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Endogeneity of household size and income in the estimation of equivalence scales from satisfaction data

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Abstract

Analyses of income distributions across households crucially depend on equivalence scales. They define income increments necessary to keep a household's living standard constant as it is joined by additional adults or children. Such scales have frequently been estimated using income satisfaction data, yet under the assumption that household income, size, and structure are exogenous. The present paper is the first to relax this assumption and consider the possible endogeneity of income and family size in income satisfaction. This involves an empirical analysis of data from the German Socio-Economic Panel (SOEP) using fixed-effects regressions with heteroscedasticity-based instruments. Our results confirm that endogeneity is relevant in regressions of income satisfaction; equivalence weights, however, appear not to be biased significantly. Accounting for endogeneity in income and family size has virtually no implications for distribution and poverty analyses.

JEL Codes: I32, J13, D31

Keywords: equivalence scale, income satisfaction, endogeneity, internal instruments

1 Introduction

Equivalence scales are important tools used in the analysis of economic inequality and poverty. They summarize percentage increments in expenditure needed to keep a household's welfare constant as it is joined by additional members. Among the various approaches to estimating equivalence scales, one strand in the literature uses subjective evaluations of satisfaction with own household income to proxy households' living standards. In the last 20 years, various studies using panel data on income satisfaction have shown that equivalence weights for adults and children are typically smaller than those suggested by the commonly applied OECD or square-root scale (e.g., Schwarze 2003, van Praag and Ferrer-i-Carbonell 2004, Rojas 2007, Biewen and Juhasz 2017, Buetikofer and Gerfin 2017, Borah et al. 2019, Rapp 2021). Regressing satisfaction on household income, family size and composition along with other explanatory variables, all of these studies assume that both, income and the number of household members, are exogenous.

In this paper, we question this assumption arguing that household income as well as family size may not only be a determinant but also an outcome of income satisfaction. Several studies have provided empirical evidence for a positive effect running from satisfaction to income (De Neve and Oswald 2012, Elsas 2021, Graham et al. 2004, Mishra and Smyth 2014, Prati 2017). This link may exist because of actual causality but also because of systematic misreporting. Similarly, marital status, fertility, and hence family size and composition seem to be explained by satisfaction (Stutzer and Frey 2006, Parr 2010, Le Moglie et al. 2015, Cetre et al. 2016, Mencarini et al. 2018). These studies suggest that happier individuals are more likely to get married and have children. Thus, one may expect a positive effect of satisfaction on household size.

These findings call for assessing the consequences of endogeneity of income and household size in satisfaction. To check if regression coefficients and derived equivalence scale parameters are biased when exogeneity is assumed but not met, we compare results from the conventional model to results from estimations in which instruments for household income and family size and structure are used. Because plausible external instruments are absent, we apply internal instruments as proposed by Lewbel (2012). In an empirical application using data from the German Socio-Economic Panel (SOEP v38.1), we find that there is considerable heteroscedasticity in income, family size and family structure across the respondent's age and the interviewer's birth year distribution. This can be used to construct strong internal instruments that we apply in the identification of household equivalence scales.

Our estimation results suggest that endogeneity of household income and of household size and structure attenuates the estimated effects in the satisfaction regression. Therefore, the true costs of additional household members and the benefits of higher household income are understated. Effects on the estimated equivalence scale are negligible, though. They appear more sizeable only if households whose children have just passed the scale-relevant age threshold are excluded from the estimation sample.

The paper is structured as follows. Section 2 introduces the conventional model used to estimate equivalence scales from income satisfaction data. It discusses the likely causes and consequences of endogeneity of household size and structure, and endogeneity of income in the satisfaction regression and presents our approach of using internal instruments to estimate unbiased coefficients. Section 3 introduces the data and reports descriptive statistics. Section 4 shows our estimation results and their implications for the analysis of income distributions and poverty risks. Section 5 concludes.

2 Model

There is a large strand in the economic literature that is concerned with the assessment of household economies of scale and the cost of children. Over the last five decades, contributions to this strand have made increasing use of subjective survey data to estimate income equivalence scales. Especially in the analysis of income satisfaction, important advances have been made with respect to estimation methods and model specification (e.g., Biewen and Juhasz 2017, Borah et al. 2019, Rapp 2021). Despite of differences in the underlying research designs, all of these recent studies confirm earlier findings that the consumption needs of additional household members are relatively small (e.g., Schwarze 2003, van Praag and Ferrer-i-Carbonell 2004, Rojas 2007). Borah et al. (2019) and Rapp (2021), however, disagree with the result that children's needs are significantly lower than those of additional adults.

The pivotal paper among those mentioned is that by Schwarze (2003). The author builds on Coulter et al. (1992) and proposes a linear model of latent income satisfaction that distinguishes equivalence weights according to the household members' age. In this paper, we will also use a similar linear model specification to derive our baseline results. We will then illustrate the consequences of endogeneity in household size, structure and income in this very framework. Throughout the analysis, however, we will assume cardinality of the income satisfaction data [1].

[1] Ferrer-i-Carbonell and Frijters (2004) found that results from happiness regressions are not particularly sensitive to the choice of estimators for discrete data versus those for continuous data. Their work has set the standard for estimations in empirical happiness research and our subsequent empirical analysis.

To clarify the basis of our analysis, we will now briefly introduce the fixed-effects model that Schwarze (2003) employed, where we make the additional assumption that latent equals stated satisfaction.[2] Suppose that income satisfaction s stated by respondent i at time t represents an evaluation of this household member's consumption possibilities, i.e. equivalent income, rather than unadjusted household income y. If we assume the marginal utility of equivalent income to be decreasing, income satisfaction s_{it} can be expressed as a function of log equivalent income and other potentially important control variables X_{it}. α_i represents the individual-fixed effect.

$$s_{it} = \beta_1 \ln\left(\frac{y_{it}}{EqSc_{it}}\right) + X_{it}'\gamma + \alpha_i + \varepsilon_{it} \tag{1}$$

Let the equivalence scale $EqSc$ be of the functional form $EqSc_{it} = h_{it}^{(a-bk_{it})}$, where h is the number of household members, k is the number of children and a and b are the equivalence scale parameters of interest. Parameter a represents the equivalence scale elasticity for households whose members are all adults. It determines the percentage increase in income necessary to keep the household members' financial welfare constant as the household experiences a relative increase in the number of adult household members. Parameter b describes a linear decline of the equivalence scale elasticity in the number of children at a given household size. It thus captures possibly lower needs of children compared to adults. With this formulation of the equivalence scale, the above equation can be translated into the following linear equation.

$$s_{it} = \beta_1 \ln(y_{it}) - \beta_1 a \ln(h_{it}) + \beta_1 b k_{it} \ln(h_{it}) + X_{it}'\gamma + \alpha_i + \varepsilon_{it} \tag{2}$$

Equivalence scale parameters a and b can thus be identified by dividing the negative of the coefficient on log household size and the coefficient on the interaction between log household size and the number of children by the coefficient on log household income, respectively.

