	0.003 [0.017]	0.002
$x_{0,k} \leq 0, x_{k,10} \geq 0,$ $x_{0,k} + x_{k,10} \leq 0$	41	0.060 [0.039]	0.048	47	0.082 [0.111]	0.049

Panel C: Predicted by the univariate compression model

$x_{0,10} - (x_{0,k} + x_{k,10})$	Group T19 #Obs	Mean [S.D.]	Median	Group T55 #Obs	Mean [S.D.]	Median
$x_{0,k} \geq 0, x_{k,10} \geq 0$	269	-0.054 [0.047]	-0.043	300	-0.077 [0.063]	-0.054
$x_{0,k} \leq 0, x_{k,10} \leq 0$	219	0.083 [0.080]	0.053	238	0.112 [0.109]	0.073
$x_{0,k} \geq 0, x_{k,10} \leq 0,$ $x_{0,k} + x_{k,10} \geq 0$	11	-0.056 [0.034]	-0.077	15	-0.021 [0.017]	-0.014
$x_{0,k} \geq 0, x_{k,10} \leq 0,$ $x_{0,k} + x_{k,10} \leq 0$	63	-0.028 [0.039]	-0.017	27	0.029 [0.030]	0.017
$x_{0,k} \leq 0, x_{k,10} \geq 0,$ $x_{0,k} + x_{k,10} \geq 0$	70	0.033 [0.031]	0.022	26	-0.020 [0.017]	-0.014
$x_{0,k} \leq 0, x_{k,10} \geq 0,$ $x_{0,k} + x_{k,10} \leq 0$	41	0.064 [0.041]	0.049	47	0.045 [0.079]	0.025

Notes: Means, medians and standard deviations of differences between the direct and the indirect expectations over the ten-year period for different constellations of the signs of reported expectations. In each panel, the first/second row is for respondents who expect that the DAX will increase/decrease in both sub-periods. The third/fourth row is for respondents who expect the DAX will increase in the first sub-period and will decrease in the second sub-period, and that it will increase/decrease in the entire ten-year period. The fifth/sixth row is similar but for those who expect a decrease first and an increase afterwards. Panel A presents the statistics of the actual data. Panel B displays the predictions by the generic compression model of Section 5.1. Panel C presents the predictions by the one-parameter model of Section 5.2. The categorization is always based on the actual data.

between weights and reports:

$$\frac{x_{0,10}}{\kappa(0,10)} = \frac{x_{0,k}}{\kappa(0,k)} + \frac{x_{k,10}}{\kappa(k,10-k)} \tag{9}$$

Rearranging (9) and adding an error term ϵ, we obtain an estimable model of reported expectations for all individuals i over the three different time horizons,

$$x_{0,10;i} = \frac{\kappa(0,10)}{\kappa(0,k)} x_{0,k;i} + \frac{\kappa(0,10)}{\kappa(k,10-k)} x_{k,10;i} + \epsilon_i \; . \tag{10}$$

Estimating this model delivers estimates of the ratios of compression factors, for general instantaneous expectations.

Panel A of Table 4 reports the estimation results for this model from linear regressions with error terms clustered at the household level. Column (1) displays the estimates of the original specification. All estimates are significantly smaller than one, providing evidence for sub-additivity. The estimate of $\frac{\kappa(0,10)}{\kappa(0,5)}$ is greater than $\frac{\kappa(0,10)}{\kappa(0,1)}$ ($p = 0.0365$), suggesting that expectations over longer periods receive relatively lower weights, and thus are compressed more, than expectations over shorter periods. The estimate of $\frac{\kappa(0,10)}{\kappa(5,5)}$ is greater than $\frac{\kappa(0,10)}{\kappa(0,5)}$, suggesting that expectations over later periods are compressed more than expectations over earlier periods, though the difference is only weakly significant ($p = 0.0907$). We perform similar robustness checks as in Section 4.2, allowing for measurement error of one percentage point per report (Column (2) of Table 4), compounding error (Column (3)), or both (Column (4)). In these robustness checks, the coefficient ratios tend to be closer to the rational benchmark (unity) than in the original specification, but still lie well below it. The p-values of the tests on equality between different ratios presented in the lower half of Panel A give fairly strong evidence that the compression factor decreases in the length of the period, and weaker evidence that the compression factor also decreases in the front-end delay.

We can now use the estimates of Column (1) to generate the model's prediction about the targeted moments. Panel B of Table 3 presents the predicted differences between the direct and indirect expectations, with the layout of the different sub-groups of expectations being the same as in Panel A. Essentially all patterns in the data can be replicated by the model.

