Financial Convergence in the New EU Member States

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1 Introduction

In this paper we explore the issue of financial convergence in the new EU member states (NMS). For the purposes of our analysis the countries falling into the category NMS are Bulgaria, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Slovakia and Slovenia, i.e. all countries that joined the EU in the last decade, except Cyprus and Malta. The degree to which the financial markets of these countries have become more similar is interesting for several reasons. First, all of the countries considered were centrally planned economies with largely undeveloped financial services. The transition path to a market economy for some of them was very uneven in the beginning due to delayed structural reforms and bad macroeconomic policies. As a result some countries achieved markedly better outcomes in terms of increase of per capita income and price stability than others, despite the similarity in initial conditions. These differences might have influenced also the process of integration of their financial sectors with those of the EU states. Secondly, the removal of barriers to factor mobility (a prerequisite for an economic union) implies convergence of the factor returns. Since capital tends to be relatively more mobile than labour,
for example, the price equalization should take place more rapidly. Indeed
the capital flows to the financial sectors of the central and east European
countries increased substantially between the late 1990s and 2007 when the
US subprime mortgage market crisis began to unfold. This massive inflow
pushed down the interest rates and bond yields to levels very similar to,
and in some cases even lower than the corresponding returns in the more
developed EU countries. In this regard, it is important to assess to what ex-
tent these developments are sustainable and what is the equilibrium spread
between the returns in the individual countries and the returns in the euro
area for example. The state of financial integration is also important from a
policy making point of view. Well integrated markets are characterized by
faster and more complete transmission of changes in the monetary policy.
This is particularly relevant for countries with currency boards (three out
of ten in the sample) which do not set interest rates but rather "import"
the monetary policy of the ECB.

The analysis in this paper is based on two approaches to measuring
convergence which have been originally developed in the growth literature
– cross-sectional measures and time-series measures. The cross-sectional
measures have been applied in the context of financial integration in earlier
works. However, the interpretation of the empirical results in the literature
have not always been precise. Here we point to some weaknesses in the ex-
sting conditions for convergence and definitions of the speed of convergence
and propose new ones. Using data on interest rates and bond yield spreads
we estimate their equilibrium levels and calculate the speed of convergence
implied by the model. In addition, we adopt a time-series based technique
for testing for convergence which does not require that the economy is in
equilibrium and allows for structural changes in the process of transition to
equilibrium.

2 Related literature

The interest in economic convergence emerged from the work on growth
theory. One important implication of the neoclassical growth model is that
the rate of increase of per capita output is inversely related to the initial level
of this variable. This implies that if all economies have the same steady state
and differ only with respect to initial conditions, less developed economies
will grow faster than the rich ones and will eventually catch up with the
latter. This feature is typically referred to as absolute convergence. When differences in the underlying parameters of the economies are allowed, the steady state of per capita income is no longer the same for all countries and in this case the relevant concept of convergence is conditional convergence. Within the framework of conditional convergence an economy tends to grow faster the further below its own equilibrium it starts. Barro and Sala-i-Martin [4] focus their analysis on the following equation which is a log-linearized approximation of the Solow growth model with Cobb-Douglas production function:

\[ \ln y(t) = e^{-\beta t} (\ln y(0) - \ln y^*) + \ln y^*, \]

where \( y^* \) denotes the steady state value. In this setup the value of the (positive) parameter \( \beta \) determines the speed of adjustment to equilibrium. This parameter appears also in the discretized version of the growth equation which is used for empirical estimation and it has become common to refer to this type of convergence as \( \beta \)-convergence. The concept of \( \beta \)-convergence has been criticized on a number of grounds. Perhaps one of the best known critiques comes from Milton Friedman. In his article [9] Friedman cited Hotelling that the true test of convergence is a decline in the variance among individual observation. This description of convergence has become popular as \( \sigma \)-convergence [5]. For an extensive review of the literature on the various tests of convergence the reader is referred to [14].

The idea of convergence, that originated in relation with economic growth, was later applied to a wide range of problems, including the integration of financial markets. There is a well developed strand in the literature that tries to test and find evidence for the financial convergence among countries and especially, within a monetary union. Since the adoption of the Maastricht Treaty, which set the principles for further integration in the EU and the establishment of common currency, there has been substantial interest in testing the financial convergence hypothesis within the EU and the euro area. The expectations to find convergence are based on the effects of the removal of national barriers to flow of capital, explicit and market driven harmonization of regulations and supervisory standards and the mechanisms for coordination and convergence of macroeconomic policies of the national authorities within the union. Although the economic union should allow all factors of production to move freely across the borders, capital flows cross the borders much more easily than people or goods. Hence, one would ex-
pect to see a faster convergence in the financial markets as compared to the labour and goods markets.

