On the efficiency of spam mailing and portal advertising‡

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Abstract

The aim of this paper is to provide a simple approach that is able to endogenise the welfare reducing or welfare enhancing result of informative advertising. It is then possible to analyse the welfare effect of a technology shock that reduces unit information costs, and to explain whether electronic junk mails or portal advertising will cause welfare gains or losses in a competitive environment.

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1 Introduction

Advertising is often assumed to be a bad that reduces customers’ utility. Machlup (1980, p. 130)\(^1\) uses colourful words to illustrate this disutility of advertising: “The existence of such unwanted knowledge will hardly be contested by anybody who has his radio [...] program rudely interrupted by long-winding commercials [...] . Some of the jingles which advertise the wonderful qualities of this or that poduct [...] may stick with the musical memory of some unhappy listeners like wads of chewing gum to the shoe soles of unhappy pedestrians and resist all efforts to remove them.”

In recent years, new tedious sources of advertising have been added to television and radio commercials: As Lueg (2003) emphasises, an average customer has received some 1400 electronic junk mail messages (“spam”) in the year 2003. In total, spam is estimated to cause damages of some 3 billion EUR p. a. in Europe and 8 billion EUR in the U. S. Also portal advertising has emerged, causing a nuisance to surfers on the internet.\(^2\)

However, from an information economists’ point of view, advertising may also be welfare increasing, since it provides information about sellers, product attributes and prices and so allows beneficial trades to occur.\(^3\) The aim of this paper is to provide a simple approach that is able to endogenise the welfare reducing or welfare enhancing result of informative advertising. It is then possible to analyse the welfare effect of a technology shock that reduces unit information costs, and to explain wether electronic junk mails or portal advertising will cause welfare gains or losses in a competitive environment.

2 The model

Consider a market that has been described by Butters (1977), where a large number of producers and consumers trade a homogenous good. All sellers use an identical technology to produce the homogenous good at constant marginal costs \(c\). All

\(^{1}\)Quoted in Kulenkampff (2000, p. 154).

\(^{2}\)See, e. g. Barros et al. (2002) for references.

\(^{3}\)See Schmalensee (1986) for a comprehensive survey of the literature related to this topic.
consumers have an identical willingness to pay a price $p_m$ to buy one unit each. However, they are initially uninformed and do not know, where to buy the products. To inform the buyers, sellers send out advertisements at random. Each advertisement is regarded as a binding offer to buy the product at price $p$. If a customer has received more than one advertisement, he will accept the cheapest offer, as long as this price offer does not exceed the reservation price $p_m$, while customers who have not received any advertising, are not able to buy the product. If a consumer receives two or more offers quoting the same price, he chooses one of them randomly.

Let $w_H$ be the cost of sending one advertisement to one randomly and independently selected consumer, and let $w_L$ be the cost of “reading" that advertisement. While the sender of an advertisement (i. e. the producer or, in the expanded model, the internet portal) bears $w_H$, the reading cost $w_L$ is borne by the receiver of the ad, i. e. the customer. Hence, total social costs of advertising add up to $w = w_H + w_L$ per advertisement.

The pricing decision of a company will depend on the advertisement cost it bears and the probability that an advertising announcing this price $p$ will generate a sale. Note that this probability depends on how many other advertisements quoting a price lower than $p$, an average customer is expected to receive. Assume that this number is characterised by an advertising price distribution function $Z(p)$, showing the expected number of ads per customer sent out at a price less than or equal to $p$. Hence, the total amount of advertising per consumer is given by $Z(p_m)$.

Advertising is successful, whenever the consumer who reads this ad, has not received any other advertising which offers the product at a lower price. For a large number of advertisements and consumers, and a given per-customer advertising price distribution $Z(p)$, this probability $\pi(p)$ can approximately be calculated using the Poisson distribution:

$$\pi(p) = e^{-Z(p)} \quad (1)$$

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4Readers should not take this expression too literally: It involves any costs borne by the reader of the ad, e. g. costs to identify and discard the message.