In the first step of our empirical analysis, we will conduct linear fixed effects regressions of Eq. (2). We argue, however, that the results from this replication may be biased. This is because endogeneity of income, household size and the number of children in income satisfaction implies that the crucial explanatory variables will be correlated with the error term in our regression model. Individual fixed effects and relevant control variables could help to mitigate this problem,

[2] Arguing that latent satisfaction cannot be observed, Schwarze (2003) estimates an ordered probit model on pooled cross-sections and a binary probit model with individual fixed effects.

but fixed effects can only cure time-constant endogeneity and observable controls could themselves be endogenous. Crucial coefficients may thus be expected to suffer from endogeneity bias, which could potentially affect the estimated equivalence scales as well. In the following, we will discuss possible reasons and consequences of endogeneity and present our hypotheses regarding the direction of bias in the respective coefficients and the resulting equivalence scale. After that, we will introduce our approach to empirically estimate the extent of this bias with the help of internal instruments.

2.1 Endogeneity Bias

2.1.1 Endogeneity of household size changes

Suppose that income satisfaction is an appropriate measure of individual household members' consumption possibilities and that there are no perfect economies of scale. Under these conditions, an increase in the number of household members at a given income causes a decrease in satisfaction. Hence, the coefficient on log household size ($-\beta_1 a$) in Eq. (2) can be expected to be negative. Now suppose that income satisfaction not only depends on household size but that it also predicts it. This may be the case (1) when there are true causal effects of income satisfaction on household size changes, (2) when income satisfaction adjustments precede household size changes due to anticipation effects, and/or (3) when there is a spurious relationship via individual characteristics affecting both, survey responses and household size changes. The latter possibility is unproblematic as far as time-invariant characteristics are concerned, and the estimation controls for individual fixed effects. If, for instance, more optimistic individuals generally report higher levels of satisfaction, ceteris paribus, and are also more likely to form a couple or decide to have children, this will not pose a problem in fixed effects estimations of the effect of household size on income satisfaction. But if certain traits become apparent in changes rather than in levels of response and household formation behavior, Eq. (2) will not be able to account for the spurious relation from household size to income satisfaction. Impulsive individuals or enthusiasts, for example, may react to changes in their living conditions more strongly in terms of both, expressed satisfaction levels and family formation behavior. Household size will thus partly be endogenous. If unobserved personality traits drive income satisfaction and family size changes in the same direction, higher income satisfaction will predict household size increases. We cannot know with certainty, however, that there are no other unobserved stressors or individual characteristics that neutralize or reverse this relation.

A more interesting case in terms of theoretical predictions is when income satisfaction has a direct, causal effect on household formation or when anticipation effects lead income satisfaction to change well before household size adjustments are realized.

Consider first, for example, two partners moving in together. While it is conceivable that especially financially dissatisfied individuals would choose to cohabit to benefit from economies of scale, it is much more likely that the opposite is true (Stutzer and Frey 2006, Peetz and MacDonald 2025). It might be easier for financially satisfied individuals to date and engage in a serious relationship. They may perceive the costs of moving or getting married to be less pressing. Furthermore, an anticipation effect is very likely. The prospect of enjoying economies of scale when living together may affect income satisfaction before the new household is actually formed. All these effects point towards higher income satisfaction predicting growth in household size by one adult. The negative effect of an additional adult on income satisfaction is thus very likely to be underestimated.

Next, think about a couple separating: The anticipation of having scale economies vanish in the event of separation or union dissolution may precede the actual change in household size. Thus again, the link from satisfaction to household size seems to be positive, with lower income satisfaction preceding a decline in household size by one adult member.

The same bias could occur in the event of one partner's death: If this event was preceded by severe illness, it is likely that increased medical and care expenses would have affected the income satisfaction of all adults in the household negatively beforehand. Again, we would find that the positive effect of having fewer household members at given incomes would be underestimated, because income satisfaction was impaired well before the respective event.

The second type of household size changes crucial to our estimations involves the number of children. An increase in this number may be caused by the birth of a child or by children moving in. A decrease will be observed as a child moves out of the parental home (primarily when parents separate) or as the child turns 14 years old and thus is considered an adult. [3]

We suspect a causal relationship between lagged income satisfaction and the birth of a child. Given that parents wish to provide a financially stable and secure environment to their offspring, being satisfied with own economic conditions may be one prerequisite for planned pregnancies (research on life satisfaction and fertility suggests a positive effect Parr (see e.g. 2010), Le Moglie et al. (see e.g. 2015), Cetre et al. (see e.g. 2016), Mencarini et al. (see e.g. 2018). Given the considerable delay in births after fertility decisions, it is a priori unclear whether current-period

[3] The age threshold chosen in this study is based on the frequently employed OECD-scale, which assigns individuals 14 years and older a weight of 0.5 and younger children a weight of 0.3.

income satisfaction remains higher at birth. Auto-regressive processes may induce satisfaction to be persistently higher, whereas regression to the individual-specific mean or the anticipation of the child's cost may imply significant declines in income satisfaction from the previous to the current period. The predictive content of current-period income satisfaction for the birth of a child thus remains unclear.

While there is a biologically dictated time lag between changes in income satisfaction and the birth of a child, changes in satisfaction may be much more contemporaneously linked to children moving in and out. Similar to when a partner joins a household, having children move in may be especially likely in times of great income satisfaction, especially so when accompanying an adult. Similarly, children will be more likely to leave the household when income satisfaction is low, because most children under the age of 14 leave the household (instead of passing the age threshold to count as an adult) when parents separate. We can therefore reasonably expect greater current-period satisfaction to predict increases in the number of children through this channel.

When the number of children declines mechanically only because of them aging, the corresponding change in the number of children at a given household size clearly is not caused by changes in income satisfaction.

Overall, we therefore expect a small, but positive effect of income satisfaction on the observed number of children.

Taking our expectations regarding changes in the number of adults and children together, we thus derive the following hypothesis:

Hypothesis 1 *Higher income satisfaction predicts increases in the number of household members. If this link is not accounted for, the negative effect of household size on income satisfaction (coefficient* $\beta_1 a$ *in Eq.* (2)*) will be underestimated.*

The number of children enters our regression also via its interaction with log household size. The associated coefficient will also be biased if the extent of endogeneity in household size and the number of children differs.

To see this, suppose for a moment that the number of adults is exogenous but that higher income satisfaction leads to an increase in the number of children. If a child enters the household through birth, the increase in household size and the number of children due to higher income satisfaction will thus be identical. In this case, all bias will be captured by the coefficient on household size. Increases in satisfaction due to a higher number of children at a given household size, as indicated by the coefficient of the interaction term, will be correctly estimated. If, however, income satisfaction had a positive effect on the number of individuals in the household but no distinct effect on the number of children in the household, the extent of endogeneity in household size and the number of children would differ. The negative impact of additional household members would be underestimated and part of that bias would be captured by the interaction effect, thus indicating smaller than actual satisfaction increases to be obtained from additional children compared to additional adults (or at a given household size).