Table 4: Estimates of the compression models

Panel A: General compression model

	(1)	(2)	(3)	(4)
Allow small measurement error	No	Yes	No	Yes
Allow compounding error	No	No	Yes	Yes
$\kappa(0,10)/\kappa(0,1)$	0.147	0.295	0.177	0.326
	(0.069)	(0.074)	(0.070)	(0.075)
$\kappa(0,10)/\kappa(1,9)$	0.866	0.880	0.868	0.881
	(0.032)	(0.030)	(0.031)	(0.029)
$\kappa(0,10)/\kappa(0,5)$	0.392	0.459	0.458	0.526
	(0.094)	(0.096)	(0.081)	(0.081)
$\kappa(0,10)/\kappa(5,5)$	0.683	0.724	0.663	0.705
	(0.091)	(0.090)	(0.083)	(0.081)
Observations	1,314	1,314	1,314	1,314
R^2	0.66	0.70	0.67	0.72
P-value of two-sided test:				
$\kappa(0,10)/\kappa(0,1) = \kappa(0,10)/\kappa(0,5)$	0.037	0.180	0.009	0.070
$\kappa(0,10)/\kappa(0,5) = \kappa(0,10)/\kappa(5,5)$	0.091	0.123	0.167	0.215

Panel B: Univariate compression model

	(1)	(2)	(3)	(4)
Allow small measurement error	No	Yes	No	Yes
Allow compounding error	No	No	Yes	Yes
α	0.909	0.739	0.844	0.680
	(0.088)	(0.080)	(0.082)	(0.075)
Observations	1,314	1,314	1,314	1,314
R^2	0.65	0.70	0.67	0.71

Note: Coefficients displayed for linear regressions (Panel A) and non-linear least square estimation (Panel B). Standard errors clustered at the household level in parentheses.

5.2 A simple parametric compression model

The generic compression model is very flexible and does not impose functional forms for the compression factors. In the following we examine whether a simple parametric variant of this model can also capture the observed data patterns. This univariate compression model only allows the length of the (sub-)period to affect the compression term. In particular, we consider

$$\kappa(\tau) = \tau^{-\alpha}. \tag{11}$$

The corresponding regression model is given by

$$x_{0,10;i} = x_{0,k;i}\left(\frac{k}{10}\right)^{\alpha} + x_{k,10;i}\left(\frac{10-k}{10}\right)^{\alpha} + \epsilon_i, \tag{12}$$

where the special case $\alpha = 0$ reflects the case of additive expectations.[16] The model can be estimated using a Non-linear Least Squares estimator.

Panel B of Table 4 displays the estimates with and without allowing for measurement error and compounding error. In all columns, the estimates of α are positive, consistent with sub-additive expectations. With the estimate in Column (1), we can predict the compression term, κ, as well as the total compression over the horizon τ, $\tau\kappa$. The corresponding predictions are reported in Table A.14, which illustrates that $\kappa(\tau)$ is decreasing in τ and $\tau\kappa(\tau)$ is increasing in τ.

Using these estimates, we can again generate the predicted differences between the direct and indirect expectations and compare these patterns to the data patterns reported in Panel A of Table 3. The predictions are reported in Panel C of Table 3 and, just as for the general model, they reveal strikingly similar features as the data. That is, even the simple, single-parameter model of time weighting can predict most patterns of non-additivity that appear in our data.

6 Conclusion

This paper investigates the expectations of a representative sample of Germany's adult population about stock market growth over various periods. The analysis does not question the accuracy of expectations but documents a novel inconsistency of expectations, namely a violation of the simple benchmark of additivity, which requires expectations over a longer horizon to be consistent with expectations over shorter sub-periods that cover the same horizon. The non-additivity of expectations in our sample is pervasive and affects the majority of respondents. Robustness checks reveal that the finding of non-additive expectations is not driven by simple reporting errors or errors of compounding. The bias is systematic, with a strong tendency towards sub-additivity, and is less

[16] If $\alpha < 1$ holds in (11), $\tau\kappa(\tau)$ is increasing in τ, so that a respondent whose underlying growth expectation is constant reports a larger price change for a longer period.

prevalent among young respondents and respondents with higher levels of education. We also document that non-additivity of expectations is associated with substantially lower levels of investment and savings.

The results from estimating the simple behavioral model that we propose – mapping true expectations into reported expectations with a compression factor that represents time perception – suggest that a simple compression factor that depends on the length of time over which expectations are formed can rationalize the data patterns reasonably well. While developing a microfoundation for this compression is beyond the scope of our paper, perceptual noise that increases with the length of time horizons might provide a candidate explanation, paralleling recent work on perceptual noise in risk taking (see, e.g., Woodford, 2012; Khaw and Woodford, 2021; Frydman and Jin, 2021). Such an explanation would also provide a link to previous work on expectations over different horizons, which has found that long-term expectations exhibit greater variability and sensitivity to new information than short-term expectations (Giglio and Kelly, 2018; Bordalo et al., 2019). Our finding that time perception rather than time delays of the starting point appears to matter for the observed patterns of expectations connects the analysis of expectations formation to the sub-literature on intertemporal choice that has pointed to the possible role of concavity in subjective perceptions of time (e.g. Read, 2001; Zauberman et al., 2009). This suggests several directions for future research. An obvious next step is the wider application of time compression models to data on intertemporal choice behavior, e.g., along the lines of Dohmen et al. (2012, 2022), to explore its external validity and obtain measurements of non-constant time perception across different domains (behavior and expectations). More evidence for expectations in different contexts and over different time horizons would also be useful. It would provide a wider empirical basis for discussing possible microfoundations, or drivers, of time compression in expectations.