5As a matter of fact, there will be no advertising at prices $p > p_m$, as a price like this would exceed the customers’ willingness to pay.
Note there will always remain some customers who are still uninformed and thus not in the position to buy the traded good. However, this proportion of uninformed customers declines when $Z(p)$ grows large. Given the assumptions stated above, it is now possible to investigate the market outcome for three different scenarios:

(i) Firstly, I will assume that reading costs do not exist. Thus, firms bear the total social cost of advertising, when sending advertisements randomly to the customers. This case has been investigated by Butters (1977) and will serve as reference to the other scenarios, because its outcome is efficient.

(ii) Secondly, I will presume that reading costs do exist and are borne by the customers who receive the advertisements. It is then possible to evaluate the welfare loss that arises from the external effect of sending advertisements. I will show that a reduction of the unit information cost $w_H$ will lead to an increasing welfare loss, if the reading cost $w_L$ does not change.

(iii) Thirdly, I will assume portal advertising. That is, producers do not engage in direct mailing, but place their advertising on the website of an internet portal and pay a price per view $w_I$ to the portal operator. All internet portals are assumed to provide the same editorial content and bear the same unit cost $w_H$ to provide the advertisement, that is, to transmit the advertisement information to the viewer.

### 3 The market result of the basic model

Now, consider the first case where the firms bear total advertising costs $w = w_H + w_L$. Butters (1977, p. 470) argues that firms will engage in additional price offers $p$, until the average profit of an advertisement equals zero. If the average profit was beyond, firms would have an incentive to increase their number of advertisements. This implies zero expected profits $G(p)$ for any price $p$:

$$G(p) = \pi(p)(p - c) - w = 0$$  \hspace{1cm} (2)

Simple transformation leads to

$$\pi(p) = \frac{w}{p - c}$$  \hspace{1cm} (3)
Note that both conditions (3) and (1) have to be fulfilled. Hence, the equilibrium advertising price distribution is as follows:

\[
Z(p) = \begin{cases} 
\ln(p_m - c) - \ln w & \text{iff } p \geq p_m \\
\ln(p - c) - \ln w & \text{iff } c + w \leq p < p_m \\
0 & \text{iff } p < c + w 
\end{cases}
\]  

(4)

All prices between \(c + w\) and \(p_m\) will be advertised. However, the probability of success declines from 1 (for a price offer \(p = c + w\)) to \(w/(p_m - c)\) for \(p = p_m\). To calculate the normalised total welfare \(W\) (i.e. the net surplus per consumer), one has to regard the gains from trade \(p_m - c\), which occur with probability \(1 - \pi(p_m)\), and the total information costs, \(w \cdot Z(p_m)\):

\[
W = p_m - c - w - w \ln(p_m - c) + w \ln w
\]

(5)

It is easy to see that the market equilibrium maximises \(W\): Assume that an additional advertisement is sent. This will be welfare increasing if an uninformed customer is reached—which happens with probability \(\pi(p_m) = w/(p_m - c)\). Hence, expected gains of an additional advertisement are \((p - c)[w/(p - c)]\) and equal to its social unit cost \(w\).

4 The efficiency of spam mailing

I do now consider the case where reading costs exist: I assume that every consumer has to bear a unit reading cost \(w_L\) when he receives an advertisement, no matter whether he will accept this price offer or not. However, the sellers ignore this reading cost when they decide about their advertising strategy. As a consequence, they send too many ads: The lower bound of the advertising price distribution lies at \(c + w_H\), whereas a lower bound of \(c + w = c + w_H + w_L\) would be efficient. Hence, the total amount of advertising \(Z^*(p_m) = \ln(p_m - c) - \ln w_H\) exceeds the efficient value \(\hat{Z}(p_m) = \ln(p_m - c) - \ln(w_H + w_L)\). Thus, the equilibrium surplus is given by

\[
W^* = p_m - c - w_H - (w_H + w_L) \ln[(p_m - c)/w_H]
\]