A comprehensive methodology for analyzing the process of financial market integration in the European Union is provided by Adam et al. [1]. The authors suggest to use a variety of indicators, based on price and returns data as well as indicators based on quantities (besides the $\beta$ and $\sigma$ convergence measures) to infer about the degree of integration of capital markets in the EU. The paper by Baele et al. [2] steps on the methodology developed in [1] and complements it with some additional indicators for integration, like the response of interest rates to common news vs. local news. Also, analysis of the corporate bond market is included. They find that in the period 1993-2003 the degree of integration has increased in all markets except for the credit market. The money market is found to be the most integrated among the markets notwithstanding the differences that still remain between the various segments of the money market. As far as the new EU member states are concerned, a recent study by Baltzer et al. [3] provides valuable insights about the developments of the financial markets in these countries. Following Baele et al. [2] the authors estimate price-based, news-based and quantity-based measures of integration. They find that the financial markets in the NMS are considerably less integrated than the countries in the euro area. Yet, the process of integration is under way and the EU accession has given a significant impetus to it. Integration is more visible in the money and banking markets, both among NMS and vis-à-vis euro area, whereas in the bond markets it seems to be more relevant for the large economies.

The backbone of the financial convergence analysis in the above cited papers is the following panel regression:

$$(1) \quad \Delta r_{i,t} = \alpha_i + \beta r_{i,t-1} + \sum_{l=1}^{L} \gamma_l \Delta r_{i,t-l} + \epsilon_{i,t},$$

where $r_{i,t}$ denotes the spread between the return on some asset in country $i$ and the return of the benchmark asset, $\Delta$ is the difference operator, $\epsilon_{i,t}$ is the error term and $\alpha_i$ is a country-specific constant. The main parameter of interest in this equation is $\beta$. Negative and statistically significant estimate of this parameter implies that returns in countries with high returns tend to decrease more rapidly. This specification was considered by Evans and
Karras [8] in their study of economic growth.

Along with the $\beta$ and $\sigma$ measures, which are based on cross-sectional data, there are a number of studies which employ time-series tests of convergence. The time-series notion of convergence requires that the forecasts of the variable of interest $y$ in countries $i$ and $j$, given the available information $I_t$ at time $t$, should be equal, i.e. \( \lim_{T \to \infty} E(y_{j,t+T} - y_{i,t+T} | I_t) = 0 \) (see for example [7]). This approach to convergence is associated with a number of assumptions, e.g. zero mean stationarity of the difference of the variables.

Datta [6] noted that co-integration based tests, which assume structural stability, would fail to detect convergence if the countries are still in the process of converging. She proposed an alternative approach to capture the idea that the cross-country differences reflect the fact that the steady-state has not been reached yet. Furthermore, the process may not be "smooth", i.e. there could be structural changes which could lead to a rejection of the hypothesis of convergence. To overcome this shortcoming Datta suggests estimating a time-varying parameter model where the parameter $\beta_t$ changes over time following a first-order autoregressive process. In doing so she uses a definition of convergence as catching up: if $y_{i,t} > y_{j,t}$ then $E(y_{i,t+k} - y_{j,t+k} | I_t) < y_{i,t} - y_{j,t}$. The model used to infer about convergence is written in a state-space form:

\[
(2) \quad y_t = X_t \beta_t + \epsilon_t \\
(3) \quad \beta_t = M \beta_{t-1} + \nu_t.
\]

Here $X_t$ is a matrix of exogenous variables and $M$ is a diagonal matrix. The time-varying parameters $\beta_t$ are estimated with the Kalman filter. The "convergence as catching up" concept has been applied for a sample of OECD countries by estimating the following equation:

\[
\ln Y_i(t) = \beta_0(t) + \beta_1(t) \ln Y_{US} + \epsilon_i(t),
\]

where $Y_i$ and $Y_{US}$ denote the per-capita GDP of country $i$ and US, respectively. Convergence in this setup is measured by the difference $(Y_{US} - \hat{Y}_i)$, i.e. the difference between the actual value for the reference economy and the fitted value from the time-varying parameter (TVP) model for country $i$. A declining trend of this difference signals convergence.
3 Some caveats in the interpretation of results