In reality, we expect both the number of adults and the number of children to be endogenous. The number of children is by definition less endogenous, because children "become" adults when they pass the age threshold; this alters the number of children exogenously. There is no reason to believe that given income satisfaction changes will increase the number of children more than they will increase the total number of household members as this would imply a replacement of adults by children. Hence, we expect that endogeneity in the number of children is smaller than endogeneity in household size. This leads us to the following hypothesis to be tested empirically:

Hypothesis 2 *Higher income satisfaction predicts increases in the number of children. If this link is not accounted for, the positive interaction effect of the number of children with household size on income satisfaction (coefficient* $\beta_1 b$ *in Eq.* (2)*) will be underestimated.*

2.1.2 Endogeneity of income

Just as we can expect family size and structure to be predicted by income satisfaction, we must account for the possibility that reported household income may be endogenous.

Most recent studies that analyze endogeneity of income in income satisfaction presume that reporting behavior or recall ability (Elsas 2021, Prati 2017) are causing this endogeneity. Comparing survey to register data, Prati (2017) shows that people who are satisfied with their wage over-report, whereas those relatively less satisfied tend to under-report their wage. Elsas (2021) finds a similar effect for annual household income and life satisfaction. Her analysis is based on an income measure that refers to the year before the interview and is generated from detailed data on all household members' income over the entire previous year. For our estimation of equivalence scales, another income measure is preferable: the so-called income screener, which refers to the same point in time when the number of individuals in the household is surveyed. It measures monthly net household income in one single item that is reported only by the household head. We therefore expect measurement error or recall bias to be even more pronounced in this income measure. Assuming the existence of classical measurement error in monthly net household incomes, Borah and Knabe (2018) propose a correction using an alternative, constructed income measure in their estimation of equivalence scales using income satisfaction data. Their results suggest that the bias in estimated equivalence scales introduced by measurement error is small but their finding depends crucially on the measurement error being non-systematic. This assumption is challenged by Prati's (2017) results and hence a re-examination of the effect of endogeneity in income, in particular due to measurement error, in the estimation of equivalence scales from income satisfaction data is required.

If incomes were indeed over-/under-reported in times of greater/smaller income satisfaction, we would expect the following hypothesis to hold:

Hypothesis 3 *Higher income satisfaction predicts increases in measured household incomes. If this link is not accounted for, the coefficient* β_1 *in Eq.* (2) *will be overestimated.*

Endogeneity of household income may not only be the outcome of systematic measurement error, it could also stem from an actual causal effect, whereby individuals who are more satisfied with their income strive less for further income. Less satisfied individuals, on the other hand, may be more motivated to strive for growing income and thus be more productive. If this was the source of endogeneity, the positive effect of income on satisfaction would be biased downwards. Another source of endogeneity of income in the satisfaction regression could be confounding factors, such as the experienced disutility of labor. People who perceive strong disutility of labor will c.p. work less and earn less and will on the other hand enjoy their income less, because it is earned at a higher price. Disutility of labor would hence impact negatively on both income and income satisfaction and thus bias estimates of the income effect downwards.

In case of such a negative causal link or spurious relation between income satisfaction and productivity or effort (over-weighting the hypothesized measurement error), we will expect the following competing hypothesis to be confirmed:[4]

Hypothesis 4 *Lower income satisfaction predicts increases in measured household incomes. If this link is not accounted for, the coefficient* β_1 *in Eq.* (2) *will be underestimated.*

2.1.3 Bias in equivalence scale parameters

As mentioned above, the equivalence scale parameters will be identified by dividing the coefficients on family structure by the coefficient on household income. Given that Hypothesis 1 is true, the negative effect of household size on income satisfaction (coefficient $\beta_1 a$ in Eq. (2)) will

[4] Both presented arguments refer to income production, which is not the only source of income for every household. Some receive supplementary social benefits, some do not actively produce income but fully depend on social benefits or retirement income. For these households, a negative link from income satisfaction to household incomes may be of less or no relevance.

be underestimated. If the coefficient on income was without bias, this would mean that we would also underestimate the equivalence scale parameter a. As outlined in the previous subsection, this may be naive (especially when taking into account that household income and family size and structure are naturally correlated). Depending on the direction and size of this bias, the equivalence scale elasticity in adult equivalents will be over- or underestimated. If the positive effect of income on satisfaction was overestimated (i.e. Hypothesis 3 was true), parameter a would clearly be biased downwards. The consequences of an underestimation of the positive effect of household income (as formulated by Hypothesis 4) would depend on the relative size of this bias. Only if it was small compared to the bias in the coefficient on household size, parameter a would be underestimated. Otherwise, the bias could also run in the opposite direction.

Hypothesis 2 suggests that the positive interaction effect of the number of children with household size (coefficient $\beta_1 b$ in Eq. (2)) will be underestimated. Again, suppose there was no bias in the income coefficient. In this case, the true linear decline of the equivalence scale elasticity in the number of children would be larger, suggesting lower material needs of children than conventionally estimated. This result would also hold if the income effect was overestimated (i.e. Hypothesis 3 was true). With a negative bias in the income coefficient, the bias in parameter b would again depend on its relative size.

Given the absence of a clear prior regarding the bias in the income coefficient, we hence cannot predict the direction of bias in the equivalence scale parameters. This calls for an empirical assessment, in which we employ internal instruments to solve the problem of endogeneity. The construction of these instruments is outlined in the following subsection.

2.2 Internal instruments approach

A first cure for endogeneity problems in satisfaction regressions is the application of panel fixed effects, which we also apply in our analysis. Yet, if endogeneity is not time-constant at the individual level but time-varying, further steps will be needed. The standard approach would be to find suitable external instruments for household size, the number of children and income. An obvious instrument for income in wellbeing equations is windfall income, such as lottery wins or inheritances (Meer et al. 2003, Lindahl 2005), yet lottery wins are rare events and inheritances typically occur later in the family life cycle, when children will already have moved out of the household. Another typical instrument is industry- or occupation-wide variation in earnings (Luttmer 2005, Vendrik 2013, Kaiser 2018). This instrument, however, is not very suitable for our purpose, as it is valid especially for permanent income rather than for shorter period incomes

(Vendrik 2013). The number of children has been instrumented using twinning and the sex (mix) of the first child(ren) (Angrist and Evans 1998, Angrist et al. 2010, Black et al. 2005, Bonsang and Skirbekk 2022).

We suspect that income, household size and the number of children are all endogenous at the same time. Consequently, we would have to find external instruments for all three variables in the same observations, which is either very demanding or completely infeasible. If no external instruments are available, an alternative strategy can be employed, where instruments are constructed from a subset of the model's exogenous variables (Lewbel 2012). This approach identifies structural parameters by exploiting heteroscedasticity in the model's data (details are explained in the Appendix A2). In the context of life satisfaction measures, Elsas (2021), Le Moglie et al. (2015), Otrachshenko et al. (2023), for example, applied this approach.