(6)
From (5) we can derive the welfare maximum by inserting \( w_L + w_H \) for \( w \):

\[
\hat{W} = p_m - c - w_H - w_L - (w_H + w_L) \ln\left(\frac{p_m - c}{w_H + w_L}\right)
\]  

(7)

The welfare loss due to the reading cost externality is given by \( \Delta W = \hat{W} - W^* \):

\[
\Delta W = (w_H + w_L) \ln\left(\frac{w_H + w_L}{w_H}\right) - w_L
\]

(8)

Note that this welfare loss can be estimated by a simple Taylor approximation:

\[
0 \leq \Delta W \leq \frac{w_L^2}{w_H}
\]

(9)

While the welfare loss from excessive advertising does not depend on the potential gains from trade, the relation between the unit information cost \( w_H \) and the unit reading cost \( w_L \) is crucial. As soon as the reading costs account for a higher proportion of total advertising costs, this leads to an increase of the welfare loss. On this account, the welfare loss of “spam” advertising has risen after new information technology has been introduced: Unit information cost has declined dramatically, while the cost of “reading” an advertisement have roughly stayed the same. Spammers nowadays bear less than 0.1% of the social costs of advertising. The policy implication is to ban electronic junk mailing, since there exists a more efficient way to inform the customers about products.

5 The efficiency of portal advertising

Now, consider the third scenario: Advertisements are not directly sent by the producers, but placed on internet portals. Internet users—that is, the customers—are

\[\text{From standard textbook mathematics, it follows that } \ln\left(\frac{x+y}{y}\right) \leq \frac{x}{y} \text{ and } \ln\left(\frac{x+y}{y}\right) \geq \frac{x}{(x+y)} \text{ generally holds for any } x, y > 0\]

\[\text{I use the taylor approximation here, because the resulting formula is more “eye catching”. However, the main results of this section will remain unchanged, regardless whether the Taylor approximation is used or not.}\]

\[\text{This proposition is affirmed by the derivatives of } \Delta W \text{ with respect to } w_H \text{ and } w_L: The results are } \partial \Delta W / \partial w_L = \ln(w_H + w_L) > 0 \text{ and } \partial \Delta W / \partial w_H = \ln\left(\frac{w_H + w_L}{w_H}\right) - w_L/w_H < 0.\]

\[\text{Yet, this is a crude estimation based on some figures found in the internet. See e. g. the website of John Levine for more information and hyperlinks.}\]
assumed to choose one portal each, read the advertising on it and benefit from editorial content. As in the case of direct advertising, I assume that the advertising information is transmitted randomly to the customer. This could be established when we assume that visitors surfing a portal, read different pages and ads that are installed on these pages. In their portal choice, consumers behave perfectly rational: They regard the disutility of reading the ads as well as the expected surplus from a potential buy.

5.1 Model assumptions

To be more exact, I assume a three stage model for portal advertising: In the first stage, internet portals $I$ set their ad-rates $w_I$. The ad-rate denotes the price the advertiser has to pay each time a consumer visits the respective website that contains the advertisement.\footnote{In reality, this is the most common buying model—however, the ad-rate is expressed by the “CPM”, i. e. the cost of 1000 ad impressions.} In the second stage, the firms deside on their advertising strategy. In the last stage, consumers choose one portal.

To keep calculations simple, I assume perfect competition between two internet portals. That is, they provide the same editorial content $r$ (e. g. news, stories and hyperlinks), and the expected consumer surplus depends on both, editorial content and advertising:

$$U_I = q_I(w_I, w_L, p_m - c) + v(r) \quad (10)$$

While $v(r)$ is fixed in the context of this model\footnote{Still, it might be interesting to look at the portals’ decision on $r$ in an expanded version of the model.}, the portals implicitly decide on $q_I$ when they set the advertisement prices $w_I$. One should note that this additional information rent $q_I$ a consumer receives by visiting a portal and reading the ads, is similar to what I have called the “net surplus per consumer” in the basic model. While in that basic case, the net consumer surplus is equivalent to the total welfare (because firms just earn zero profits), gains from trade are now shared by customers and internet portals. To calculate the correct value of $q_I$, one has to bear in mind
that the producers are charged \( w_I \) for portal advertisements, and not the (lower) unit information cost \( w_H \). They internalise this cost \( w_I \), when calculating a price offer. As a consequence, \( q_I \) yields

\[
q_I = p_m - c - w_I - (w_I + w_L) \ln[(p_m - c)/w_I]
\]