Before we proceed with the data description and the estimation of the models it is necessary to make some clarifications regarding the interpretation of the results from the tests of convergence based on equation (1). In [1], [2] and [3] the parameter $\beta$ is interpreted as the speed of convergence.\footnote{This interpretation is also found in [12], where the authors mention that they have borrowed the model formulation from the study on unit-root testing in panel data by Levin and Lin [13].} Below we shall show that the interpretation of $\beta$ as speed of convergence in the framework of model (1) is misleading. We start from the simplest version of model (1), where $\beta$ has indeed the meaning of speed of convergence, and then show that for the more general specification this is not the case. Consider equation (1), where all $\gamma_l = 0$ and ignore the stochastic term (or alternatively take the expectation). Thus, equation (1) reduces to

\begin{equation}
\Delta r_t = \alpha_i + \beta r_{t-1},
\end{equation}

Suppose further that $\alpha_i = 0$. Then, the above is equivalent to the following first-order linear difference equation:

$$r_t = (1 + \beta) r_{t-1}.$$ 

The solution of this equation is:

$$r_T = (1 + \beta)^T r_0.$$ 

Clearly, whenever $|1 + \beta| < 1$, which implies $-2 < \beta < 0$, the spread will tend to zero as $T \to \infty$ and $\beta$ is a meaningful measure of the speed of convergence. Now go back to model (1) and assume that it has been estimated with one lag of $\Delta r$. In other words, we consider the following equation:

$$\Delta r_t = \alpha_i + \beta r_{t-1} + \gamma \Delta r_{t-1}$$

$$r_t = \alpha_i + (1 + \beta + \gamma) r_{t-1} - \gamma r_{t-2}.$$
This is already a second-order difference equation and by setting $q_t = r_{t-1}$ it is equivalent to the following system:

$$(5) \quad \begin{pmatrix} r_t \\ q_t \end{pmatrix} = \begin{pmatrix} \alpha_i \\ 0 \end{pmatrix} + \begin{pmatrix} 1 + \beta + \gamma & -\gamma \\ 1 & 0 \end{pmatrix} \begin{pmatrix} r_{t-1} \\ q_{t-1} \end{pmatrix}.$$ 

Denote

$$A = \begin{pmatrix} 1 + \beta + \gamma & -\gamma \\ 1 & 0 \end{pmatrix}$$

and

$$b = \begin{pmatrix} \alpha_i \\ 0 \end{pmatrix}.$$ 

Thus, we study the system

$$y_t = Ay_{t-1} + b,$$

the solution of which at time $t = T$ is

$$y_T = A^T y_0 + \sum_{t=0}^{T-1} A^t b.$$ 

It is well known that the properties of the solution will depend on the eigenvalues of the matrix $A$. In this simple case we can compute these eigenvalues as solutions to the characteristic equation:

$$\det(A - \lambda I) = 0.$$ 

The two eigenvalues are:

$$\lambda_{1,2} = \frac{1 + \beta + \gamma \pm \sqrt{(1 + \beta + \gamma)^2 - 4\gamma}}{2}.$$ 

It is easily seen that the stability property of the solution depends substantially on the parameter $\gamma$. Even if $\beta < 0$, unless additional assumptions are made, the dynamics may be divergent or oscillatory for some values of $\gamma$.

The above argument suggests that negative $\beta$ alone does not guarantee convergent behaviour, neither it can be interpreted as speed of convergence.
If the matrix \((I - A)\) is invertible, then the equilibrium solution of the system (5) is given by

\[
y^* = (I - A)^{-1}b.
\]

The stability property of the equilibrium depends on the absolute values (the moduli) of the eigenvalues \(\lambda_i(A)\) of the matrix \(A\). Thus, the equilibrium is stable if and only if all eigenvalues of \(A\) have moduli smaller than 1. (Recall that for a complex eigenvalue \(\lambda_i\) the modulus is defined as \(|\lambda_i| = (a^2 + b^2)^{1/2}\), where \(a\) and \(b\) are the real and imaginary part of \(\lambda_i\), respectively.)