Lewbel instruments are suitable if endogeneity can be attributed to a common factor. In our application this involves a time varying common factor in income satisfaction and income, another common factor in income satisfaction and household size, and a third one in income satisfaction and number of children in the household. Common factors can be omitted variables, but also simultaneity, and even measurement error can be modeled as a common factor (Lewbel 2012). In Section (2.1) we discussed that simultaneity and systematic measurement error in the case of income are potential sources of endogeneity in our model, while endogeneity in household formation and size are more likely attributed to anticipation effects (which are formally equal to simultaneity) or unobserved variables.

The intuition behind this identification strategy follows from linear regression mechanics: Instrument exogeneity relies on the fact that residuals of models with strictly exogenous regressors are orthogonal to the outcome. Variables are thus needed that are exogenous to the endogenous variables in our satisfaction regression and exogenous to our outcome of interest, i.e. income satisfaction.

Instrument relevance, on the other hand, relies on the fact that heteroscedasticity contains information about the outcome. Heteroscedastic residuals from a regression of the endogenous variables (income, number of household members and number of children in the household) on strictly exogenous regressors hence contain the information that is necessary to instrument the endogenous regressors.

Instrument construction thus requires variables that are (1) exogenous to the outcome of interest and (2) exogenous to the endogenous regressors and (3) related to heteroscedasticity in the residuals from regressions of the endogenous on the exogenous regressors. In our case these conditions are met by the respondent's age and age squared and the interviewer's birth year

(heteroscedasticity analyses are presented in the Appendix A3, other relevant test statistics are provided with the estimation results.)

These instruments are - as in conventional instrumental variable estimations - used to explain variation of the endogenous regressors in the structural equation, Eq. (2). They are typically less efficient than external instruments (Lewbel 2012), but since we have more instruments than endogenous regressors we can perform overidentification tests, to assess instrument exogeneity.

3 Data

3.1 Sample

Data for this analysis come from the German SOEP (Goebel et al. 2019) release v38.1 (doi:10.5684/soep.core.v38.1eu). The sample covers data from 1985 to 2021. Subsamples that oversample high income households and refugees have been excluded.

We do not use each respondent's first year in the panel because Frick et al. (2006) have demonstrated that data quality increases from the second year in the panel, especially for income data. Our sample is further restricted to respondents who live in private households, and whose households include no other adults than the household head, partner and adult children[5]. For the estimations, however, we use data only from the household head and the partner. Only for our analyses of the income distribution, we expand the sample to represent actual household sizes (in Section 4.2). Households with more than seven children are also dropped; likewise, families with members who need special care, because this usually causes extraordinary financial need. Finally, we exclude household in the outer percentiles of each year's per-capita income distribution. This leads to a sample of 364,038 observations of 47,395 individuals in 32,449 households.

3.2 Variables

As our income measure we use the current net monthly household income as stated by the household head, deflated by the consumer price index to warrant the inter-temporal comparability of income.

Household composition at the time of the interview is captured by two variables: The number of individuals in the household and number of children younger than 14. We choose this age threshold because the OECD scale applies the same. This will allow a comparison of our results with the most widely used equivalence scale.

[5] Households with children older than 30 years are excluded from the analysis.

For construction of the instruments we use interviewer's birth year and the respondent's age and age squared.

Region fixed-effects are defined according to the broader categories: East and West Germany.

Other individual characteristics contained in vector X in Eq. 2 are survey wave squared and cubed, the number of hospital overnight stays in the previous year, the marital status (single, married, separated or widowed) and home ownership.

The set of control variables including those for instrument construction was chosen according to the Hansen J Test and the Kleibergen-Paap F statistic in the instrumtental variable estimation. To account for individual fixed effects in our analysis, data is demeaned for all estimations - either manually, e.g. for the heteroscedasticity analysis (see Appendix A3) or internally, as in all fixed effects estimations.

4 Results

In what follows, we will compare results from estimating a model assuming the strict exogeneity of household size, the number of children and household income with one using internal instruments for these three crucial variables. This will allow us to assess the consequences of endogeneity for both, regressions of income satisfaction and equivalence scale parameters that are derived from those estimates. In a second step, we will restrict our attention to a sample excluding households with children slightly above the threshold of being considered an adult (i.e., children aged 14 to 17) to see if a presumably higher degree of endogeneity implies stronger bias in the estimates.

4.1 The consequences of endogeneity for satisfaction regressions

Our main results can be found in the first two columns of Table 1. These are based on estimating the model presented in Eq. (2) on the full estimation sample outlined above.

In column 1, we apply the conventional approach assuming household size, the number of children and income to be exogenous. Accordingly we run a conventional fixed-effects regression of income satisfaction on all explanatory variables mentioned in Table 1. Standard errors are clustered at the individual level. All controls are found to be significantly related to income satisfaction. We find that higher log household income is linked to higher levels of income satisfaction. Individuals appear to be less satisfied the more household members there are at given levels of income. The interaction term of log household size and the number of children is positively related to income satisfaction. As such, our results are in line with earlier studies using

Table 1
FE-OLS and FE-IV regressions of income satisfaction

	Whole sample				Reduced sample (see section 4.3)			
	(1)		(2)		(3)		(4)	
Log(y)	1.912	***	3.168	***	1.922	***	2.847	***
	(0.012)		(0.147)		(0.013)		(0.132)	
Log(h)	–0.868	***	–1.433	***	–0.948	***	–1.574	***
	(0.016)		(0.130)		(0.018)		(0.148)	
k*log(h)	0.085	***	0.193	***	0.111	***	0.254	***
	(0.004)		(0.023)		(0.005)		(0.033)	
Nights in hospital	–0.002	***	–0.001	***	–0.002	***	–0.001	***
	(0.000)		(0.000)		(0.000)		(0.000)	
Divorced/separated	–0.080	***	0.003		–0.124	***	–0.107	**
	(0.027)		(0.047)		(0.028)		(0.049)	
Married	0.093	***	–0.062		0.091	***	0.003	
	(0.021)		(0.054)		(0.022)		(0.054)	
Widowed	0.192	***	0.070		0.131	***	–0.066	
	(0.032)		(0.066)		(0.033)		(0.074)	
Home owner	–0.035	***	–0.127	***	–0.051	***	–0.113	***
	(0.012)		(0.022)		(0.013)		(0.022)	
Interviewer birthyear	0.003	***	0.003	***	0.003	***	0.003	***
	(0.000)		(0.001)		(0.000)		(0.001)	
Age	–0.082	***	–0.160	***	–0.078	***	–0.136	***
	(0.004)		(0.012)		(0.005)		(0.011)	
Age, squared	0.000	***	0.001	***	0.000	***	0.001	***
	(0.000)		(0.000)		(0.000)		(0.000)	
East Germany	–0.220	***	–0.038		–0.205	***	–0.065	
	(0.044)		(0.086)		(0.047)		(0.087)	
Survey wave, squared	–0.000	**	0.002	***	–0.000	**	0.001	***
	(0.000)		(0.000)		(0.000)		(0.000)	
Survey wave, cubed	0.000	***	–0.000		0.000	***	0.000	
	(0.000)		(0.000)		(0.000)		(0.000)	
Constant	–11.001	***			–11.411	***		
	(0.814)				(0.885)			
K-P F-Statistic			61.2				68.0	
Hansen J			4.126				4.829	
Prob>Chi squared			0.660				0.566	
Observations	364,038		364,038		313,127		313,127	
Individuals	47,395		47,395		44,970		44,970	

Source: SOEP v38.1, own calculations.

