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Structural Vector Autoregressions with Smooth Transition in Variances

The Interaction Between U.S. Monetary Policy and the Stock Market*

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Abstract. In structural vector autoregressive analysis identifying the shocks of interest via heteroskedasticity has become a standard tool. Unfortunately, the approaches currently used for modelling heteroskedasticity all have drawbacks. For instance, assuming known dates for variance changes is often unrealistic while more flexible models based on GARCH or Markov switching residuals are difficult to handle from a statistical and computational point of view. Therefore we propose a model based on a smooth change in variance that is flexible as well as relatively easy to estimate. The model is applied to a five-dimensional system of U.S. variables to explore the interaction between monetary policy and the stock market. It is found that previously used conventional identification schemes in this context are rejected by the data if heteroskedasticity is allowed for. Shocks identified via heteroskedasticity have a different economic interpretation than the shocks identified using conventional methods.

Key Words: Structural vector autoregressions, heteroskedasticity, smooth transition VAR models, identification via heteroskedasticity

JEL classification: C32

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1 Introduction

Structural vector autoregressions (SVAR) are popular tools for empirical macroeconomic analysis. The underlying model is a basic reduced form linear vector autoregression (VAR) as advocated by Sims (1980). The standard structural VAR approach derives identifying restrictions for the structural shocks and imposes them on the reduced form of the model. Such restrictions usually come from economic theory related to the variables involved. However, there is a growing strand of literature that proposes to use features of the data to help with the identification of structural shocks. More specifically, distributional assumptions (Lanne and Lütkepohl (2010)) as well as heteroskedasticity (Rigobon (2003), Lanne, Lütkepohl and Maciejowska (2010)) may be useful for identification purposes. In this paper we use heteroskedasticity for the identification of shocks. This approach is attractive because changes in the volatility of different macroeconomic time series are broadly documented and discussed by Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Blanchard and Simon (2001), and Stock and Watson (2003) among others.

Alternative approaches for modelling changes in volatility have been used in this context. For example, Rigobon (2003), Rigobon and Sack (2003) Lanne and Lütkepohl (2008), and Bacchiocchi and Fanelli (2012) use simply a deterministic shift in the variances while Normandin and Phaneuf (2004) and Bouakez and Normandin (2010) model the changes in volatility by a vector GARCH process and Lanne et al. (2010) propose a Markov switching (MS) mechanism for changes in volatility. The GARCH and MS approaches have the disadvantage that estimation of the models is very involved and so far reliable estimation methods are available only for small models with three or four variables and a moderate number of lags and volatility states at best. On the other hand, assuming an exogenous change in variance as in Rigobon (2003) and others is also not very attractive because, in practice, gradual changes in volatility seem more plausible in many situations. Therefore, in this study we propose an intermediate approach. More precisely, we consider the SVAR with heteroskedastic residuals modelled by a smooth transition function. A discussion of smooth transition models with heteroskedasticity can be found in Yang (2014). In the smooth transition literature it is more common to model non-linearity in the mean equation (e.g., Hubrich and Teräsvirta (2013)). However we use this idea to take into account heteroskedasticity present in the data.

Our current setup has a number of advantages compared to other volatility models used in this context. If the transition function is parameterized parsimoniously, the parameters are relatively easy to estimate. A well de-

veloped toolkit for the statistical analysis of smooth transition regression models is available and can potentially be adopted for the purposes of identification of structural shocks. The estimation of the model as set up in the present paper benefits from currently available computational power and can be performed with reasonable computation time. The timing of the change in volatility is determined by the data and does not have to be imposed exogenously by the analyst. Depending on the parametrization of the transition function, the timing of the transition to a new state may be estimated. By a suitable choice of the transition variable, the change in volatility regimes may be endogenized, that is, it may be linked to relevant economic variables. Given these advantages of models with smooth transition in the variances, they may be strong competitors of MS- and GARCH-SVAR models or models with exogenously imposed heteroskedasticity regimes.

We use the smooth transition VAR model to investigate the interaction between monetary policy and the stock market based on a five-dimensional system of U.S. variables. The relation between monetary policy and the stock market has been analyzed with VAR models by a number of authors. Our benchmark study is Bjørnland and Leitemo (2009). Previous studies vary in the identifying assumptions for shocks of interest and the data they use. Generally they find some interdependence between monetary policy and the stock market. However, the magnitude of the effects of monetary policy shocks on the stock market differ widely in the various studies. A direct comparison of alternative identifying assumptions is usually difficult because the authors typically use just-identifying restrictions that cannot be tested with statistical tools in a conventional SVAR framework. In that situation, using heteroskedasticity as an additional identification device seems plausible and we apply that tool to check the identifying assumptions used by Bjørnland and Leitemo (2009). Since their system is five-dimensional, it is difficult to handle with MS- or GARCH-SVAR models. In contrast, our smooth transition model works well and we can use it to check the identifying restrictions used by Bjørnland and Leitemo (2009). It turns out that they are rejected. While we also find a strong interaction between monetary policy and the stock market, we argue that the data suggest a quite different interpretation of stock market shocks than that found by other authors in a conventional SVAR setting.

The remainder of the paper is organized as follows. Section 2 briefly discusses conventional VAR and SVAR models as well as identification of structural shocks. Section 3 sets up the smooth transition SVAR model and explains how it can be used for identification purposes. A suitable estimation procedure is discussed as well. An empirical example analyzing the relation between U.S. monetary policy and the stock market is discussed in Section

4. The last section summarizes the conclusions from our study.

2 The Baseline Model

The baseline model is a VAR of order p (VAR(p)) of the form

$$y_t = \nu + A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t, \quad (1)$$

where $y_t = (y_{1t}, \dots, y_{Kt})'$ is a vector of observable variables, the A_i are $(K \times K)$ coefficient matrices, ν is a $(K \times 1)$ constant term and the u_t are K -dimensional serially uncorrelated reduced form residuals with mean zero and covariance matrix Σ_u .

The structural residuals are denoted by ε_t . They have zero mean and are serially uncorrelated. Typically they are also assumed to be instantaneously uncorrelated, that is, $\varepsilon_t \sim (0, \Sigma_\varepsilon)$, where Σ_ε is a diagonal matrix. Sometimes it is actually assumed that the variances of the structural shocks are normalized to one so that Σ_ε is an identity matrix.