The profit of an internet portal depends on its market share (i.e. the number of customers \( x(q_I) \) visiting that portal), the amount of advertising per consumer which is given by the advertising distribution function \( Z(p_m) \) (please notice the analogy to the basic model), and the mark up on the unit information cost of advertising \( w_I - w_H \). In addition, the fixed cost \( r \) for providing editorial content might be taken into consideration. However \( r \) will be ignored for the further analysis, since it is assumed to be a sunk cost. Hence, the profit of an internet portal \( I \) can be written as

\[
G_I = x_I(q_I)(w_I - w_H) \ln[(p_m - c)/w_I]
\]

With this information on the consumers’ and the internet portals’ payoff functions, I am now in the position to calculate the subgame perfect market equilibrium.

### 5.2 The market equilibrium

The market equilibrium is solved by backward induction. In the last step, customers decide on which portal to visit. Since both portals offer the same editorial content, they select the internet portal that offers a higher expected information rent \( q_I \) to their visitors. If both portals offer the same \( q_I \), customers select one portal by random. Note that \( q_I \) depends on the amount of advertising on the respective internet portal: If there was no advertising on that portal, a visitor could not gain from any additional trade. On the other hand, an internet portal full of advertising would be tedious to read, with reading costs that might well exceed the expected gains from trade.

Hence, the advertising policy of the sellers seems to be crucial for the success of an internet portal. However, the average number of ads that a consumer will find on his preferred portal, is a result of the advertisement price \( w_I \) a portal charges
to the producers. Since the producers to not posses any market power, they just adapt their advertising policy to the market conditions.\textsuperscript{12} Thus, the second stage advertising decision of the producers is pre-determined by the the first stage decision of the internet portals on ad-rates \( w_I \).

In general, players aim to maximise their objective funtion. Take a look at the objective function of an internet portal (see equation 12): Its value depends on the number of visitors and the mark up on the unit information cost. It is important to know that the number of visitors is not a continous function. As in the discussion of the last step has been explained, consumers select the portal that offers the highest net expected utility. Hence, the portal operators engage in a Bertrand-like competition on customers. As a result, they adopt the objective of the consumers in order to maximise the customers net expected utility \( q_I \). Note that both portals will set the same ad-rate, because they maximise the same objective function (that one of the consumers).

Doing this, they face two constraints: The ad-rate \( w_I \) has to cover the unit information cost \( w_H \) (else it would cause losses for the internet portals to offer advertising; I will call this the “zero profit constraint”); and advertising has to generate a positive surplus for the visitors reading the ads. Else the customers would select a portal that does not provide any advertising (this I will call the “consumer participation constraint”).

Taking this into consideration, we receive the following maximisation problem:

\[
\begin{align*}
\max_{w_I} q_I &= p_m - c - w_I - (w_I + w_L) \ln[(p_m - c)/w_I] \\
\text{s. t.} \quad & w_I^* \geq w_H \\
& q_I^* \geq 0
\end{align*}
\]

Now, consider the unrestricted solution \( w_I^* \) to this maximisation problem. It is given impliciteely by the first order condition

\[
\frac{w_L}{w_I^*} - \ln \frac{p_M - c}{w_I^*} = 0.
\]