Notes: Significance levels * 0.10 ** 0.05 *** 0.01. Cluster-robust standard errors in parentheses.

the same approach. As argued above, we believe that these estimates may be biased, however. Therefore, we apply Lewbel's internal instruments, making use of heteroscedasticity in our three crucial regressors with respect to the respondent's age and interviewer's year of birth. The corresponding estimation makes use of the Stata-ado ivreg2h, written by Baum and Schaffer (2012) calling the GMM estimator. In the bottom of column 2 in Table 1, we report statistics regarding the quality of our internal instruments. The Kleibergen-Paap rank F statistic is used to assess if our instruments are sufficiently strong. We follow the rule-of-thumb that it should exceed 50 (Keane and Neal 2022). Since three instruments are used for each endogenous regressor, it is possible to apply Hansen's J-test to indicate if the instruments (as well as all other variables in the estimation) are endogenous. It tests the joint null that all instruments (the constructed instruments and the exogenous covariates) are uncorrelated with the error term of the second stage regression. Hansen's J-test should be insignificant; else the (instrumented or other) regressors are still endogenous.

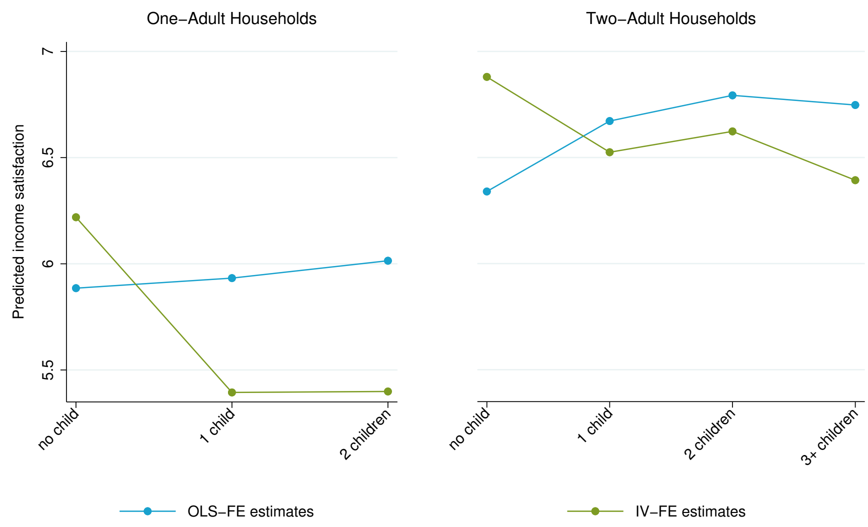
The coefficients in column 2 reflect our main results, with standard errors clustered at the individual level. Turning to the first three coefficients, we again find that log household income as well as the interaction term $k * log(h)$ are positively related to income satisfaction, whereas log household size is negatively associated with it. Each of these coefficients obtained from the instrumental variables regression is larger in magnitude than its counterpart in column 1. Thus, our results indicate that the endogeneity of regressors in column 1 may lead to biased results.

First of all, we find that the negative effect of additional household members has been underestimated by the conventional model. This confirms our Hypothesis 1, implying a positive link of income satisfaction to household size.

Secondly, the positive effect of having more children at a given household size (as captured by the coefficient on the interaction term $k * log(h)$) appears to be underestimated, too. This confirms Hypothesis 2 and suggests that higher income satisfaction predicts increases in the number of children.

Finally, we see that the positive income effect is also underestimated in the conventional fixed effects estimation. This is clearly in line with hypothesis 4 and with Vendrik (2013), Kaiser (2018), Luttmer (2005), yet in contradiction to findings from Prati (2017) and Elsas (2021). Divergence from the results in Elsas (2021) are easily understood: the outcome of interest there is life satisfaction while it is income satisfaction here. The former encompasses more aspects of life and is typically only weakly associated with income. Contradiction to Prati's (2017) calls for further examination.

Figure 1
Binned scatterplots of predicted income satisfaction against household composition



Source: SOEP v38.1, own calculations. 364,038 person year observations of 47,395 individuals.
Notes: Means of predicted values of income satisfaction from baseline and IV estimations in Table 1.

Further, when individuals evaluate their household income they refer to both income and needs, and the household's financial needs depend on the number of household members and the number of children. Meaning that when instrumentation changes the estimated effects of household composition on income satisfaction, the estimate for the income effect adapts to this change.

Our finding that family size, structure and income are endogenous to income satisfaction may have important implications for the study of welfare distributions based on income satisfaction. It may be misleading to compare the average income satisfaction of household types in order to draw conclusions regarding their relative welfare levels. Our research shows that income satisfaction also predicts what type of household an individual lives in and how much household income is reported. If we exclude the links running into these directions, we may draw different conclusions regarding the relative income satisfaction experienced by individuals living in certain types of household. To see this, the left graph of Figure 1 reports the predicted income satisfaction of households with one or two adults without and with different numbers children. As can be seen, our OLS estimates would suggest the predicted income satisfaction of couples with children to be higher than that of childless couples. Similarly, singles without children appear to have a lower predicted income satisfaction than singles with children. These results switch when we apply our IV regressions to account for endogeneity. Childless singles and couples without children are predicted to experience greater income satisfaction. Single parents' predicted income satisfaction turns out to be strikingly low.

4.2 Implications of endogeneity for the analyses of income distributions

Table 2
Scale parameters from estimates in Table 1 with bootstrapped confidence intervals