The structural residuals are typically obtained from the reduced form residuals, u_t , by a linear transformation,

$$u_t = B\varepsilon_t \quad \text{or} \quad \varepsilon_t = B^{-1}u_t. \quad (2)$$

The matrix B contains the instantaneous effects of the structural shocks on the observed variables. Given the relation between the reduced form residuals and the structural residuals, the matrix B has to satisfy $\Sigma_u = B\Sigma_\varepsilon B'$. In other words, in principle B can be any matrix satisfying $\Sigma_u = B\Sigma_\varepsilon B'$. The relation between the reduced form and structural residuals does not uniquely determine the matrix B and, hence, the structural innovations are not uniquely determined without further assumptions.

The conventional approach is to impose further restrictions on B directly to make it unique. These restrictions may be zero restrictions indicating that a certain shock does not have an instantaneous effect on one of the variables or it may be implied by a restriction on the long-run effects of a structural shock or by other kinds of information. The matrix of long-run effects of structural shocks is given by

$$\Xi_\infty = (I_K - A_1 - \dots - A_p)^{-1}B,$$

assuming that the inverse exists. That condition is satisfied for stable, stationary processes without unit roots. For integrated and cointegrated processes the long-run effects matrix is related to the cointegration structure of the model (see, e.g., Lütkepohl (2005)). For our purposes it is sufficient to

know that the matrix of long-run effects can be computed from the reduced form and structural parameters and imposing restrictions on that matrix implies restrictions on B .

Typically the restrictions on B just-identify the structural model and, hence, the structural shocks. In other words, there are just enough restrictions for uniqueness of B and no more. If there are two competing sets of just-identifying assumptions or theories implying just-identifying restrictions, they lead to identical reduced forms and cannot be tested against the data. Hence, the conventional setup is often uninformative regarding the validity of specific economic theories. In the next section it is discussed how heteroskedasticity can be used to improve the situation in this case.

3 SVAR Model with Heteroskedastic Residuals

3.1 Smooth Transition in Variances

Suppose u_t is a heteroskedastic error term with smoothly changing covariances,

$$E(u_t u_t') = \Omega_t = (1 - G(\gamma, c, s_t))\Sigma_1 + G(\gamma, c, s_t)\Sigma_2 \quad (3)$$

where Σ_1 and Σ_2 are distinct covariance matrices and $G(\gamma, c, s_t)$ is a transition function. It depends on a parameter (vector) γ and c as well as a transition variable s_t . This quantity can be a stochastic variable which determines the transition to another volatility state or it may be a deterministic function of time. In the current paper we use a logistic transition function proposed by Maddala (1977) with time being the transition variable, i.e., $s_t = t$ so that

$$G(\gamma, c, s_t) = (1 + \exp[-\exp(\gamma)(t - c)])^{-1} \quad (4)$$

with the term $\exp(\gamma) > 0$ for positive and negative values of γ . Notice that $0 < G(\gamma, c, s_t) < 1$. Thus, Ω_t is a convex combination of two positive definite matrices and, hence, it is also a positive definite matrix.

The transition of the volatility from the covariance matrix Σ_1 to Σ_2 can be used for identification purposes. There exists a decomposition

$$\Sigma_1 = BB' \text{ and } \Sigma_2 = BAB', \quad (5)$$

where $\Lambda = \text{diag}(\lambda_1, \dots, \lambda_K)$ is a diagonal matrix with positive diagonal elements (see Lütkepohl (1996, Section 6.1.2 (10)) or Theorem 7.6.4 in Horn

and Johnson (2013)). Apart from changes in sign of the columns of B this decomposition is in fact unique for a given ordering of the λ_i if these quantities are all distinct (Lanne and Lütkepohl (2008)). Thus, if the B matrix from (5) is used to transform the reduced form residuals, the covariance matrices of the structural shocks are I_K and Λ for the initial and final regimes, respectively. The diagonal elements of the Λ matrix can thus be interpreted as variances of structural shocks in the final regime relative to the initial regime.

An important advantage of using this transformation matrix B is that the uniqueness condition and, hence, the identification condition for the shocks can be checked with standard statistical tests. For example, Lanne et al. (2010), Lütkepohl and Netšunajev (2014) and Netšunajev (2013) use Wald and likelihood ratio tests for this purpose in a similar context.

Using B as transformation matrix to obtain $\varepsilon_t = B^{-1}u_t$ throughout the sample implies that the structural shocks have the same instantaneous effects regardless of the residual volatility and are normalized such that they have unit variance when $E(u_t u_t') = \Sigma_1$. If the uniqueness conditions for B are satisfied, any restrictions imposed on B in a conventional SVAR framework become over-identifying and can be tested against the data. Lanne et al. (2010) use likelihood ratio tests for this purpose.

Of course, the structural shocks obtained in this way may be quite distinct from economic shocks of interest. Still the potentially unique decomposition of covariance matrices provides additional information that can help identifying structural shocks of interest. If the shocks happen to be the same as the economically identified shocks, they are over-identified in our framework and, hence, different sets of restrictions become testable. In addition, identification through heteroskedasticity can be used to identify only some of the shocks by using data properties and imposing those restrictions that are not rejected by the data for identifying economically interesting shocks.

3.2 Estimation

We refer to the model specified in (1), (3), (4), (5) as a smooth transition structural VAR (ST-SVAR) model. It can be estimated via maximum likelihood (ML) under the assumption of normality of the residuals. The log-likelihood function for the model is

$$\log L = \text{constant} - \frac{1}{2} \sum_{t=1}^T \log \det(\Omega_t) - \frac{1}{2} \sum_{t=1}^T u_t' \Omega_t^{-1} u_t \quad (6)$$

where $u_t = y_t - \nu - A_1 y_{t-1} - \dots - A_p y_{t-p}$ and Ω_t is given by (3) and (5).

The model is nonlinear and has many parameters to be estimated. Therefore we use a grid search over γ and c . For a given pair $\{\gamma, c\}$, estimation proceeds in the following two steps.

Step 1: For given starting values of the VAR parameters $\{\nu, A_1, \dots, A_p\}$ the structural parameters $\{B, \Lambda\}$ are estimated by maximizing the log likelihood function using nonlinear optimization. This step may be done subject to economic restrictions on the B or Ξ_∞ matrices.