\textsuperscript{12}Interestingly, not all producers do turn away from a portal that charges a higher unit ad-rate \( w_I \), because an advertisement succeeds with a higher probability, due to a less amount of advertising (and less advertising of the competitors) on that portal.
Unfortunately, it is not possible to present an explicit solution to \( w^*_I \). However, equation (14) can be simplified by expressing \( p_m - c \) as a multiply \( k \) of the reading cost \( w_L \), i.e. \( k \) is defined as \((p_m - c)/w_L\)—it is then possible to implicitly determine the relative ad-rate \( w^*_I/w_L \):

\[
\frac{w_L}{w^*_I} - \ln \frac{w_L}{w^*_I} = \ln k
\]  

(15)

As can be seen easily, the equilibrium value \( w_I \) is only influenced by the potential gross gains from trade in terms of reading cost \( kw_L \), and the reading cost \( w_L \). Obviously, the minimum value of the left hand side of this equation is 1. This value is received for \( w_L = w_H \), i.e. when the equilibrium ad-rate equals the reading cost.

As a consequence, for any spread \( p_m - c \leq ew_L \), no interior solution exists and zero advertising maximises \( q_I \). For the border case \( p_m - c = ew_L \), a solution exists where the internet portals charge an ad-rate \( w_I \) equal to the reading cost \( w_L \). For higher values of \( k \), there are two relations \( w_L/w_I \) to solve the first order condition. However, the second order condition holds only for the case that the reading cost exceeds the ad-rate, that is \( w^*_I < w_L \). The other solution to the first order condition \( w^*_I > w_L \) marks the minimum value of the objective function.

Until now, the two constraints \( w^*_I \geq w_H \) and \( q^*_I \geq 0 \) have been neglected. First, have a look at the consumer participation constraint: Intuitively, this constraint is hurt when the gross potential benefit from reading an ad is small in comparison to the reading cost \( w_L \).\(^{13}\) On the other hand, reading will be profitable if potential gains from trade are large compared to the reading costs. As I have mentioned above, this relative benefit is denoted by \( k \). Now, denote \( \bar{k} \) such that \( q_i(\bar{k}) = 0 \). Using equation (11) and (14) and the definition for \( k \), \( \bar{k} \) is implicitly given by

\[
\frac{w_L}{w^*_I} - \ln \left( 1 + \frac{w_L}{w^*_I} + \frac{w^2_L}{(w^*_I)^2} \right) = 0
\]

\[
1 + \frac{w^*_I}{w_L} + \frac{w_L}{w^*_I} = \bar{k}
\]

(16)

(17)

This equation system can be solved numerically.\(^{14}\) As result, \( \bar{k} = 3.35 \), that is, the potential gains from trade have to be more than three times as high as the reading

\(^{13}\)For example, consider the case where \( p_m - c = w_L \): Even if the product was offered at a low price close to \( c \), the customer will harm from reading one ad, and his utility will be reduced further, when he has to read more than one advertisement.

\(^{14}\)Notice that two equations are sufficient to solve the system, since \( \bar{k} \) depends on the relative
cost of an advertisement in order to generate a positive surplus of advertising. At
this value of \( k \), the portals charge an ad-rate \( w_I^* = 0.56w_L \). At first sight, it looks
astonishing that the gains from trade have to significantly exceed the reading costs
of an ad. However, notice that it cannot be assured that every consumer receives
exactly one ad; actually, due to the distribution of ads by random, several portal
visitors will suffer from reading more than one advertisement, while others do not
receive any advertising information.

Note that the second constraint, namely the zero profit constraint, is not binding:
As internet portals use the electronic communication technology to distribute the
advertisement, unit information cost \( w_H \) is (close to) zero and negligible compared
to the reading cost of an advertisement, i.e. \( w_H << w_L \). As a consequence, por-
tals will earn positive (per consumer) profits \( (w_I - w_H) \ln[(p_m - c)/w_I] \) from the
advertisements they distribute, as long as \( w_I \) is strictly positive. Since both portals
offer the same ad-rates, they share the market for advertising and for portal visiting
equally.