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Appendix

Appendix I Sample description

Table A.1: Distributions of reported price changes (percentage)

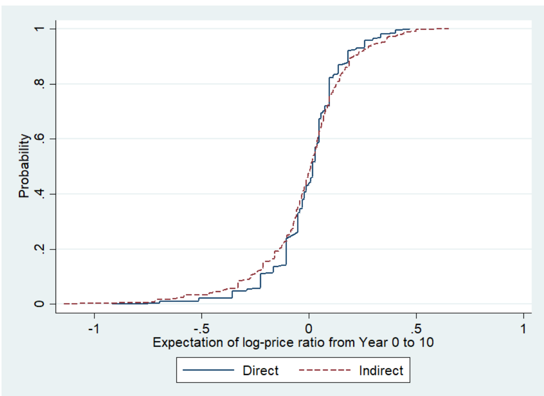
Group	Period	N	Mean	(S.D.)	Min	1%	25%	Median	75%	99%	Max
	0-1	662	-0.7	(8.5)	-30	-30	-5	0.2	4	20	30
T19	1-10	662	1.2	(15.9)	-60	-40	-6	2	10	50	60
	0-10	662	1.5	(16.6)	-60	-50	-7.5	2	10	50	60
	0-5	652	-0.6	(13.0)	-60	-40	-7	1	5	30	50
T55	5-10	652	0.8	(13.8)	-50	-40	-5	2	7.5	40	60
	0-10	652	1.0	(17.5)	-60	-50	-8	2	10	50	70

Note: Moments of expectations for different horizons.

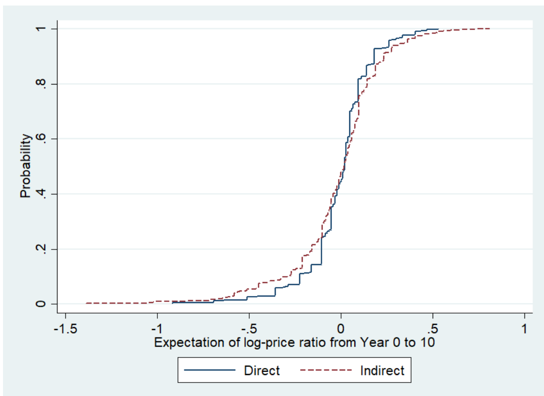
Table A.2: Distributions of direct and indirect log-price ratios over the far future

Group	Log-price ratio	N	Mean	(S.D.)	1%	25%	Median	75%	99%
T19	$x_{0,10}$	662	0.001	(0.172)	-0.693	-0.078	0.020	0.095	0.405
	$x_{0,1} + x_{1,10}$	662	-0.012	(0.219)	-0.713	-0.103	0.010	0.098	0.476
T55	$x_{0,10}$	652	-0.006	(0.184)	-0.693	-0.083	0.020	0.095	0.405
	$x_{0,5} + x_{5,10}$	652	-0.018	(0.254)	-0.868	-0.103	0.018	0.098	0.525

Note: Moments of expectations for full ten years, measured either directly or indirectly. The indirect measures (Rows 2 and 4) compute the log-price ratios from the reported expectations about the shorter time intervals, under the assumption of additivity.



(a) Group T19



(b) Group T55

Figure A.1: Empirical CDF's of direct and indirect expectations over the ten-year period

Table A.3: Statistics of background variables

Variable	Scale	Frequency	Mean	SD
Female	0,1	1,314	0.522	0.500
Age 35-49	0,1	1,314	0.194	0.396
Age 50-64	0,1	1,314	0,300	0,458
Age $\geq$ 65	0,1	1,314	0.326	0.469
German	0,1	1,293	0.967	0.179
Married or having a partner	0,1	1,312	0.583	0.493
No. minor children	$\mathbb{N}$	1,313	0.544	0.955
Unemployed	0,1	1,293	0.038	0.191
Log net income	$\mathbb{R}$	1,306	7.974	0.568
Vocational education	0,1	1,314	0.618	0.486
College education	0,1	1,314	0.299	0.458

Appendix II Details of tests of non-additivity

Table A.4 presents the numbers of respondents with additive versus non-additive sets of responses, according to the different classifications. The different columns distinguish respondents by the signs of their ten-year expectation. Column (1) reports responses by individuals with negative expectations for the next ten years, Column (2) those who expect the stock market to be constant over the 10-year horizon, and Column (3) those with positive expectations. The rows distinguish responses by their additivity properties.