For system (5) it is straightforward to calculate the equilibrium value to which the spread would converge starting from any initial value, provided that \(|\lambda_i(A)| < 1\). This value is:

\[
r^* = -\frac{\alpha_i}{\beta}.
\]

It turns out that in the more general formulation with an arbitrary number of lags \(L\), the equilibrium is the same. This fact is pointed out in [12] (p. 15) but no reference or proof is given there. We show how it can be derived in the Appendix.

An important question is how to measure the speed of convergence. As discussed above, the value of the coefficient \(\beta\) is not a good measure of the speed of convergence. Consider two versions of model (1) with \(L = 1\), where \(\alpha_i\) and \(\beta\) are the same, but the estimated values of \(\gamma\) differ. To be more concrete, suppose that \(\alpha = 0.01, \beta = -0.1, \gamma^{(1)} = 0.1\) and \(\gamma^{(2)} = 0.4\), where the superscripts \((1)\) and \((2)\) denote the two different values of the parameter \(\gamma\) used in the model. The evolution of the spread is displayed in Figure 1. As one can see, both series converge to the steady state equilibrium of 0.1 but the series, corresponding to \(\gamma = 0.4\), approaches this value faster. This is so because the largest eigenvalue of the matrix \(A^{(2)}\) is smaller in absolute value than the the largest eigenvalue of the matrix \(A^{(1)}\). To see how the speed of convergence depends on the eigenvalues of the system matrix, assume that \(\alpha_i = 0\) and that the matrix \(A\) has distinct real eigenvalues. The latter implies that \(A\) can be represented as \(A = QDQ^{-1}\), where \(Q\) is a non-singular matrix, composed of the eigenvectors \(v_i\) of \(A\), and \(D\) is a diagonal matrix, the elements of which are the eigenvalues of \(A\). The general solution to the system of difference equation (in the two-dimensional case) can be written as
Figure 1: Different speed of convergence for different values of $\gamma$

$$y_t = c_1 \lambda_1^t v_1 + c_2 \lambda_2^t v_2,$$

where $c = (c_1, c_2)' = Q^{-1} y_0$. If $|\lambda_i| < 1$, $i = 1, 2$, one can see that as time increases, the dynamics of the system will be governed by the larger eigenvalue since it will tend to zero more slowly. Furthermore, the smaller the modulus of the larger eigenvalue, the faster the system tends to equilibrium. The above arguments remain valid for an arbitrary system. If the system is not homogeneous, make the transformation $z_t = y_t - y^*$ and consider the system $z_t = A z_{t-1}$. If $A$ has repeated or complex eigenvalues, the matrix $D$ is the normal Jordan form of $A$ (see [10]). The preceding discussion suggests a natural choice for measure of the speed of convergence – the modulus of the largest eigenvalue of the system matrix $A$. We shall use precisely this measure in the remaining of the paper.
4 Data and Estimation results

The data used for testing for financial convergence of the new EU member states comprise bond yields in accordance with the EU convergence criterion and three types of interest rates – unsecured overnight interbank interest rate, deposit and lending rates for the period 1999-2008. Interest rates and bond yields refer to instruments in local currency. Interest rates on deposits and credits are for the flow of newly attracted deposits and newly extended loans. Bonds yields are for ten years government bonds denominated in local currency. For Estonia, where there are no ten-year government bonds, the data refers to five-year corporate loans denominated in local currency. The bond yields are taken from Eurostat and the source of the interest rate data is IMF’s International Financial Statistics. Since the series on lending and deposit rates for the euro are available only until 2003, for the rest of the sample period in order to construct a single measure for lending and deposit rates we use weighted averages of the interest rates on various loans and deposits as reported by European Central Bank.