Step 2: For the updated structural parameters the VAR part of the model is reestimated. Note that given the transition parameters $\{\gamma, c\}$ and structural parameters $\{B, \Lambda\}$ the model is linear in the VAR part. For that reason the vectorized VAR coefficients $b := \text{vec}(\nu, A_1, \dots, A_p)$ can be estimated with a weighted least squares procedure with the weights given by Ω_t^{-1} , that is,

$$\hat{b} = [(Z' \otimes I_K) W_T (Z \otimes I_K)]^{-1} (Z' \otimes I_K) W_T y,$$

where

$$W_T := \begin{pmatrix} \Omega_1^{-1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \Omega_T^{-1} \end{pmatrix}.$$

is a $(KT \times KT)$ block-diagonal weighting matrix. Moreover, $y := (y'_1, \dots, y'_T)'$ is a $(KT \times 1)$ data vector, and each row of the $(T \times (1+Kp))$ data matrix Z contains a leading one for the constant as well as lagged observations:

$$(1, y'_{t-1}, \dots, y'_{t-p}).$$

These two steps are iterated until there is no improvement in the log-likelihood value.

As mentioned earlier, a grid search is performed for the parameters of the transition function $\{\gamma, c\}$ and for each pair of values the previously described estimation is carried out. In a first round the grid for c is over a subset of the integers $\{1, \dots, T\}$ and for γ a grid in steps of 0.1 in the interval $[-3.5, 3.5]$ is used. The range of γ is chosen such that the full range of transition functions from very flat to very steep functions is covered. In a final step the grid is refined in the neighbourhood of the values minimizing the likelihood function in the first round grid search. Thereby the ML estimators of all parameters are found. The standard errors are obtained as square roots of

the inverted information matrix. The information matrix is estimated using the outer product of the numerical first order derivatives (see Hamilton (1994, p. 143)). This procedure is computationally demanding but not infeasible.

In structural VAR analysis impulse responses are usually used for investigating the transmission process of the shocks. We construct confidence intervals around the impulse responses using a fixed design wild bootstrap procedure. The method preserves the pattern of heteroskedasticity and contemporaneous dependence of the data as noted by Goncalves and Kilian (2004). In the context of structural VAR models identified via heteroskedasticity the method was proposed by Herwartz and Lütkepohl (2011) and used by Lütkepohl and Netšunajev (2014) and Netšunajev (2013) among others. In that procedure the bootstrap samples are constructed conditionally on the ML estimates as

$$y_t^* = \hat{\nu} + \hat{A}_1 y_{t-1} + \cdots + \hat{A}_p y_{t-p} + u_t^*,$$

where $u_t^* = \eta_t \hat{u}_t$ and η_t is a Rademacher distributed random variable that assumes values -1 and 1 with probability 0.5 . We bootstrap parameter estimates conditionally on the initially estimated transition parameters $\{\gamma, c\}$ to alleviate the computational burden. Notice that computing the bootstrap impulse responses still requires nonlinear optimization of the log-likelihood and, hence, is computationally demanding. We use the ML estimates as starting values in the bootstrap replications.

4 An Application: Identifying Monetary Policy and Stock Market Shocks

As an application we reconsider a study of Bjørnland and Leitemo (2009). The authors are interested in the interdependence between U.S. monetary policy and stock prices. The topic has been of considerable interest in the literature. A brief review of some related results is given next and then our own results are presented.

4.1 Literature Review

The interaction between monetary policy and the stock market has been investigated in many studies based on SVAR models. The studies use different data and different identification of shocks. For instance, Li, Iscan and Xu (2010), Thorbecke (1997), Park and Ratti (2000), Patelis (1997) use short-run restrictions on the matrix of impact effects. Bjørnland and Leitemo (2009) criticize identification of structural shocks based on a Cholesky

decomposition used, for instance, by Millard and Wells (2003), Thorbecke (1997), and Cheng and Jin (2013) among others. In this context, ordering stock prices last implies that they react contemporaneously to all other shocks, but all other variables including the monetary policy stance react with a lag to stock market news. This may not be realistic. Building on the argument of money neutrality, Lastrapes (1998) imposes long-run identifying restrictions. Rapach (2001) also uses long-run restrictions to analyze money supply, portfolio and other shocks. Bjørnland and Leitemo (2009) consider an identification that combines contemporaneous as well as long-run restrictions. However, in their setup the restrictions just-identify the shocks and can not be tested against the data.

In spite of different data and identifying assumptions most studies suggest that monetary policy shocks affect stock prices in an important way. Thorbecke (1997), Park and Ratti (2000), and Patelis (1997) find that a contractionary monetary policy shock leads to a decrease in stock prices. Thorbecke (1997) documents a 0.8% decrease in stock prices after an increase in the federal funds rate of one standard deviation. Similarly, Li et al. (2010) observe an immediate negative response of stock prices to a monetary policy shock as well as a relatively prolonged dynamic response with comparable magnitude to the earlier literature. Bjørnland and Leitemo (2009) notice that an analysis based on long-run restrictions (e.g., Lastrapes (1998)) yields considerably stronger effects on the stock market. Using a combination of short- and long-run restrictions they estimate a very strong and persistent decline of around 12% in stock prices after a contractionary monetary policy shock of 100 basis points. No other study finds such a strong effect.

In our approach we are able to test competing ways of identifying shocks based on Cholesky decomposition and on the combination of short-run and long-run restrictions as proposed by Bjørnland and Leitemo (2009). We use our approach in the following to check such restrictions. In particular we check the restrictions imposed by Bjørnland and Leitemo (2009).

Some studies have in fact used identification via heteroskedasticity in this context. Prominent examples are Rigobon and Sack (2003, 2004) and Chen and Velinov (2013). Rigobon and Sack (2004) find that a 25 basis points increase in the three-month interest rate results in a 1.7% decline in the S&P 500 index and a 2.4% decline in the Nasdaq index. Rigobon and Sack (2003) find stock market movements to have a significant impact on short-term interest rates. Chen and Velinov (2013) analyse a smaller version of the Bjørnland and Leitemo (2009) system. They use a Markov switching mechanism to model heteroskedasticity. It should also be noted that their study uses quarterly data and excludes commodity prices from the VAR. Hence their model is not directly comparable to Bjørnland and

Leitemo (2009). Their results indicate that the identification *à la* Bjørnland and Leitemo (2009) is rejected by the data, but a recursive identification is supported. The authors do not plot a full set of impulse responses to shocks identified through changes in variance. Hence interpretation and discussion of their finding is somewhat difficult.