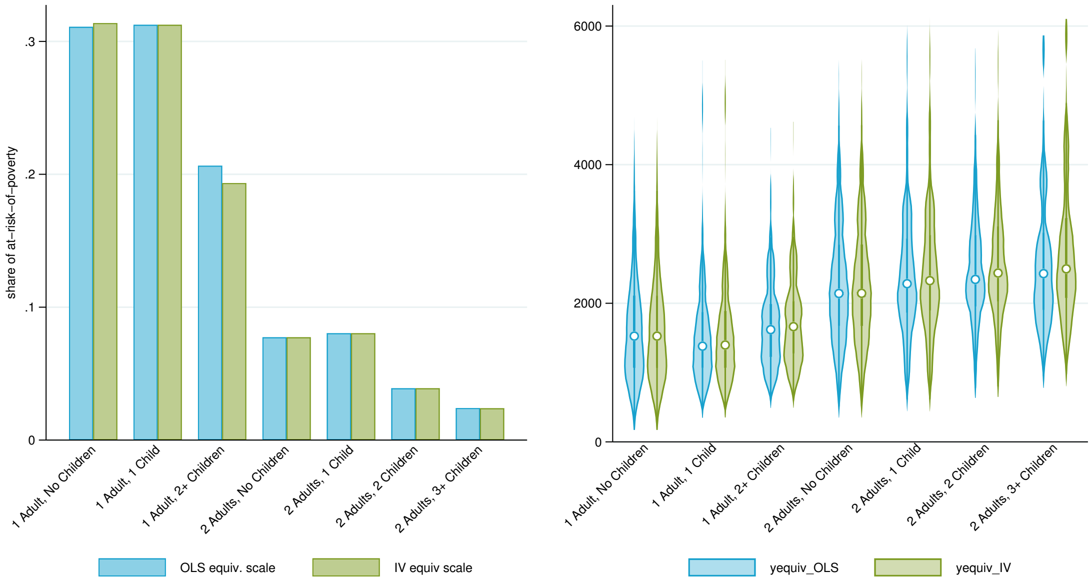
	OLS FE			IV FE		
	Observed	Confidence interval		Observed	Confidence interval	
	Whole sample estimation (364,038 observations of 47,395 individuals.)					
Scale parameter a	0.454	0.431	0.477	0.452	0.375	0.529
Scale parameter b	0.045	0.039	0.050	0.061	0.046	0.075
	Reduced sample estimation (313,127 observations of 44,970 individuals.)					
Scale parameter a	0.493	0.468	0.519	0.553	0.457	0.649
Scale parameter b	0.058	0.050	0.066	0.089	0.066	0.113

Source: SOEP v38.1, own calculations.
Notes: 500 replications clustered by person. Confidence intervals are normal-based.

As equivalence scale parameters are constructed from the regression coefficients, biased coefficients from the conventional approach could theoretically translate into biased equivalence scale parameters. The parameters of interest can be found in the top panel of Table 2. Parameter a represents the equivalence scale elasticity for households whose members all are 14 years or older. b captures the linear decline of the equivalence scale elasticity for additional children at a given household size. The coefficients obtained using the conventional approach suggest a scale parameter a of 0.454. Interestingly, the IV estimates yield a very similar parameter of $a = 0.452$. Even though the coefficients on household income and size appear to be biased when endogeneity is not taken into account, the resulting scale parameter virtually remains the same, because both biases work in opposite directions. Bootstrapped confidence intervals also suggest a great overlap between the OLS and the IV result. The decline in financial needs depending on the number of children, however, seems to be affected by the endogeneity of income, household size and structure. The OLS approach suggests that $b = 0.045$, whereas the IV regression yields a parameter $b = 0.061$. This parameter thus turns out to be approximately 35 percent larger when endogeneity is accounted for. Our results therefore suggest that the financial needs of children are somewhat lower than what has been suggested by the conventional approach, which is also confirmed by the bootstrapped confidence intervals.

When comparisons of welfare levels are made via equivalent incomes, our main results show that accounting for endogeneity has virtually no implications for distribution and poverty analyses. This is because our equivalence scale parameters using the IV approach are almost identical

Figure 2
Mean equivalized income by household type and share of individuals at risk of poverty in 2021



Source: SOEP v38.1, own calculations. 567,283 observations.

to those obtained using the conventional approach. Our estimates suggest the same weight for adults and a slightly lower equivalence weight for children. If any, differences may be expected only for households with children. Figure 2 shows that differences are negligible. Mean equivalent incomes and the underlying distribution for different types of households (left graph) are virtually identical. Only for households with 2 or more children, small changes can be observed. The share of individuals at risk of poverty[6] also remains basically unchanged for all household types, except for single parents with two or more kids. For the latter, the risk of poverty is slightly reduced due to the lower financial needs of children suggested by our IV regression (see the right of Figure 2).

Overall, it thus seems there is little reason to worry about analyses of income distributions to be greatly biased when equivalent incomes are constructed on the basis of income satisfaction, even though some crucial variables may be endogenous. In that sense, our findings support the existing strand in the literature that uses the conventional estimation approach.

[6] An individual is defined as being at risk of poverty if their equivalized household income falls below the poverty line, i.e. below 60 % of the median income, for the respective year.

4.3 More endogeneity - more bias

In the sample we have analyzed so far, one important change in household composition is exogenous in nature - the number of children is reduced as they turn 14 (while the number of household members typically stays the same). The aging of children is certainly not affected by income satisfation. Hence, endogeneity may be less of a concern when such changes affect our estimates.

Transition probabilities reported in Table A4.3 in the Appendix show that about 6% of all parent couples with children younger than 14 become a parent couple without children younger than 14 from one year to the next. Similar for single parents, from one year to the next, nearly 11% of those with children younger than 14 become single parents without children younger than 14.

Since these transitions are not endogenous, they might help us shed some more light on the validity of Hypothesis 2. If endogeneity in the number of children was a problem in the income satisfaction regression, it should be more pronounced when we exclude households whose number of children decreases only due to their aging. Therefore, we rerun our estimations on a sample without households whose children are between 14 and 17 years old.

Estimation results can be found in columns 3 and 4 of Table 1. Again, we find that the conventional model underestimates the magnitude of all our coefficients of interest. While the coefficient on the interaction term $k * log(h)$ appears to be particularly large in column 4, the relative increase from column 3 to column 4 is nearly identical to the relative difference visible when comparing columns 1 and 2 (almost 130 percent). The same is true for the coefficient on log household size. The increase in the coefficient on log household income is smaller in the restricted sample, however. In sum, this yields the equivalence scale parameters reported in the bottom panel of Table 2. When compared to the parameter estimates in the top panel of the same table, we see that the financial need for additional adults is higher when households with children between 14 and 17 are excluded from the estimation. This is independent of the estimation approach used and points towards the financial needs of teenagers (between 14 and 17 of age) still being lower than those of adults. Furthermore, we find that endogeneity may have stronger implications for the estimated equivalence scale parameters when the sample is restricted. More specifically, we observe a stronger increase in both scale parameters a (from 0.493 to 0.553) and b (from 0.058 to 0.089) than in the non-restricted sample. Thus, the subsample analysis confirms that endogeneity leads the relative income requirements of additional adults to be underestimated and those of children to be overestimated by the conventional approach.

5 Conclusion

In this study, we have estimated equivalence scale parameters under consideration of potentially endogenous income, household size and structure. We have applied the income satisfaction approach, which estimates the effect of income, the number of household members and the number of children on income satisfaction and computes the equivalence scale elasticity for additional household members and its linear decline in the number of children. This approach has frequently been applied in the previous literature, yet always under the assumption that crucial variables are exogenous.

We have argued that this assumption may be violated. Using internal instruments, we have shown that endogeneity causes an attenuation of the estimated effects of income, the number of household members and the number of children in the household. Our results support earlier studies that have found a positive effect of satisfaction on household size (Cetre et al. 2016, Le Moglie et al. 2015, Mencarini et al. 2018, Parr 2010). With respect to the endogeneity of income, our results do not confirm the hypothesis that systematic measurement or reporting errors cause an overestimation of the true income effect. Instead, we have seen that the income effect is stronger when endogeneity of household income, size and structure is taken into account, which is in line with Luttmer (2005), Vendrik (2013), and Kaiser (2018).