A quick look at the data suggests that there has been a substantial decline in the interest rates and the bond yields in most of the countries since 1999. To test the hypothesis for $\beta$-convergence we estimate model (1) for each of the price measures (the government bond yields and the three types of interest rates). Since the model specification as it appears in (1) is a general one, we need to choose the number of lags. The inclusion of the lagged values of $\Delta r$, according to Goldberg and Verboven [12], accounts for possible serial correlation in the error term. To determine the number of lags they use the so-called "top-down" approach, i.e. start with a reasonably high number of lags $L$ (in their case $L = 5$) and look at the $t$-statistic of the coefficient on the longest lag. If $|t| < 1.96$, re-estimate the equation with $L - 1$ lags. Repeat this procedure until the $t$-statistic of the coefficient on the longest lag is greater than 1.96 in absolute value. Here we adopt the same approach with respect to the choice of the number of lags, starting with $L = 5$. According to this criterion, we choose $L = 3$ for bond yields, $L = 4$ for interest rates on deposits, $L = 5$ for interbank interest rates and $L = 4$ for lending rates. In all equations, except the one for government bond yields, the estimated coefficients imply convergence to a steady-state equilibrium. In the equation for bond yields the coefficient $\beta$ is estimated at 0.05 and the largest eigenvalue of the system matrix is 1.05, implying divergent behavior. This result could reflect the fact that
bond yields generally depend on the fiscal policies pursued by the individual countries, which range from substantial deficits (e.g. Hungary) to large surpluses (e.g. Bulgaria). Moreover, since yields are in local currency, the perceived exchange rate risk could be a source of considerable yield differentials. Furthermore, the countries in the sample have very different amount of outstanding ten years government bonds, which implies differences in the liquidity and liquidity premia.

Following Baele et al. [2] and Baltzer et al. [3] we estimated equations (1) with two lags for each of the four spreads. The results were quite different in that the convergence property, established for the interest rate variables under the richer lag structures, now disappeared for some of them. Therefore, we do not report the estimation results for $L = 2$.

On Figures 3-5 in the Appendix we present simulation of the dynamics of the interest rate spreads based on the estimation of model (1). We take as initial conditions the latest available observations and simulate the dynamics for 50 periods ahead. Generally, after some irregular behavior in the beginning (due to the presence of complex eigenvalues), the spreads converge monotonously to the steady state, either from above or from below (for countries which in 2008 had smaller spreads than their equilibrium levels). Regarding the speed with which convergence takes place, it is relatively higher for deposit and interbank interest rate spreads (the largest eigenvalues being 0.968 and 0.975, respectively), whereas differentials in the lending rates appear more persistent ($\max \lambda_i = 0.993$). The banking systems of the NMS in the sample are dominated by foreign banks, which hold on average over 70% of the bank assets in these countries. In some cases, like Estonia, this percentage approaches 100%. This is a very important factor that affects the speed of convergence of deposit and overnight interbank interest rates. It is well known that within the EU the lending market is segmented and households and firms mainly borrow from the local banks, which explains the lower speed of convergence of lending rates. Also, for the lending interest rates differentials, one notices that the estimated long run equilibria in most of the countries in the sample are negative numbers. In other words, interest rates on loans in the NMS should be smaller in the long run than those in the euro area. Clearly, this is hardly plausible. This results is probably influenced by the estimation of the model parameters in a period of strong competition of euro area banks for market share in the NMS economies. In some cases this competition for market shares has
lead to underpricing of credit risks and hence to unrealistically low lending interest rates. It should be emphasized that the simulated spread evolution displayed on Figures 3-5 is based on the point estimates of the parameters in equation (1). While the estimates of $\beta$ are statistically significant at the 1% level (except for the loans equation), most of the estimates of the individual fixed effects are not statistically significant at the conventional levels. Therefore, the calculated equilibrium spreads are associated with considerable uncertainty. If one constructs confidence intervals and takes the extreme points of these intervals, the resulting equilibrium values can differ by as much as 40 percentage points (in the case of loans), rendering these estimates not particularly useful. It would be interesting to compare our results to those in Baltzer et al. [3] but unfortunately they report only the estimates of $\beta$ and not the individual effects, so the equilibrium values of the spreads cannot be computed.

In [3] the authors divide their sample into two sub-samples (before and after 2001) and compare the estimates of $\beta$. Running the panel regression for both periods they obtain negative and statistically significant (at the 10% level) estimates. For most of the indicators the estimates of $\beta$ in the two periods are not statistically different. Replicating this exercise with our data by dividing the sample into pre- and after 2004, produces very different results – the estimates of $\beta$ are not close and in some cases the signs are opposite, implying qualitative differences in the behaviour.

To measure the so-called $\sigma$-convergence we calculate the cross-sectional coefficients of variation (the standard deviation divided by the mean) for the bond yield and interest rates spreads. If the cross-sectional coefficients of variation exhibit a declining pattern over time this indicates convergence. The results are shown on Figure 2.

Until 2007 the coefficient of variation is on a declining trend for all series. After that it starts to increase which could be attributed to the effects from the global financial crisis and the rising perception of risk in these economies.