4.2 Empirical Analysis

As in Bjørnland and Leitemo (2009) a five dimensional VAR with the vector of variables $y_t = (q_t, \pi_t, c_t, \Delta s_t, r_t)'$ is considered, where

- q_t is the linearly detrended log of an industrial production index;
- π_t denotes the annual change in the log of consumer prices (CPI index) ($\times 100$);
- c_t is the annual change in the log of the World Bank (non energy) commodity price index ($\times 100$);
- s_t is the log of the real S&P500 stock price index deflated by the consumer price index to measure the real stock prices; the series is first differenced to represent monthly returns (Δs_t);
- r_t denotes the Federal Funds rate.

We use monthly data for the period 1970M1 - 2007M6 which is longer than that of Bjørnland and Leitemo (2009) who exclude the 1970s. With the exception of the commodity price index the data is downloaded from the Federal Reserve Bank of St. Louis database FRED. The commodity price index is from the World Bank.

The more generous Akaike Information Criterion (AIC) suggests a VAR(3) for our sample period 1970M1 - 2007M6. Therefore we also use VAR order 3 for the ST-SVAR model. Estimation of the unrestricted ST-SVAR(3) model is done with the relative variances λ_i ordered from smallest to largest. Some statistics for the estimated models without and with smooth transition in variance are presented in Table 1. It is obvious that the ST-SVAR model allowing for heteroskedasticity is clearly preferred by AIC and the Schwarz criterion (SC).

The transition function of the ST-SVAR(3) model is presented in Figure 1. It shows that there is a gradual change in variance from the end of the 1970s to the middle of the 1980s. This corresponds well to the beginning of the Great Moderation period. Note that the smoothed state probabilities of

Table 1: Comparison of VAR(3) Models for $y_t = (q_t, \pi_t, c_t, \Delta s_t, r_t)'$

| Model | $\log L_T$ | AIC | SC |
|------------|------------|----------|----------|
| VAR(3) | -3159.344 | 6508.689 | 6899.067 |
| ST-SVAR(3) | -2878.255 | 5976.510 | 6428.527 |

Note: L_T - likelihood function, $AIC = -2\log L_T + 2 \times \text{no of free parameters}$, $SC = -2\log L_T + \log T \times \text{no of free parameters}$.

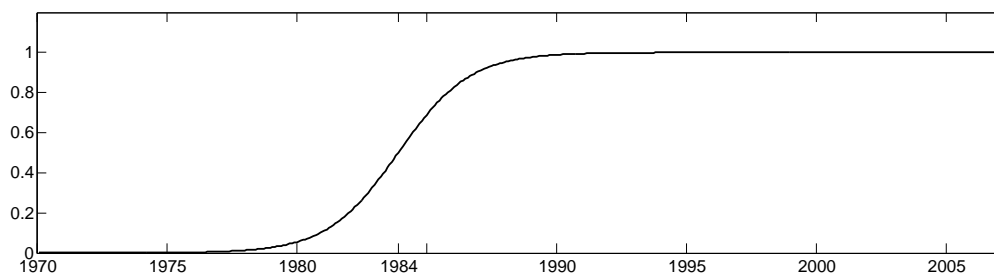


Figure 1: Transition function for the ST-SVAR model.

the model used by Chen and Velinov (2013) resemble the gradual change in 1970-2007 from one regime to another (see Chen and Velinov (2013, Figure 4.2)). This lends support for the ST-SVAR model for our sample. We admit, however, that our model does not perfectly capture the volatility changes in all residual series. Thus, based on the statistics in Table 1 our model is clearly better than one without changes in residual volatility but may leave room for further refinements. It is shown in the following that the model is quite suitable for illustrating the additional options the ST-SVAR model opens up for structural analysis. In other words, using our relatively simple model is useful for illustrative purposes.

Since we are interested in using changing variances of structural shocks for identification purposes, a central question of interest is whether we have sufficient heterogeneity in the volatility changes to get identification. The estimated λ_i of the ST-SVAR model are shown in Table 2 together with estimated standard errors.

The estimated relative variances are all below one meaning that the transition occurs from a high volatility state to a low volatility state. This is consistent with the economic narratives on the U.S. Great Moderation start-

Table 2: Estimates of Relative Variances of ST-SVAR(3) Model for Unrestricted B and Ξ_∞

| parameter | estimate | std.dev. |
|-------------|----------|----------|
| λ_1 | 0.019 | 0.002 |
| λ_2 | 0.315 | 0.057 |
| λ_3 | 0.548 | 0.088 |
| λ_4 | 0.867 | 0.154 |
| λ_5 | 0.927 | 0.172 |

ing in the mid 1980s.

The low standard errors indicate that estimation precision is reasonable, hence, the identification condition may be satisfied for the λ_i . To test formally if there is sufficient heterogeneity in the variances we use Wald tests. On the one hand, they are attractive in the present context because they are very easy to compute from the estimates of the unrestricted model. On the other hand, they are known to be unreliable for highly nonlinear null hypotheses. For the five-dimensional system 10 pairs of relative variances have to be distinct to get a full set of identified shocks. The corresponding test statistics of the relevant hypotheses are presented in Table 3. For the current model, the null hypotheses of pairwise equality is rejected at a 10% significance level for all pairs but the last. This is no surprise as the estimates in Table 2 show that the estimated λ_4 and λ_5 are close to each other. Thus, we conclude that we have some statistically identified shocks but perhaps not a fully identified structural model.

Not having a full set of identified shocks is not necessarily a problem for our approach because it is enough if there is some extra identifying information that allows us to test conventional identifying restrictions. For example, we may be able to check a recursively identified model with a triangular B matrix. Such a set of restrictions is of interest because recursively identified shocks have been used by a number of authors in the present context (e.g., Millard and Wells (2003), Cheng and Jin (2013), Park and Ratti (2000) and others).