Table A.4: Comparison between direct and indirect log-price ratios

		Negative growth ($x_{0,10} < 0$)	No growth ($x_{0,10} = 0$)	Positive growth ($x_{0,10} > 0$)	Total
	> 0	364	2	232	598
Diff	$= 0$	0	1	5	6
	< 0	211	2	497	710
Total		575	5	734	1,314

Note: Numbers of respondents with additive versus non-additive sets of responses. The different columns distinguish respondents by the signs of their ten-year expectation. The rows distinguish responses by their additivity properties.

The numbers in the table imply that only 6 out of 1,314 (row 2) respondents present additive expectations. The majority of responses is non-additive (rows 1 and 3). Among those non-additive responses, most are consistent with sub-additivity (364+2+2+497=865) than with super-additivity (no more than 232+211=443).

To allow for error in the reporting of expectations, we define an effective difference between the direct and the indirect expectations, in a way that tolerates a 1-percentage-point deviation in each of the three reported expectations. Formally,

$$\mathrm{Diff}^{\mathrm{Error}} = \begin{cases} \log(1+q_{0,10}-0.01)-\log(1+q_{0,k}+0.01)-\log(1+q_{k,10}+0.01) \\ \quad \text{if } \log(1+q_{0,10}-0.01) > \log(1+q_{0,k}+0.01)+\log(1+q_{k,10}+0.01) \\ \log(1+q_{0,10}+0.01)-\log(1+q_{0,k}-0.01)-\log(1+q_{k,10}-0.01) \\ \quad \text{if } \log(1+q_{0,10}+0.01) < \log(1+q_{0,k}-0.01)+\log(1+q_{k,10}-0.01) \\ 0 \\ \quad \text{else} \end{cases} \tag{13}$$

For instance, if an individual reports a 20% increase in both sub-periods, additivity would require the individual to report 44% for the entire ten-year period. The above definition implies that if the individual reports 21% for both sub-periods and 43% for the ten-year period, then $\mathrm{Diff}^{\mathrm{Error}} = 0$ and this response would be considered to be consistent with additive expectations.

Table A.5 presents the number of responses according to the same classification as in Table A.4, but after allowing for reporting errors. The error correction implies that the number of responses consistent with additive expectations increases substantially, to 504 of 1,314 responses. However, the majority of responses still implies non-additive expectations (rows 1 and 3). Moreover, the pattern prevails that more responses are consistent with sub-additivity (244+0+2+302=548) than with super-additivity (no more than 152+110=262).

Table A.5: Comparison between direct and indirect log-price ratios, allowing for small errors

		Negative growth ($x_{0,10} < 0$)	No growth ($x_{0,10} = 0$)	Positive growth ($x_{0,10} > 0$)	Total
$\mathrm{Diff}^{\mathrm{Error}}$	> 0	244	0	152	396
	$= 0$	221	3	280	504
	< 0	110	2	302	414
Total		575	5	734	1,314

Note: Numbers of respondents with additive versus non-additive sets of responses, allowing for a small error of one percentage point in each reported expectation. The different columns distinguish respondents by the signs of their ten-year expectation. The rows distinguish responses by their additivity properties.

Towards allowing for exponential-growth bias, we define the simple indirectly reported proportional growth ("simple" in the sense of simple interest, as opposed to compound interest; that is, cumulative effects are neglected) as

$$q_{0,10}^{\text{simple}} = q_{0,k} + q_{k,10} \;, \tag{14}$$

with $q_{0,10}^{\text{simple}}$ denoting the expectation over the ten-year period calculated by using this simple-interest shortcut. The analysis in the main text would imply that these respondents report non-additive expectations.

In order to allow for exponential-growth bias, we first calculate the simple-interest approximate of the indirect expectation:

$$x_{0,10}^{\text{Simple}} = \log(1 + q_{0,k} + q_{k,10}) \tag{15}$$

Then we define an effective indirect expectation as follow:

$$x_{0,10}^{\text{Nearer}} = \begin{cases} x_{0,k} + x_{k,10} & \text{if } x_{0,10} < x_{0,k} + x_{k,10} \le x_{0,10}^{\text{Simple}} \text{ or } x_{0,10}^{\text{Simple}} \le x_{0,k} + x_{k,10} < x_{0,10} \\ x_{0,10}^{\text{Simple}} & \text{if } x_{0,10} < x_{0,10}^{\text{Simple}} < x_{0,k} + x_{k,10} \text{ or } x_{0,k} + x_{k,10} < x_{0,10}^{\text{Simple}} < x_{0,10} \\ x_{0,10} & \text{else} \end{cases} \tag{16}$$

When the exact indirect expectation and the simple-interest approximate are on the same side of the direct expectation, the effective indirect expectation is either the exact indirect expectation or the simple-interest approximate, whichever is closer to the direct one. When the direct expectation is between the exact indirect expectation and the simple-interest approximate, the effective indirect expectation is set to the direct expectation.