As discussed in Adam et al. [1], $\beta$-convergence is a relevant measure for the speed of adjustment to the long run equilibrium value and $\sigma$-convergence measures if the spreads become more similar over time. Both measures rely on cross-sectional data. Alternatively, one can take the time series approach to convergence, which essentially postulates that as time goes to infinity the expected returns tend to equalize. As mentioned earlier, the time series concept of convergence is associated with a number of assumptions, some of
which may not hold for economies in transition. To account for the fact that these economies may not be in equilibrium and the transitional dynamics may be associated with structural changes we apply the time-varying parameter regression model proposed in [6]. We estimate the following model in state space form:

\[ r_{i,t} = \beta_{0,t} + \beta_{1,t} r_{eu,t} + \epsilon_t \]

\[ \beta_{0,t} = \beta_{0,t-1} + \nu_{1,t} \]

\[ \beta_{1,t} = \beta_{1,t-1} + \nu_{2,t}, \]

where \( r_i \) is the return in country \( i \) and \( r_{eu} \) is the return in the euro area. The difference between this model and the model (2)- (3) is that we restrict the matrix \( M \) to be the identity matrix, i.e. we consider the case when the time-varying parameters follow a unit root process. This choice is justified by the fact that main issue of interest here is the stability of the coefficients \( \beta_i \) and the pattern of their variation, rather than estimating the specific stochastic
process that generates them. As Garbade [11] argues, a more appropriate setup for explaining the variation in the regression coefficients would be to think of a structural model with exogenous explanatory variables rather than considering more complicated stochastic processes.

The results from the estimation of the TVP models for interest rates on loans are presented in the Appendix. The estimates of the unknown parameters $\text{Var}(\epsilon)$, $\text{Var}(\nu_1)$, $\text{Var}(\nu_2)$ and the inferences on the regression coefficients have been obtained by applying the Kalman filter, using routines included in the Matlab Econometric Toolbox developed by James LeSage.

There does not seem to be convergence of bond yields and of the inter-bank interest rates, except for Poland and Romania and to some extent for Slovakia and Slovenia. For the latter countries convergence is also observed with respect to the interest rates on deposits. Regarding the interest rates on loans, the TVP measure suggests that convergence was ongoing in most of the countries until some time in 2007 when the process was terminated, perhaps due to the global turbulence. This is particularly evident in the estimates for the Baltic countries, and to some extent for Bulgaria.

5 Conclusions

The analysis of financial convergence in the new EU member states reveals a mixed picture. The application of the $\beta$-convergence concept suggests that the spreads of the interbank interest rates and the interest rates on bonds tend to move towards a long run equilibrium. For government bonds yields the convergence hypothesis is not supported by the data which may reflect the heterogeneity of fiscal policies in the EU. The $\sigma$-convergence measure in general indicates convergence until 2007. After that, most likely due to the effects of the global financial crisis and the associated uncertainty, the coefficient of variation of spreads increases. The effects of the financial crisis are also evident in the time-series measure of convergence based on estimation of a time-varying parameter model. In some countries a tendency of decline in spreads was observed initially which was later reversed. One possible extension to the work presented in this paper is to estimate the panel data model (1) with country-specific coefficients $\beta_i$. This would allow for differences in the speed of convergence among countries and would possibly yield more reliable results when computing the equilibrium levels of the spreads.
Also, conditional on data availability the analysis can be extended to include other asset prices or a more detailed breakdown of interest rates by instrument. The use of aggregated deposit and lending rates may disguise specific features of the credit and deposit markets.

References


APPENDIX

Below we show how the equilibrium vale of the spreads in model (1) can be computed. Consider the following system of first-order difference equations:

(6) \[ x_t = \hat{A}x_{t-1} + \hat{b}, \]

where the matrix \( \hat{A} \) and the vector \( \hat{b} \) are defined as follows:

\[
\hat{A} = \begin{pmatrix}
1 + \beta + \gamma_1 & \gamma_2 - \gamma_1 & \ldots & \gamma_L - \gamma_{L-1} & -\gamma_L \\
1 & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & 0 & 0 \\
0 & 0 & \ldots & 1 & 0 \\
\end{pmatrix},
\hat{b} = \begin{pmatrix}
\alpha_i \\
0 \\
\vdots \\
0 \\
\end{pmatrix}.
\]

Proposition 1 Suppose that the matrix \( I - \hat{A} \) is non-singular and the matrix \( \hat{A} \) has eigenvalues \( \lambda_i(\hat{A}) \), \( i = 1, 2, \ldots, L+1 \), such that \( |\lambda_i| \leq 1 \). Then, the stable equilibrium value \( \bar{x} \) of \( x_t \) is equal to \( -\alpha/\beta \).