Of course, from the point of view of our analysis it is of particular interest to test the restrictions specified by Bjørnland and Leitemo (2009) and, hence, to check whether they are in line with the data. Their restrictions can be

Table 3: Tests for Equality of λ_i for Unrestricted ST-SVAR Model

| H_0 | Wald statistic | p -value |
|-------------------------|----------------|------------------------|
| $\lambda_1 = \lambda_2$ | 26.463 | 2.686×10^{-7} |
| $\lambda_1 = \lambda_3$ | 35.720 | 2.277×10^{-9} |
| $\lambda_1 = \lambda_4$ | 29.806 | 4.673×10^{-8} |
| $\lambda_1 = \lambda_5$ | 27.498 | 1.572×10^{-7} |
| $\lambda_2 = \lambda_3$ | 4.729 | 0.029 |
| $\lambda_2 = \lambda_4$ | 10.731 | 0.001 |
| $\lambda_2 = \lambda_5$ | 10.583 | 0.001 |
| $\lambda_3 = \lambda_4$ | 2.814 | 0.093 |
| $\lambda_3 = \lambda_5$ | 3.687 | 0.054 |
| $\lambda_4 = \lambda_5$ | 0.061 | 0.805 |

visualized in the following way:

$$B = \begin{bmatrix} * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & * \\ * & * & * & * & * \end{bmatrix} \quad \text{and} \quad \Xi_\infty = \begin{bmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & 0 \\ * & * & * & * & * \end{bmatrix}, \quad (7)$$

where the asterisks indicate unrestricted elements and zeros denote elements restricted to zero. The shocks of particular interest are the shocks ordered fourth and fifth. This identification suggests that the last shock is the monetary policy shock and it has no immediate impact on industrial production, inflation and commodity prices as well as no long-run effect on stock prices. The shock ordered fourth is viewed as the stock price shock and it has no contemporaneous effect on the real side of the economy. Note that the first three shocks are not of interest for the current analysis. They can be identified arbitrarily to perform a conventional SVAR analysis.

In the following we consider alternative sets of restrictions for the initial effects matrix B and the long-run effects matrix Ξ_∞ and check their compatibility with the data.

- M1: B lower triangular (recursive identification);
- M2: B and Ξ_∞ restricted as in (7) (Bjørnland-Leitemo identification);
- M3: only the two last columns of B and Ξ_∞ restricted as in (7);
- M4: only B restricted as in (7).

We have estimated ST-SVAR(3) models with these types of restrictions. No restrictions are imposed on the λ_i when restricted structural models with constraints on B and/or Ξ_∞ are estimated. Assuming that there is enough heterogeneity in the variances of the structural shocks to obtain curvature in the likelihood function we can test the restrictions by LR tests. Some results are shown in Table 4 where most models are tested against the unrestricted ST-SVAR(3).

Table 4: Tests for Identifying Restrictions in ST-SVAR Models

| H_0 | H_1 | LR statistic | df | p -value |
|-------|---|--------------|----|----------------------|
| M1 | ST-SVAR with unrestricted B | 23.395 | 10 | 0.009 |
| M2 | ST-SVAR with unrestricted B, Ξ_∞ | 35.845 | 10 | 8.9×10^{-5} |
| M3 | ST-SVAR with unrestricted B, Ξ_∞ | 30.909 | 7 | 6.4×10^{-5} |
| M4 | ST-SVAR with unrestricted B, Ξ_∞ | 22.491 | 9 | 0.0074 |
| M2 | M4 | 13.354 | 1 | 2.5×10^{-4} |

The p -values of all tests are very small and, hence, the tests reject at conventional significance levels. Thus, as a first conclusion it is clear that heteroskedasticity provides sufficient information to check the restrictions. The first test shows that a recursively identified model has no support from the data and the second test makes the same point for the restrictions used by Bjørnland and Leitemo (2009). The last test in Table 4 indicates that their long-run restriction is clearly rejected as an additional restriction in a model with the same impact restrictions as in their model. The third and fourth tests in the table show that rejection is not just due to the restrictions imposed to identify shocks that are not of interest in the present context. Thus, there is strong evidence against the identifying restrictions imposed in a conventional analysis of the interaction between monetary policy and the stock market. The restrictions are clearly rejected by the data once heteroskedasticity is taken into account.

Given that the identifying restrictions are rejected it is of interest to see the impact of the different identifying restrictions on the impulse responses and compare them to impulse responses obtained without imposing the restrictions. Of course, there is one problem with impulse responses computed from our ST-SVAR model identified via heteroskedasticity. The shocks obtained by this kind of identification do not have a natural labelling and may not be interpretable as economic shocks. In Figure 2 the impulse responses are plotted with 68% confidence bands based on 1000 bootstrap replications. It turns out that there are only two shocks that can be interpreted as monetary policy and stock market shocks. Clearly, for a monetary policy shock

a minimum characteristic is that the interest rate moves on impact. There is only one shock where a significant impact effect on the interest rate is obtained, namely the first one. Likewise, a stock market shock should affect the stock index on impact and again there is just one shock where this condition is clearly satisfied, namely the fourth one. Therefore we label these shocks monetary policy and stock market shocks, respectively. In other words, the monetary policy shock is the one with the smallest relative variance (λ_1) and the stock market shock is the fourth shock with the second largest relative variance (λ_4). These are also the most plausible monetary policy and stock market shocks considering the impulse responses of the variables. We discuss the interpretation of the shocks and compare them with the conventionally identified shocks later.

Another obstacle for comparing impulse responses obtained in this way to impulse responses from a conventional analysis is the variation in the volatility of the shocks. In a conventional analysis the shocks are occasionally scaled to be of size one standard deviation. Since in our framework the standard deviation changes across the sample this scaling is problematic. Therefore, to compare shocks from a conventional analysis and from our approach, we look at shocks of one unit. For example, a monetary policy shock may be scaled to lead to a 25 or 100 basis points increase in the interest rate.

In Figures 3 and 4 we compare impulse responses of monetary policy and stock market shocks identified in three different ways:

1. A recursive ordering based on a VAR(3);
2. The Bjørnland and Leitemo (2009) identification based on a VAR(3);
3. Our approach of identification via heteroskedasticity based on the ST-SVAR(3).

In Figure 3 the impulse responses of q_t , π_t , c_t , s_t , and r_t to a monetary policy shock of 100 basis points are compared and in Figure 4 the corresponding impulse responses to a stock market shock of 1% are depicted.