Based on this effective indirect expectation, we can calculate an effective difference:

$$\text{Diff}^{\text{Nearer}} = x_{0,10} - x_{0,10}^{\text{Nearer}} \tag{17}$$

By testing whether $\text{Diff}^{\text{Nearer}} = 0$, we can test additivity with tolerance for compounding error. If a respondent reports a direct expectation that is between the exact indirect expectation and the simple-interest approximate, e.g., if she reports 20% for both sub-periods and a number between 40% and 44% for the entire ten-year period, then $\text{Diff}^{\text{Nearer}} = 0$ and this response is considered to be consistent with additive expectations.

Table A.6 presents the number of responses according to the same classification as in Table A.4, but after allowing for compounding errors depending on a positive, zero or negative value of $\text{Diff}^{\text{Nearer}}$. With this definition, the share of respondents that are consistent with additive expectations is, once again, significantly larger than in Table A.4. This indicates that a significant fraction of respondents may use the simple-interest shortcut when responding. Nevertheless, the majority of the respondents still present non-additive expectations that are consistent with sub-additivity.

Table A.6: Comparison between direct and indirect log-price ratios, allowing for compounding errors

		Negative growth ($x_{0,10} < 0$)	No growth ($x_{0,10} = 0$)	Positive growth ($x_{0,10} > 0$)	Total
$\text{Diff}^{\text{Nearer}}$	> 0	345	0	215	560
	$= 0$	70	3	109	182
	< 0	160	2	410	572
Total		575	5	734	1,314

Note: Numbers of respondents with additive versus non-additive sets of responses, allowing for exponential-growth bias. The different columns distinguish respondents by the signs of their ten-year expectation. The rows distinguish responses by their additivity properties.

To allow for combinations of reporting error and compounding error, we first define

$$\text{Diff}^{\text{Simple Error}} = \begin{cases} \log(1 + q_{0,10} - 0.01) - \log(1 + q_{0,k} + q_{k,10} + 0.02) \\ \quad \text{if } q_{0,10} - q_{0,k} - q_{k,10} > 0.03 \\ \log(1 + q_{0,10} + 0.01) - \log(1 + q_{0,k} + q_{k,10} - 0.02) \\ \quad \text{if } q_{0,10} - q_{0,k} - q_{k,10} < -0.03 \\ 0 \\ \quad \text{else} \end{cases} \tag{18}$$

Based on that, we define the effective difference which allows for both reporting error and compounding error in a similar way as (17):

$$\text{Diff}^{\text{Nearer Error}} = \begin{cases} \text{Diff}^{\text{Error}} & \text{if } 0 < \text{Diff}^{\text{Error}} \leq \text{Diff}^{\text{Simple Error}} \\ & \quad \text{or } \text{Diff}^{\text{Simple Error}} \leq \text{Diff}^{\text{Error}} < 0 \\ \text{Diff}^{\text{Simple Error}} & \text{if } 0 < \text{Diff}^{\text{Simple Error}} < \text{Diff}^{\text{Error}} \\ & \quad \text{or } \text{Diff}^{\text{Error}} < \text{Diff}^{\text{Simple Error}} < 0 \\ 0 & \text{else} \end{cases} \tag{19}$$

This definition implies that if a respondent reports an indirect expectation that is between the exact compound rate plus or minus small errors and the simple-interest approximate plus or minus small errors, making her $\text{Diff}^{\text{Error}}$ and $\text{Diff}^{\text{Simple Error}}$ have opposite signs or one of them be zero, we consider her responses to be consistent with additive expectations.

Table A.7 presents the number of responses according to the same classification as in Table A.4, but after allowing for response errors and/or compounding errors. As expected, the share of individuals with additive expectations is higher than in the previous cases (569 out of 1,314 responses). Yet, even with this extensive definition of additive expectations, the majority of individuals still have non-additive expectations and the pattern is consistent with sub-additivity.

Table A.7: Comparison between direct and indirect log-price ratios, allowing for small errors and compounding errors

		Negative growth ($x_{0,10} < 0$)	No growth ($x_{0,10} = 0$)	Positive growth ($x_{0,10} > 0$)	Total
$\text{Diff}^{\text{Nearer Error}}$	> 0	242	0	140	382
	$= 0$	229	3	337	569
	< 0	104	2	257	363
Total		575	5	734	1,314

Note: Numbers of respondents with additive versus non-additive sets of responses, allowing for both a small error of one percentage point in each reported expectation and exponential-growth bias. The different columns distinguish respondents by the signs of their ten-year expectation. The rows distinguish responses by their additivity properties.