Proof. We need to show that

\[
(I - \hat{A})^{-1} \hat{b} = \begin{pmatrix}
-\alpha_i/\beta \\
\vdots \\
-\alpha_i/\beta \\
\end{pmatrix}.
\]

For the purpose we shall calculate the matrix \( B^{-1} = (I - \hat{A})^{-1} \). Note that

\[
(I - \hat{A}) = B = \begin{pmatrix}
-\beta - \gamma_1 & \gamma_1 - \gamma_2 & \ldots & \gamma_L - \gamma_{L-1} & \gamma_L \\
-1 & 1 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & 1 & 0 \\
0 & 0 & \ldots & -1 & 1 \\
\end{pmatrix},
\]

Denote the transpose of \( B \) as \( B^T \) and recall that \( (B^{-1})^T = (B^T)^{-1} \). The matrix \( (B^T)^{-1} \) can be computed easily through Gauss-Jordan elimination. Recall that the Gauss-Jordan method consists of concatenating the identity matrix \( I \) to the matrix \( B \) and using elementary row operations on the matrix
$[B^T | I]$ to obtain the matrix $[I | (B^T)^{-1}]$ if possible. So we first construct the matrix $[B^T | I]$:

\[
B^T | I = \begin{pmatrix}
-\beta - \gamma_1 & -1 & \ldots & 0 & 0 & 1 & 0 & \ldots & 0 & 0 \\
\gamma_1 - \gamma_2 & 1 & \ldots & 0 & 0 & 0 & 1 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\gamma_{L-1} - \gamma_L & 0 & \ldots & 1 & 1 & 0 & 0 & \ldots & 1 & 0 \\
\gamma_L & 0 & \ldots & 0 & 1 & 0 & 0 & \ldots & 0 & 1 
\end{pmatrix}
\]

Denote by $R_j$ the $j^{th}$ row of $[B^T | I]$. One can easily check that by adding $R_j$ to $R_{j-1}$, starting from the last row, one obtains the following matrix:

\[
B^T | I = \begin{pmatrix}
-\beta & 0 & \ldots & 0 & 0 & 1 & 1 & \ldots & 1 & 1 \\
\gamma_1 & 1 & \ldots & 0 & 0 & 0 & 1 & \ldots & 1 & 1 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\gamma_{L-1} & 0 & \ldots & 1 & 0 & 0 & 0 & \ldots & 1 & 1 \\
\gamma_L & 0 & \ldots & 0 & 1 & 0 & 0 & \ldots & 0 & 1 
\end{pmatrix}
\]

Next, multiply $R_1$ by $\gamma_i/\beta$, $i = 2, \ldots, L$ and add it to the $i^{th}$ row. Finally, divide $R_1$ by $-\beta$. Thus,

\[
(B^{-1})^T = \frac{1}{\beta} \begin{pmatrix}
-1 & -1 & \ldots & -1 & -1 \\
\gamma_1 & \gamma_1 + \beta & \ldots & \gamma_1 + \beta & \gamma_1 + \beta \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\gamma_{L-1} & \gamma_{L-1} & \ldots & \gamma_{L-1} + \beta & \gamma_{L-1} + \beta \\
\gamma_L & \gamma_L & \ldots & \gamma_L & \gamma_L + \beta 
\end{pmatrix}
\]

and hence,

\[
x^* = B^{-1} \hat{b} = \begin{pmatrix}
-\alpha_i/\beta \\
\vdots \\
-\alpha_i/\beta 
\end{pmatrix}.\]
Figure 3: Simulated dynamics of interbank interest rate spreads
Figure 4: Simulated dynamics of deposit interest rate spreads
Figure 5: Simulated dynamics of loan interest rate spreads
Figure 6: TVP measure of convergence for bond yields
Figure 7: TVP measure of convergence for deposit interest rates
Figure 8: TVP measure of convergence for interbank interest rates
Figure 9: TVP measure of convergence for lending interest rates