The response of the output variable q_t to a monetary policy shock is rather standard for the conventional SVAR models. There is a lagged negative effect reaching its lowest point at around the second year after the shock. On the contrary, the ST-SVAR model produces a significantly positive reaction on impact that dies out after several months. Even though the point estimates of the impulse responses are negative after nearly two years, the reaction is insignificant at the given confidence level. Looking at the response of inflation π_t , one can observe a small initial increase for both conventional

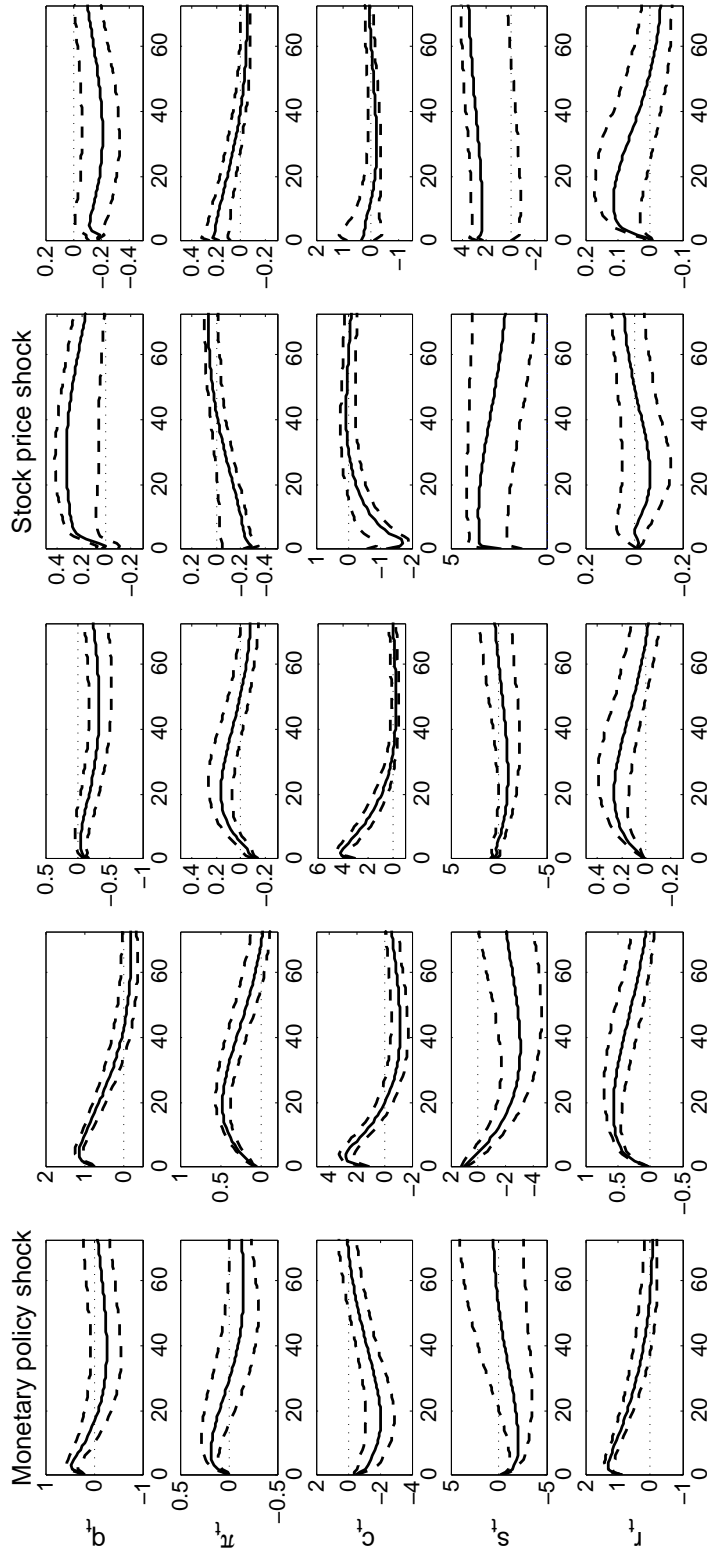


Figure 2: Impulse response functions for fully unrestricted ST-SVAR model. Solid line - point estimate of the response, dashed line - 68% confidence bands based on 1000 bootstrap replications.