Table A.8 displays the results of the tests of additivity which allow for compounding errors. Similar with Table 1, the first two rows show that the alternative hypothesis (4) is favored for the groups with non-negative expectations, while the last two rows show that the alternative hypothesis (5) is favored for the non-positive groups. Therefore, the results show taht the pattern of sub-additivity is robust to compounding errors.

Table A.8: Tests of additivity, allowing for compounding errors

Sign of $x_{0,10}$	Treatment	$\text{Diff}^{\text{Nearer}}$ defined in (17) > 0	$= 0$	< 0	All	z stat	p-value
Non-negative	T19	98	42	199	339	-5.514	0.0000
	T55	49	70	213	332	-10.148	0.0000
Non-positive	T19	171	30	68	269	7.182	0.0000
	T55	174	43	49	266	8.450	0.0000

Note: Wilcoxon signed-rank tests on the differences between the direct and the effective indirect expectations over the far future. Observations where the direct and the effective indirect expectations are of opposite signs are excluded.

Table A.9 displays the results of the tests of additivity by comparing the direct and the indirect expectations over the second sub-period. In Panel A, no errors are allowed. The first two rows of Panel A show that the direct expectation $x_{k,10}$ is greater for the non-negative groups, which is consistent with the hypothesis (4). The last two rows show that the direct expectation is farther away from zero for the non-positive groups, which is consistent with the alternative hypothesis (5).

Panel B displays the results of similar tests but allow for compounding errors. The exclusion of inconsistent observations as well as the test are based on an effective indirect expectation $x_{k,10}^{\text{Nearer}}$ defined in a similar way as in (16) and an effective difference $\text{Diff}_{k,10}^{\text{Nearer}}$ defined in a similar way as in (17). Again, the results reject the null hypothesis of additivity and are in favor of sub-additivity.

Table A.9: Tests of additivity by comparing direct and indirect expectations over the second sub-period

Panel A: No errors allowed

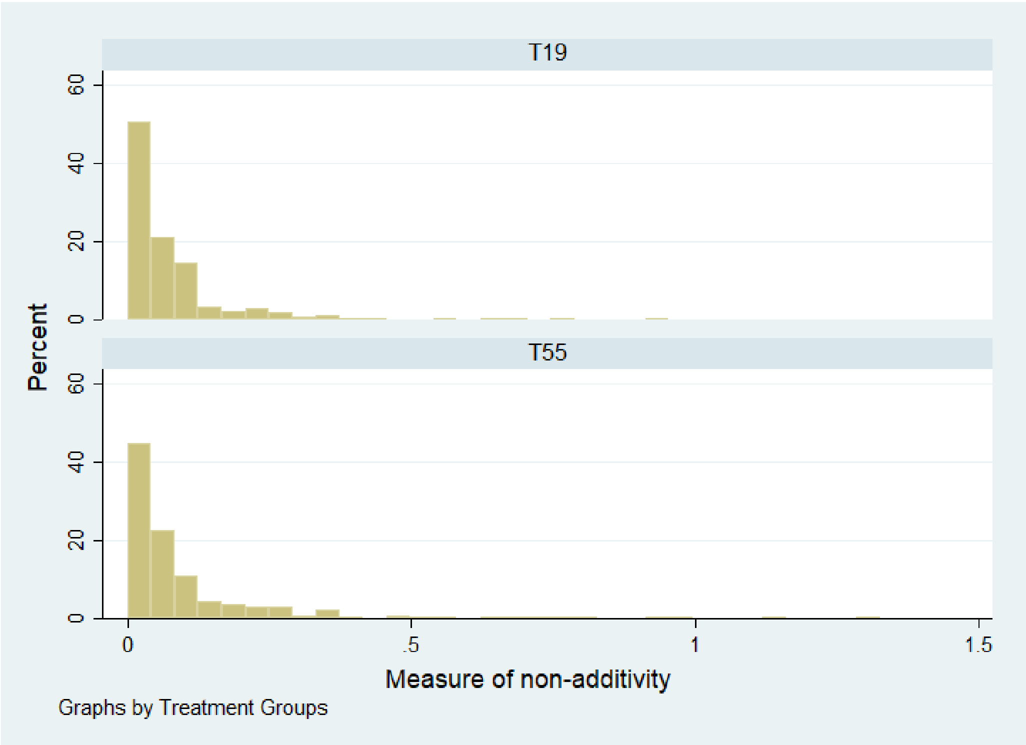
Sign of $x_{k,10}$	Treatment	$x_{k,10} - (x_{0,10} - x_{0,k})$ > 0	$= 0$	< 0	All	z stat	p-value
Non-negative	T19	195	5	124	324	2.419	0.0155
	T55	230	1	72	303	7.577	0.0000
Non-positive	T19	87	2	153	242	-5.051	0.0000
	T55	78	1	148	227	-6.074	0.0000