Moreover, we have contributed to research on the empirical estimation of equivalence scales from satisfaction data. Scale parameters rely on the income coefficient as well as on the estimated effect of the number of household members (and children). It appears that equivalence weights for additional adults do not change very much when using the novel estimation method. The adjustment factor for children turns out to be slightly larger. Taken together, these small changes provide support for earlier studies using the income satisfaction approach to estimating equivalence scales. Even though they have not accounted for endogeneity of household size and income, their estimates of the equivalence scale appear robust and largely unbiased. Unsurprisingly, the small changes in equivalence scale parameters that our analysis suggests have only minor implications for comparisons of mean equivalized income and poverty risks across household types. Again, this may be interpreted in favor of earlier studies using the income satisfaction approach.

We have proposed a robustness check that excludes households with children close to the age threshold of 14 years (who are therefore considered an adult). We have argued that this may imply an aggravation of endogeneity. This subsample analysis has confirmed that equivalence weights for adults may be underestimated and those for children may be overestimated conventionally.

Our results could serve as a starting point for further research. It would be valuable to identify the exact reasons for endogeneity of household size and income. While satisfaction and fertility decisions have been empirically linked, analyses of how satisfaction affects partners moving in or out and adult children's decisions to leave their parental homes are needed. Similarly, the source of endogeneity of income and its implications for equivalence scale estimates could be further assessed using administrative income data. Finally, other options to instrument the potentially endogenous regressors need to be explored and applied in estimations of equivalence scales.

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Appendix

A1 Sample descriptives

Table A1.1
Descriptive Statistics

	Mean	Standard deviation between	within	Min	Max
Satisfaction with household income	6.525	1.893	1.399	0	10
Real monthl. household income	2736.65	1382.07	674.70	253.31	24509.8
No. of persons	2.697	1.272	0.553	1	9
No. of children	0.518	0.857	0.464	0	7
Nights in hospital	1.600	5.639	6.570	0	365
Interviewer birthyear	1945.257	10.999	5.935	1907	2004
Respondent's age	49.969	15.818	5.163	18	100
Survey wave	22.985	9.815	5.163	2	38
	Percent overall	between	within	Min	Max
Single	13.74	22.77	83.32	0	1
Separated	10.66	14.67	74.78	0	1
Married	69.00	70.30	91.05	0	1
Widowed	6.61	7.69	78.65	0	1
Home owner	47.19	49.92	86.91	0	1
Living in East Germany	22.56	22.12	97.32	0	1

Source: SOEP v38.1, own calculations. 364,038 person-year observations of 47,395 individuals in 32,449 households.

Note: Children are here only children younger than 14 living in the respondent's household.

A2 Heteroscedasticity-based instrumentation

With respect to our application, Lewbels's (2012) approach can be described as follows.

To construct instruments for log household size in Eq. (2), the following auxiliary regression is run in a first step[7]:

$$\log h_{it} = Z'_{it}\delta + \nu_{it}, \tag{3}$$

where Z is a subset of J variables in X (the set of exogeneous regressors in Eq. (2)) that satisfy the further exogeneity assumption that $E(Z_{it}\nu_{it}) = 0$. Here, we regress log household size $\log h_{it}$ on age, age squared, and interviewer birth year. Residuals $\hat{\nu}_{1it}$, together with sample-centered values

[7] Actually, we do not conduct the estimation step by step, but use the Stata-ado 'ivreg2h' by Baum and Schaffer (2012).

of each variable z_j in Z are then used to calculate three instruments for $\log h_{it}$ in the second step:

$$hinst_{jit} = (z_{jit} - \bar{z}_j)\hat{\nu}_{it} \tag{4}$$

One instrument per exogenous variable in Z is constructed as the product of the residual and the respective demeaned variable. Instruments for our other two potentially endogenous regressors $k_{it} \log h_{it}$ and $\log y_{it}$ are generated analogously, using the same three exogenous variables age, age squared and interviewer's birth year.

The intuition of this identification strategy follows from linear regression mechanics: If satisfaction is independent of the residuals of the auxiliary regression, Eq. (3), and if the variables in Z are exogenous in the structural equation Eq. (2), then the instruments are exogenous. In that case, the instruments affect the outcome only via the endogenous regressor. To be relevant, the instrument must be related to the endogenous regressor. If the residuals in the auxiliary regression, Eq. (3), are heteroscedastic, they capture variation of the instrumented variable, which makes the instruments relevant.

As noted in Lewbel (2012) and Baum and Lewbel (2019), for each instrument to be valid, it is required that the errors of each auxiliary regression and the structural equation are uncorrelated with the exogenous controls, i.e., $\mathrm{Cov}(z_j, \varepsilon\nu) = 0$.

Parameter identification and instrument strength rely on heteroscedasticity in the errors of the auxiliary regressions that must be related to the exogenous controls, $\mathrm{Cov} z_j, \nu_{it}^2$. When heteroscedasticity exists in the linear projection of log household size (and log income and the interaction of number of children with log household size) on the exogenous variables Z, and when this heteroscedasticity is related to the exogenous variables, the instruments can identify the effect of log household size (and log income and the interaction of number of children with log household size) on income satisfaction. The basis of the instruments' relevance can therefore be tested by checking the sample covariance between variables contained in Z and the squared residuals from linear regressions of the endogenous on the exogenous variable.

A3 Heteroscedasticity in the model's data

In our analysis, we use the respondent's age, age squared and the interviewer's birth year as exogeneous controls Z in the construction of internal instruments for log household size, log household income and the interaction of log household size and the number of children. The relevance of an instrument constructed from a particular variable z_j depends on the strength of heteroscedasticity with respect to z_j. Therefore, this section inspects the degree of heteroscedasticity in our data.

Table A3.2
Squared residuals from auxiliary regressions projected on exogenous variables (z)

	Log(Income) lnY		Log(Persons) lnh		Kids*Log(Pers) kidsInLnh	
Interviewer's birth year	0.000	***	0.000	***	0.004	***
	(0.000)		(0.000)		(0.000)	
Respondent's age	–0.004	***	–0.007	***	–0.101	***
	(0.000)		(0.000)		(0.002)	
Resp.'s age squared	0.000	***	0.000	***	0.001	***
	(0.000)		(0.000)		(0.000)	
Constant	0.053	***	0.041	***	0.487	***
	(0.000)		(0.000)		(0.003)	
Chi squared	1418		3062		4025	
Prob>Chi squared	0.000		0.000		0.000	