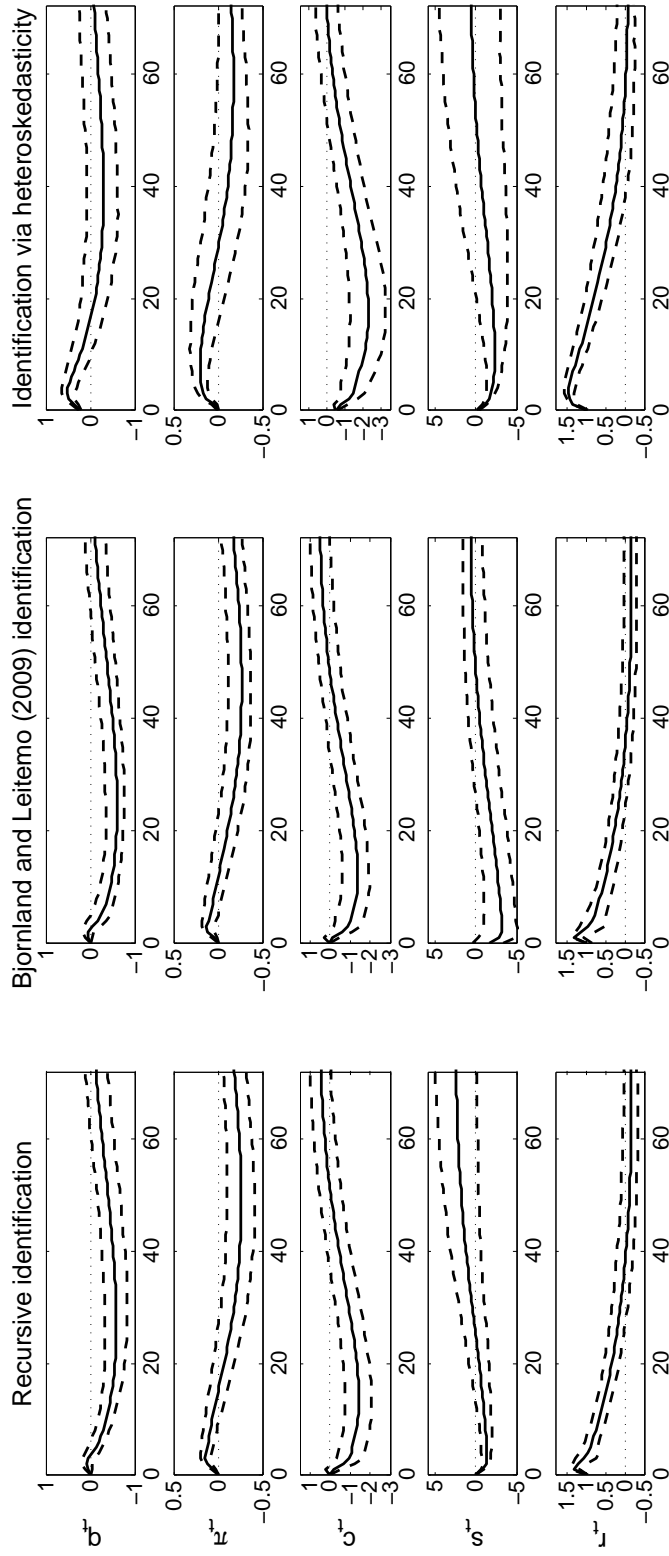


Figure 3: Responses to monetary policy shock using different identification schemes. Solid line - point estimate of the response, dashed line - 68% confidence bands based on 1000 bootstrap replications.

identification schemes, whereas the initial dynamics is more pronounced in the model identified via heteroskedasticity. The situation is different for commodity prices. They fall initially in all three models and the decrease is more pronounced in the ST-SVAR model compared to standard SVAR models.

The reaction of stock prices is of primary interest for us. Recursive identification yields a small drop that vanishes quickly and may even turn into an increase that is not significant, however. The stock prices are more responsive in the model identified in the Bjørnland and Leitemo (2009) fashion. We observe a more pronounced decrease of about 3% with the effect lasting for nearly two years. Even though we do not get the same magnitude of decline of the stock index as Bjørnland and Leitemo (2009), their observation of a more pronounced reaction than in the recursively identified scheme is confirmed when identification is done via heteroskedasticity. The reaction depicted in the last column of Figure 3 reveals a very small initial impact effect of the monetary policy shock on stock prices. However, the stock prices drop by 2.3% after a year. The persistence of the reaction is similar to what we observed in the second column. Even though we reject the long-run identifying restriction of Bjørnland and Leitemo (2009), the observed effect on the shock prices appears to be transitory. Thus, this reaction of stock returns to a monetary policy shock identified via heteroskedasticity is closer to the reaction obtained with the Bjørnland and Leitemo (2009) identification scheme than with recursive identification.

However, the monetary policy shock identified via heteroskedasticity has some economically counter intuitive properties. Namely there is a very pronounced output puzzle and a rather pronounced price puzzle. From the macroeconomic prospective one would not expect these two features for the dynamic responses to a monetary policy shock. On one hand, the price puzzle for the monetary policy shock is observed by Park and Ratti (2000) and Bjørnland and Leitemo (2009), although the latter use detrended output as suggested by Giordani (2004). On the other hand, no study finds a positive reaction of output. Comparing the monetary policy shock identified via heteroskedasticity to a variety of shocks studied in the literature one can find a sibling to our shock. To be precise, it very much resembles the money market equilibrium shock defined by Li et al. (2010) as an exogenous change in the velocity of money. This shock also leads to an increase in output, prices and interest rates on impact and a moderate decrease in stock returns (see Li et al. (2010, Figure 1)). Lastrapes (1998) studies a money supply shock and also finds a tendency for output to respond positively on impact. This leads us to think that the monetary policy shock identified via heteroskedasticity captures rather the demand side than the supply side of the

money market (compare to the reaction of variables after a money demand shock in Kulikov and Netšunajev (2013) and Favara and Giordani (2009)). Hence, labeling this shock as a ‘monetary policy shock’ in the conventional sense may be misleading. However, as discussed before, there is no other shock in the ST-SVAR(3) system that leads to an immediate increase in the Federal Funds rate. Therefore in our system and in the system analyzed by Bjørnland and Leitemo (2009) there is hardly any shock that captures the supply side of the money market. This finding sheds some light on why the identifying restrictions are not supported by the data: the restrictions do not identify the desired shock.

We now turn to the discussion of the stock price shocks shown in Figure 4. Conventionally identified SVAR models produce a lagged positive reaction of output, inflation, and the Federal Funds rate. This is in line with the findings of Bjørnland and Leitemo (2009) and Park and Ratti (2000). Note that the reaction of output dies out reasonably quickly. Several important differences can be seen when comparing the conventional SVAR with ST-SVAR stock price shocks identified via heteroskedasticity. This may indicate that the stock price shocks identified using a conventional approach have a different economic interpretation relative to the ST-SVAR stock price shocks.

In the ST-SVAR, after a stock price shock, output reacts in a much more persistent way, with the effect vanishing only after six years. This indicates that this shock may be rather seen as a “news” shock, containing information about the future that is not yet captured by the macro variables. Such a shock leads to a delayed and very persistent change in productivity (Beaudry and Portier (2006)). A shock without a longer lasting persistent effect on output is sometimes called non-fundamental in the related literature. In this sense the stock price shock is a non-fundamental one when conventional identifying restrictions are used because in the first and second columns of Figure 4 it has a short and transitory effect on output.

Jaimovich and Rebelo (2009) show in a theoretical model that a “news” shock may have an immediate effect on certain variables (especially consumption). This is ruled out by Bjørnland and Leitemo (2009) but possible in our ST-SVAR approach. Indeed the initial effect of a stock price shock identified via heteroskedasticity is significant for inflation and commodity prices, meaning that the asset prices influence consumption. This is further evidence in favour of labeling the ST-SVAR stock price shock as a “news” shock.