Panel B: Allowing for compounding errors

Sign of $x_{k,10}$	Treatment	$\text{Diff}^{\text{Nearer}}_{k,10}$ > 0	$= 0$	< 0	All	z stat	p-value
Non-negative	T19	166	46	112	324	2.557	0.0105
	T55	172	75	56	303	7.316	0.0000
Non-positive	T19	71	28	143	242	-5.297	0.0000
	T55	52	34	141	227	-6.800	0.0000

Note: Wilcoxon signed-rank tests on the differences between the direct and the effective indirect expectations over the far future. Observations where the direct and the effective indirect expectations are of opposite signs are excluded.

Appendix III Details about correlation with background variables



Note: Non-additivity is measured by the absolute difference between one's direct and indirect expectations. The upper panel dsiplays the distribution of the measure for the Treatment Group 19, and the lower panel for the Treatment Group 55.

Figure A.2: Distributions of absolute difference between one's direct and indirect expectations

In the following regression analysis of correlations with demographic variables and financial outcomes, we use the respective percentile ranks of the absolute difference between one's direct and indirect expectations within each treatment group as measure of non-additivity.

Table A.10: Correlation with Demographic Characteristics

	Non-additivity: percentile rank		
	OLS		
VARIABLES	(1)	(2)	(3)
Female	-0.009	-0.008	-0.013
	(0.015)	(0.015)	(0.015)
Age 35-49	0.018	0.031	0.031
	(0.028)	(0.029)	(0.029)
Age 50-64	0.046*	0.049*	0.052**
	(0.024)	(0.025)	(0.025)
Age $\geq$ 65	0.079***	0.081***	0.080***
	(0.024)	(0.027)	(0.027)
German	-0.049	-0.044	-0.036
	(0.044)	(0.044)	(0.044)
Married or having a partner		-0.018	-0.018
		(0.018)	(0.019)
No. minor children		-0.009	-0.011
		(0.011)	(0.011)
Unemployed		0.043	0.038
		(0.044)	(0.043)
Log net income		-0.004	0.001
		(0.017)	(0.017)
Vocational education			-0.070**
			(0.033)
College education			-0.076**
			(0.035)
Observations	1,293	1,283	1,283

Note: Linear regressions of percentile ranks of measures of non-additivity on demographic characteristics. Coefficients are displayed. Standard errors clustered at the household level in parentheses.
*** p<0.01, ** p<0.05, * p<0.1.

Table A.11: Correlation with Financial Outcomes, full table

	Having investment			Amount of investment			Saving regularly		
		probit			OLS			probit	
VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Non-additivity: quantile rank	-0.195***	-0.180***	-0.174***	-0.262	-0.259	-0.266	-0.085*	-0.067	-0.060
	(0.050)	(0.048)	(0.047)	(0.233)	(0.229)	(0.222)	(0.046)	(0.041)	(0.041)
Female	-0.065***	-0.046**	-0.035	-0.417***	-0.349***	-0.264**	-0.001	0.021	0.030
	(0.024)	(0.023)	(0.024)	(0.112)	(0.110)	(0.111)	(0.021)	(0.018)	(0.018)
Age 35-49	0.056	0.042	0.044	1.008***	0.826***	0.860***	0.053	0.035	0.035
	(0.047)	(0.047)	(0.047)	(0.236)	(0.250)	(0.246)	(0.046)	(0.044)	(0.044)
Age 50-64	0.099**	0.077*	0.076*	1.797***	1.740***	1.754***	-0.043	-0.102***	-0.108***
	(0.044)	(0.046)	(0.046)	(0.188)	(0.198)	(0.200)	(0.041)	(0.038)	(0.038)
Age $\geq$ 65	0.147***	0.160***	0.163***	1.855***	2.024***	2.068***	-0.044	-0.079*	-0.080*
	(0.044)	(0.048)	(0.047)	(0.189)	(0.218)	(0.218)	(0.042)	(0.041)	(0.041)
German	0.321***	0.275***	0.264***	0.955	1.050*	0.960*	0.227***	0.133	0.122
	(0.088)	(0.099)	(0.096)	(0.687)	(0.593)	(0.515)	(0.077)	(0.090)	(0.086)
Married or having a partner		-0.038	-0.029		-0.023	0.036		-0.022	-0.021
		(0.033)	(0.033)		(0.144)	(0.142)		(0.030)	(0.031)
No. minor children		-0.058***	-0.055***		0.027	0.024		-0.099***	-0.097***
		(0.019)	(0.019)		(0.096)	(0.092)		(0.017)	(0.017)
Unemployed		-0.254***	-0.247***		-0.448	-0.413		-0.259***	-0.253***
		(0.088)	(0.088)		(0.607)	(0.564)		(0.058)	(0.058)
Log net income		0.272***	0.254***		0.685***	0.552***		0.336***	0.329***
		(0.031)	(0.032)		(0.176)	(0.173)		(0.027)	(0.028)
Vocational education			0.109**			0.259			0.096**
			(0.052)			(0.257)			(0.040)
College education			0.162***			0.848***			0.106**
			(0.055)			(0.266)			(0.046)
Observations	1,293	1,283	1,283	532	530	530	1,279	1,271	1,271

Note: Marginal effects displayed for Probit estimation, and coefficients for linear regressions. Standard errors clustered at the household level in parentheses. */**/*** indicate significance at 10%, 5%, and 1% level.

