Conservatism and Liquidity Traps*

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Abstract

In an economy with an occasionally binding zero lower bound (ZLB) constraint, the *anticipation* of future ZLB episodes creates a trade-off for discretionary central banks between inflation and output stabilization. As a consequence, inflation systematically falls below target even when the policy rate is above zero. Appointing Rogoff's (1985) conservative central banker mitigates this deflationary bias away from the ZLB and enhances welfare by improving allocations both at and away from the ZLB.

Keywords: Deflationary Bias, Inflation Conservatism, Inflation Targeting, Liquidity Traps, Zero Lower Bound

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1 Introduction

In light of the liquidity trap conditions currently prevailing in many advanced economies, an increasing number of economists and policymakers has called for a re-assessment of central banks' monetary policy frameworks.[1] This paper contributes to this task by examining the desirability of Rogoff's (1985) inflation conservative central banker in an economy with an occasionally binding zero lower bound (ZLB) on nominal interest rates.

Rogoff (1985) considered a model where in the absence of a commitment device monetary stabilization policy suffers from a credibility problem that results in excessive inflation—the so-called inflation bias (Kydland and Prescott, 1977; Barro and Gordon, 1983). He showed that in this environment society can be better off if the central bank is less concerned with output gap stability relative to inflation stability than is society. The credibility problem of discretionary monetary policy at the ZLB is of the opposite nature: Expected inflation is too low and the inability of the central bank to increase inflation expectations further depresses current inflation.[2] We consider the latter credibility problem in a New Keynesian model where monetary policy is delegated to a discretionary central bank which decides each period about the short-term policy rate. To focus on the role of the ZLB, we abstract from the original inflation bias problem. Society's welfare can then be approximated by the negative of the weighted sum of inflation and output volatility.

In this model, the ZLB makes the first-best equilibrium unattainable and can be a huge drag on society's welfare.[3] We find that the appointment of an inflation conservative central banker reduces the welfare costs of discretionary policymaking induced by the ZLB. The mechanism behind our result is as follows. In an economy in which future shocks can push the policy rate to the lower bound, the anticipation of lower inflation and output gives forward-looking households and firms incentives to reduce consumption and prices even when the policy rate is above the ZLB. The central bank cannot fully counteract these incentives. When the central bank is concerned with both inflation and output stabilization, it faces a trade-off between the two objectives, implying deflation and a positive output gap in those states where the ZLB is not binding. Following the terminology of Nakov (2008), we will refer to this deflation when the policy rate is above zero as *deflationary bias*.

A central banker who puts comparatively more weight on inflation stabilization mitigates the deflationary bias away from the ZLB at the cost of a potentially higher output gap. Viewed in isolation, this is welfare-reducing because it shifts inflation and output gap realizations away from the welfare-implied target criteria. However, lower deflation and higher output gaps away from the ZLB also reduce expected real interest rates and increase expected marginal costs at the ZLB, mitigating deflation and output declines there. This in turn implies that a smaller positive output gap is required to stabilize inflation away from the ZLB, setting in motion a positive feedback loop.

We prove analytically the optimality of placing zero weight on output stabilization for our

[1] See, for example, Blanchard, Dell'Ariccia, and Mauro (2010); Tabellini (2014); Williams (2014, 2016).
[2] See Krugman (1998) and Eggertsson (2006).
[3] See Adam and Billi (2007).

baseline sticky-price model where the economy is only subject to a two-state natural real rate shock and we quantify the welfare gains from inflation conservatism numerically in a more elaborate continuous-state model with additional shocks, nominal price and wage rigidities and endogenous inflation inertia. In the quantitative model, the central bank's optimal weight on output stabilization—while strictly smaller than society's weight on output stabilization—is typically slightly above zero when we account for cost-push shocks. The welfare gains from inflation conservatism are non-negligible, and can be quite large, e.g. up to 80%, for reasonable parameterizations.

The desirability of inflation conservatism in the presence of the ZLB is compared with several other institutional configurations that were originally studied in the context of the classic inflation bias problem of discretionary policymaking.[4] Imposing an optimized output or inflation target on the central bank, or assigning a simple linear contract that rewards the central bank for positive inflation rates or positive output gaps also helps to reduce the welfare costs of discretionary policymaking induced by the ZLB. Depending on the model variant, the gains from these alternative institutional arrangements may be larger than those from inflation conservatism. However, an appealing feature of inflation conservatism is its robustness. Conservatism is desirable both in the context of the deflationary bias problem considered here and in the context of the classical inflation bias problem. In contrast, the sign of the optimized target or contract parameter is sensitive to whether the economy suffers from the deflationary bias or the inflation bias problem.

An additional contribution of our paper is to show that the ZLB makes discretionary monetary policy prone to equilibrium multiplicity.[5] In our baseline two-state model, there can be two Markov-Perfect equilibria, and we provide analytical characterizations of the conditions for equilibrium existence.