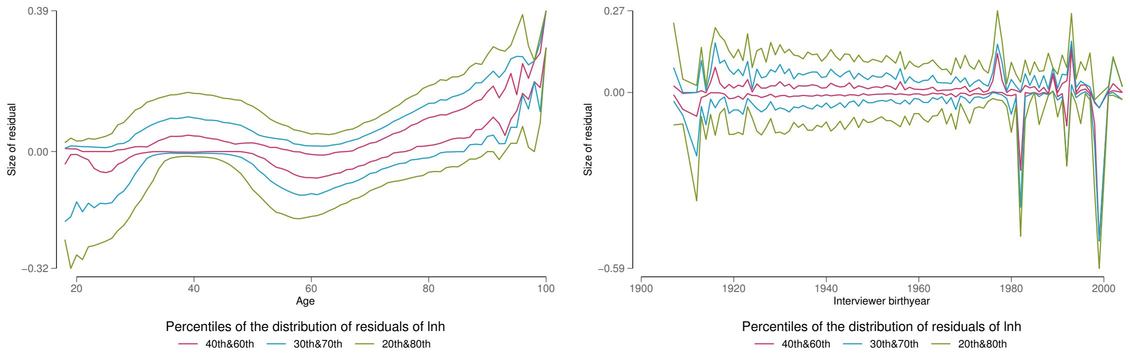
Source: SOEP v38.1, own calculations. 364,038 person year observations of 47,395 individuals
Notes: Significance levels * 0.10 ** 0.05 *** 0.01. Standard errors in parentheses.
Dependent variables are squared residuals from fixed effects regressions,
exogenous variables are within transformed.

Table A3.2 presents estimates of the association between the three exogenous controls and the squared residuals of our endogenous regressors. Residuals are based on regressions of our demeaned endogenous variables on demeaned exogenous controls, i.e. the respondent's age, age^2 and the interviewer's birth year. As can be seen, all exogenous controls bear a statistically significant relation with the squared residuals of interest (even though some of the coefficients visually appear to be zero due to their scale). The two bottom lines of the table report results from the Breusch-Pagan test statistics, assuming a normal distribution of the error terms. These confirm that our data is heteroscedastic.

The same result is supported by plots of the distribution of residuals at different levels of the exogenous controls in Figure A3.1, A3.2 and A3.3. The distribution of the residuals of our three endogenous regressors narrows at earlier interviewer's birth years (see right panel of Figure A3.1, A3.2 and A3.3). The distribution of residuals for the number of children also narrows over the respondent's age (see left panel of Figure A3.2). For household size and income the tendency is less clear (see left panel of Figure A3.1 and A3.3). But it still appears that the exogenous controls and squared residuals are sufficiently associated to allow for our model's identification based on Lewbel's internal instruments.

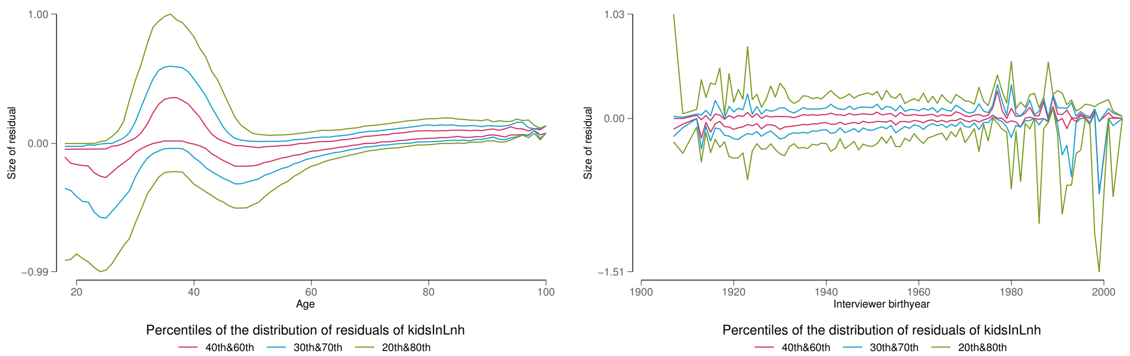
It thus appears that the exogenous controls and squared residuals are sufficiently associated to allow for identification based on Lewbel's instruments.

Figure A3.1
Age- and interviewer birth year related heteroscedasticity in number of household members (log.)



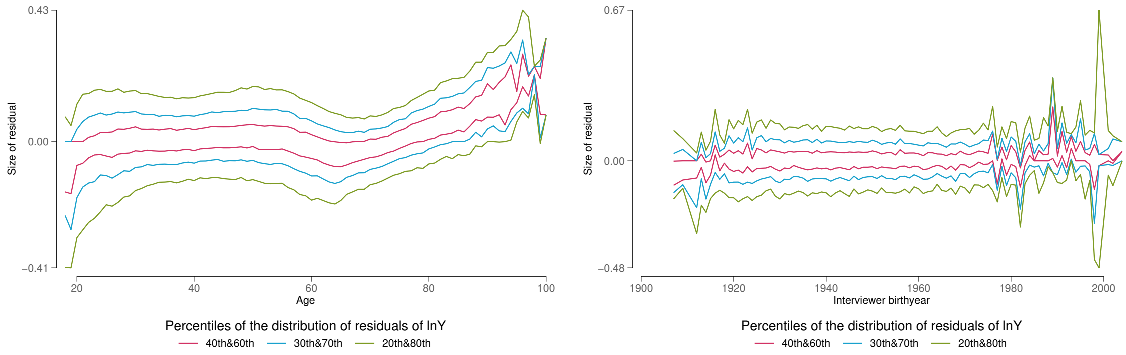
Source: SOEP v38.1, own calculations. 366,616 person year observations of 47,764 individuals.
Notes: Residuals from fixed effects regressions of log household size on the respondent's age, age squared, and the interviewer's birth year.

Figure A3.2
Age- and interviewer birth year related heteroscedasticity in the interaction of log household size and the number of children



Source: SOEP v38.1, own calculations. 366,616 person year observations of 47,764 individuals.
Notes: Residuals from fixed effects regressions of $k_{it}\ln(h_{it})$ on the respondent's age, age squared, and the interviewer's birth year. Children are here children of age<14.

Figure A3.3
Age- and interviewer birth year related heteroscedasticity in household income (log.)



Source: SOEP v38.1, own calculations. 366,616 person year observations of 47,764 individuals.
Notes: Residuals from fixed effects regressions of log household income on the respondent's age, age squared, and the interviewer's year of birth

A4 Probabilities of changes in household composition

Table A4.3
Year-to-year transition probabilities for household types

Rows show initial and columns show final values.

	Household Typology						
Household Typology	1	2	3	4	5	6	Total
1 One-person HH	94.65	4.09	0.23	0.25	0.68	0.11	100.00
2 Couple w/o children	2.43	94.35	0.04	0.02	2.88	0.29	100.00
3 Single parent w/ children<14	0.70	0.29	78.83	10.97	8.43	0.79	100.00
4 Single parent w/o children<14	9.59	1.24	0.30	85.69	0.19	2.99	100.00
5 Parent couple w/ children<14	0.78	0.19	1.31	0.17	91.44	6.11	100.00
6 Parent couple w/o children<14	0.54	8.40	0.01	1.02	0.26	89.77	100.00
Total	25.24	30.58	3.42	4.00	23.76	13.00	100.00

Source: SOEP v38.1, own calculations. 230,874 household-year observations of 32,449 households.
Note: Children are not restricted to minors. Probabilities are not normalized for missing periods.