Table A.12: Correlation with Financial Outcomes, Sub-additive and Super-additive Expectations Separated

	Having investment probit		Amount of investment OLS		Saving regularly probit	
VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
Non-additivity: quantile rank	-0.211***	-0.183***	-0.490*	-0.488**	-0.100**	-0.065
* Sub-additive expectation	(0.054)	(0.051)	(0.253)	(0.234)	(0.050)	(0.044)
Non-additivity: quantile rank	-0.167***	-0.156***	-0.003	-0.016	-0.060	-0.050
* Super-additive expectation	(0.058)	(0.055)	(0.275)	(0.271)	(0.053)	(0.047)
Vocational education		0.109**		0.239		0.097**
		(0.052)		(0.254)		(0.040)
College education		0.161***		0.823***		0.106**
		(0.055)		(0.265)		(0.046)
Controls A	Yes	Yes	Yes	Yes	Yes	Yes
Controls B	No	Yes	No	Yes	No	Yes
Observations	1,293	1,283	532	530	1,279	1,271

Note: Marginal effects displayed for Probit estimation, and coefficients for linear regressions. Controls A include gender, age groups, and being German. Controls B include marital status, number of minor children, unemployment, and log net income. Standard errors clustered at the household level in parentheses.*/**/*** indicate significance at 10%, 5%, and 1% level.

Table A.13: Correlation with Financial Outcomes, Positive and Negative Expectations Separated

	Having investment		Amount of investment		Saving regularly	
	probit		OLS		probit	
VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
Non-additivity: quantile rank	-0.202***	-0.201***	-0.423	-0.377	-0.072	-0.084
* Positive expectation	(0.066)	(0.063)	(0.316)	(0.299)	(0.062)	(0.055)
Non-additivity: quantile rank	-0.168**	-0.136*	0.072	-0.025	-0.096	-0.041
* Negative expectation	(0.074)	(0.071)	(0.333)	(0.343)	(0.064)	(0.058)
Negative expectation	-0.068	-0.061	-0.422*	-0.326	-0.004	-0.008
	(0.056)	(0.054)	(0.245)	(0.246)	(0.051)	(0.045)
Vocational education		0.113**		0.281		0.095**
		(0.052)		(0.260)		(0.040)
College education		0.165***		0.869***		0.105**
		(0.055)		(0.269)		(0.046)
Controls A	Yes	Yes	Yes	Yes	Yes	Yes
Controls B	No	Yes	No	Yes	No	Yes
Observations	1,293	1,283	532	530	1,279	1,271

Note: Marginal effects displayed for Probit estimation, and coefficients for linear regressions. Controls A include gender, age groups, and being German. Controls B include marital status, number of minor children, unemployment, and log net income. Standard errors clustered at the household level in parentheses.*/**/*** indicate significance at 10%, 5%, and 1% level.

Appendix IV Predictions of the uni-variate compression model

Table A.14 presents the compression factors, the total time weights and the relative time weights for time periods with various lengths. The results in Row 1 imply that the compression factor is decreasing in the length of period, which is consistent with sub-additivity. Row 2 demonstrates that the total weight for a period is increasing in the length of period, suggesting that the expectation of the price change in a period of time reported by a respondent who expects a constant growth rate would be increasing in the length of period. Row 3 displays the weights of a period with a certain time length relative to a ten-year period. They can be compared to the OLS estimates in Panel A of Table 4.

Table A.14: Compression factors predicted by the univariate compression model

τ	1	5	9	10
$\kappa(\tau)$	1	0.232	0.136	0.123
	-	(0.033)	(0.026)	(0.025)
$\tau\kappa(\tau)$	1	1.158	1.222	1.234
	-	(0.164)	(0.236)	(0.250)
$\frac{\kappa(10)}{\kappa(\tau)}$	0.123	0.533	0.909	1
	(0.025)	(0.032)	(0.008)	-

Note: Row 1 displays the predicted compression factors for various lengths of periods. Row 2 presents the total weights for a period with a certain length of time. Row 3 displays the time weights of a period of 10 years relative to a period of τ years, which are comparable to the OLS estimates in the generic compression model. Delta method for inference of standard errors.