5.3 Welfare analysis

To calculate the net benefit from portal advertising \( W_I \), consider the total amount
of portal advertising \( Z_I(p_m) = \ln(p_m - c) - \ln(w_I) \) which determines the probability
of trade to occur, and the unit cost of advertisement, \( w = w_H + w_L \). Hence the
equilibrium net benefit from portal advertising can be written as:

\[
W_I^* = p_m - c - w_I - (w_H + w_L) \ln[(p_m - c)/w_I]
\]  

(18)

In order to obtain the welfare loss of portal advertising \( \Delta W_I = \hat{W} - W_I \), the
equilibrium value is compared to the efficient solution (see equation 7):

\[
\Delta W_I = (w_H + w_L) \ln(w_H + w_L) - (w_H + w_L) \ln w_I - (w_H + w_L) + w_I
\]  

(19)

In a further step, \( w_H + w_L \) is replaced by \( (w - w_I) + w_I \). It is then straightforward
to estimate the welfare loss by a Taylor-approximation similar to equation (8):

\[
0 \leq \Delta W_I \leq (w - w_I)^2/\hat{w}_I
\]  

(20)
In the last section, I have pointed out that $w_H$ is close to zero and the reading cost $w_L$ is accountable for the bulk of total unit advertisement cost. Keeping this in mind, it is straightforward argumentation that the welfare loss is reduced by portal advertising—to see this, replace $w$ by $w_L$ and compare the Taylor-approximation (20) with equation (8).

5.4 Stilized results

I am now in the position to summarize the results of the portal model and compare them to the results I have received for spam mailing:

(i) When deciding about the ad-rate, internet portals consider the reading costs of their visitors and so internalise them partly. Thus portals set an ad-rate $w_I$ higher than the zero unit information cost $w_H$ and earn positive profits. Note that this result can only be applied to advertising intermediaries that use modern information technology. Else, $w_H$ may well be high enough to be a lower bound for $w_I$ (otherwise the zero profit constraint would be violated).

(ii) Although reading costs are partially considered by the portals, the market equilibrium is not efficient, since we observe too much advertising: As has been shown, $w_I < w_L$. Thus advertising is still too cheap, because the ad-rate is lower than the social cost of an advertisement. Why do portals not internalise the complete reading costs of their customers, although they aim to maximize their objective function? Note that there is no market interaction between the portals and their visitors, that is, the portal carriers are not able to pay the customers for visiting their homepage. Actually, when setting their ad-rate $w_I$, they decide implicitly on both, the total welfare (i.e. the net gains from trade) and the distribution of that gains among the customers and themselves. And a higher ad-rate $w_I$ means that the portals receive a larger part of the welfare gains, while customers lose due to higher product prices.

\footnote{A higher welfare loss due to portal advertising (compared to direct mailing) can be ruled out even for a positive unit information cost $w_H$: As can be seen by direct inspection, $\Delta W_I < \Delta W$ iff $w_I > w_H$. $w_I < w_H$ would hurt the zero profit constraint of the information intermediaries. Note that it makes no difference, whether to use the approximated or the exact values.}
To avoid this, the portals reduce the ad rate $w_I$ beneath the welfare maximizing value $w_L$.

(iii) Still, the result is more efficient than in the case of spam mailing, because spammers do not consider reading costs at all. In case of spam mailing, the welfare loss of reading costs may exceed the potential gains from trade. Internet portals, instead, will only engage in advertising when positive welfare effects are guaranteed.

6 Concluding remarks

I have shown that electronic junk mailing is a rather inefficient way to communicate product information to the customers, while portal advertising provides a more efficient alternative. Given the unique result I have received for “spam”-mailing, the lesson for public authorities should be clear: Junk mailing should be banned, as long as it is not possible to introduce a tax on spam advertising—a tax like this should cover the reading cost of some 3c per unit advertisement.

Authorities may also think about introducing a (lower!) tax on portal advertising. Yet, market power will also prevent internet portals from charging too low ad-rates: Notice that the portals will charge higher rates for their in case of market power, in order to keep a larger part of trade surplus. However, a strong monopoly may even charge ad-rates that exceed social costs. As a consequence, advertising activity may be inefficiently low.
References