Furthermore, the central bank monitors the stock prices as indicators for suitable monetary policy. For that reason the Fed reacts with a lag to a non-fundamental stock price shock by reducing the money supply and balancing the booming stock market (see columns 1, 2 of Figure 4, Bjørnland and Leitemo (2009), and Park and Ratti (2000)). The observed magnitude

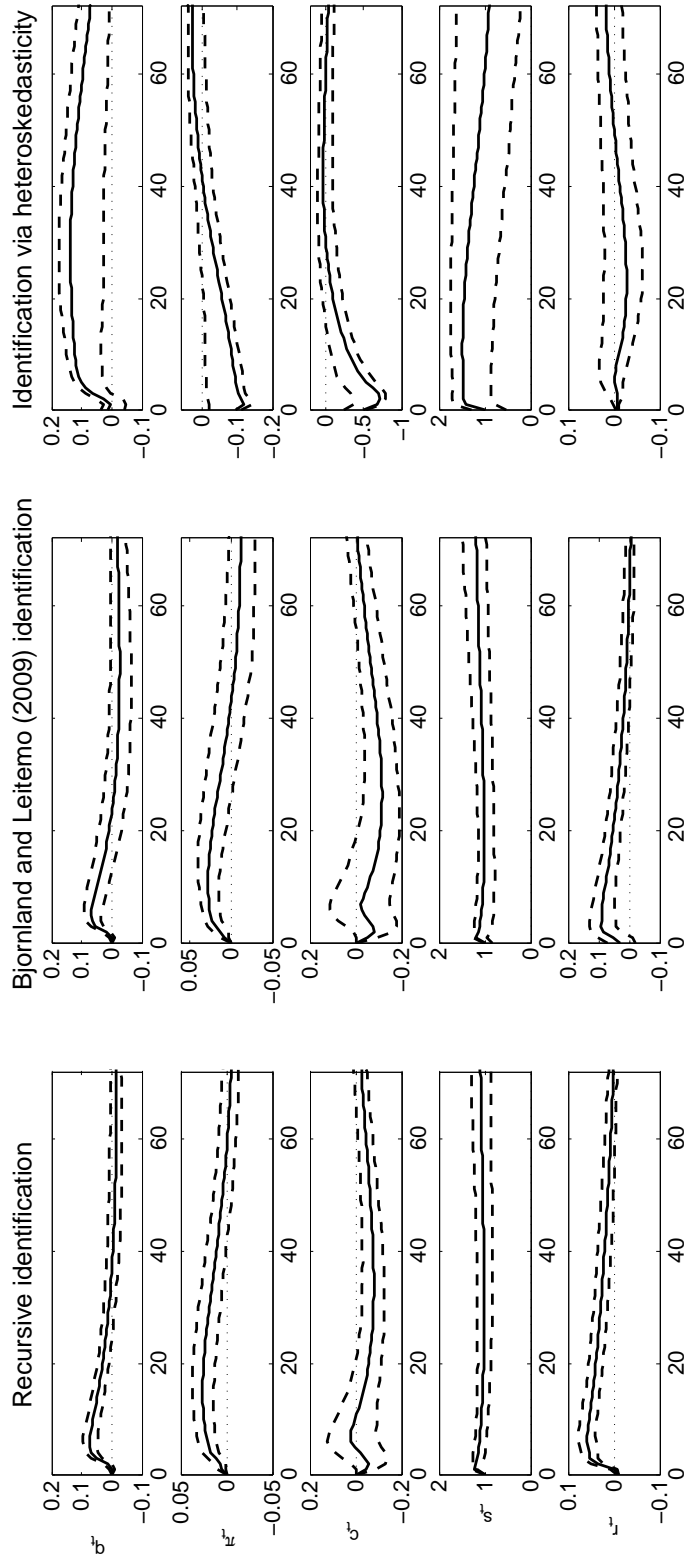


Figure 4: Responses to stock price shock using different identification schemes. Solid line - point estimate of the response, dashed line - 68% confidence bands based on 1000 bootstrap replications.

of the reaction of the interest rate in the first two columns of Figure 4 is also consistent with Bjørnland and Leitemo (2009) and is around 5 - 10 basis points. On the contrary, there is not much of a change in the Federal Funds rate in response to the ST-SVAR stock price shock. In other words, there is not much of a central bank reaction to such a shock. Hence the Fed anticipates that the increase in output is driven by fundamentals, say technology improvements, and not a bubble component of stock prices.

Summing up, these arguments make us think that the economic nature of the conventional SVAR and ST-SVAR stock market shocks is very different. While conventional SVAR models identify a “sunspot” shock, the ST-SVAR stock market shock has a “news” interpretation. This shows the importance of the identifying assumptions and that it makes sense to take into account as much information as possible. In particular, it is worth taking advantage of identifying information in the volatility of the shocks.

5 Conclusions

In the present paper we set up a SVAR model with smooth transition in the variances of the residuals. The model is an alternative to other ways of modelling changes in variance in VAR models such as Markov switching or multivariate GARCH models. Our ST-SVAR model has the advantage of being reasonably easy to estimate. Moreover, a well developed toolkit for the statistical analysis of smooth transition regression models is available that is adoptable for the present models.

We show how the model can be used to identify shocks in SVAR analysis and to test conventional identifying restrictions. Although we utilize only one transition between volatility states, it is possible to extend the model by adding further transition terms. Such an extension seems technically feasible if there are enough data in each volatility state. Moreover, it is also possible to allow a level term to change during the sample period by attaching the transition function also to that term. As long as the VAR coefficients are time invariant, impulse response analysis can be performed as discussed in the present paper.

As an illustration of the ST-SVAR approach we analyze the relation between monetary policy and the stock market using a system considered by Bjørnland and Leitemo (2009). The estimated model suggests a rather smooth transition from a high volatility state to a low volatility state at around 1984. This is consistent with the economic narratives on the Great Moderation in the U.S. being in place from the mid 1980s. The main question of interest, however, is the interaction between the U.S. stock market

and monetary policy. For the SVAR analysis Bjørnland and Leitemo (2009) propose a combination of short- and long-run identifying restrictions and contrast their results with a recursive identification scheme.

Our model allows to test the identification of Bjørnland and Leitemo (2009) as well as competing Cholesky based recursive identification. We reject both the identification based on recursive ordering of the variables and on a combination of short- and long-run restrictions. Our approach allows us to study the impulse responses of the unrestricted ST-SVAR model. They reveal why the data is not supporting the conventional restrictions. We find that in a conventional SVAR the monetary policy shock represents the supply side of the market. Using our model we find that there is a monetary shock representing the demand side of the money market which has pronounced impact effects on the variables. We document a maximum of 3% decline in stock prices after a 100 basis points increase in the interest rate. The stock price shock identified in a standard SVAR can be interpreted as a non-fundamental shock. In contrast, in the ST-SVAR model we find a stock price shock that exhibits characteristics of a “news” shock (Beaudry and Portier (2006)). Specifically, we do not observe any reaction of the central bank to the “news” shock. Thus, when identification via heteroskedasticity is used, the interaction between monetary policy and the stock market is seen in a quite different light than in a conventional analysis.

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