Our paper is related to a set of papers that examine various ways to improve allocations at the ZLB in time-consistent manners. Eggertsson (2006), Burgert and Schmidt (2014), and Bhattarai, Eggertsson, and Gafarov (2015) consider economies in which the government can choose the level of nominal debt and show that an increase in government bonds during the liquidity trap improves allocations by creating incentives for future governments to inflate. In a model in which government spending is valued by the household, Nakata (2013) and Schmidt (2013) show that a temporary increase in government spending can improve welfare whenever the policy rate is stuck at the ZLB. Schmidt (2016) examines the desirability of fiscal policy delegation regimes in the context of the ZLB. He finds that fiscal authority that cares less about government consumption stability relative to output gap and inflation stability than society mitigates the time-inconsistency problem and increases welfare. A key characteristic of these proposals is that they involve additional policy instruments and require coordination of monetary and fiscal authorities. The approach studied in our paper only requires that the central bank is maximizing its assigned objective.[6]

[4] See, e.g., Persson and Tabellini (1993), Walsh (1995), and Svensson (1997).

[5] See also Armenter (2014) and Nakata (2014). The fact that the ZLB can give rise to equilibrium multiplicity was first shown by Benhabib, Schmitt-Grohe, and Uribe (2001) in the context of simple monetary policy rules. They did not consider discretionary policy.

[6] Some studies examine other time-consistent ways to better stabilize inflation and output in the model with the ZLB constraint without relying on additional policy instruments. See Nakata (2014) for an approach based on

Finally, our work is also related to a set of papers that examine the desirability of Rogoff's conservative central banker in settings other than the original model with inflation bias. Clarida, Gali, and Gertler (1999) showed that the appointment of a conservative central banker is also desirable in a New Keynesian model, in which the presence of persistent cost-push shocks creates a stabilization bias in discretionary monetary policy—that is, an inferior short-run trade-off between inflation and output stabilization compared with the time-inconsistent Ramsey policy. Adam and Billi (2008), Adam and Billi (2014), and Niemann (2011) examined the benefit of conservatism in versions of New Keynesian models augmented with endogenous fiscal policy. However, all of these studies have abstracted from the ZLB constraint.

The remainder of the paper is organized as follows. Section 2 describes the baseline model and the government's optimization problem, and defines the welfare measure. Section 3 presents the main results on inflation conservatism. Section 4 compares inflation conservatism to other institutional configurations that aim to mitigate the credibility problem of discretionary monetary policy. Section 5 provides a quantitative analysis of inflation conservatism based on a bigger continuous-state model calibrated to the U.S. economy. Section 6 concludes.

2 A simple model

This section presents the baseline model, lays down the policy problem of the central bank and defines the equilibrium. The basic structure of this model is also at the heart of the more elaborate model that we use in our quantitative analyses.

2.1 Private sector

The private sector of the economy is given by the standard New Keynesian structure formulated in discrete time with infinite horizon as developed in detail in Woodford (2003) and Gali (2008). Following the majority of the literature on the ZLB, we put all model equations except for the ZLB constraint in semi-loglinear form. This allows us to derive closed-form results.

The equilibrium conditions of the private sector are given by the following two equations:

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1} \tag{1}$$

$$y_t = E_t y_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} - r_t^n \right), \tag{2}$$

where π_t is the inflation rate between periods $t-1$ and t, y_t denotes the output gap, i_t is the level of the nominal interest rate between periods t and $t+1$, and r_t^n is the exogenous natural real rate of interest. Equation (1) is a standard New Keynesian Phillips curve and equation (2) is the consumption Euler equation. The parameters are defined as follows: $\beta \in (0,1)$ denotes the representative household's subjective discount factor, $\sigma > 0$ is the intertemporal elasticity of

reputation, and Billi (2013) and Nakata and Schmidt (2016) for alternative monetary policy delegation schemes.

substitution in consumption, and κ represents the slope of the New Keynesian Phillips curve.[7]

We assume that the natural real rate follows a two-state Markov process. In particular, r_t^n takes the value of either r_H^n or r_L^n where we refer to $r_H^n > 0$ as the high (non-crisis) state and $r_L^n < 0$ as the low (crisis) state. The transition probabilities are given by

$$\text{Prob}(r_{t+1}^n = r_L^n | r_t^n = r_H^n) = p_H \tag{3}$$

$$\text{Prob}(r_{t+1}^n = r_L^n | r_t^n = r_L^n) = p_L. \tag{4}$$

p_H is the probability of moving to the low state in the next period when the economy is in the high state today and will be referred to as the *frequency* of the contractionary shocks. p_L is the probability of staying in the low state when the economy is in the low state today and will be referred to as the *persistence* of the contractionary shocks.

2.2 Society's objective and the central bank's problem

We assume that society's value, or welfare, at time t is given by the expected discounted sum of future utility flows,

$$V_t = u(\pi_t, y_t) + \beta E_t V_{t+1}, \tag{5}$$

where society's contemporaneous utility function, $u(\cdot,\cdot)$, is given by the standard quadratic function of inflation and the output gap,

$$u(\pi, y) = -\frac{1}{2}\left(\pi^2 + \bar{\lambda} y^2\right). \tag{6}$$

This objective function can be motivated by a second-order approximation to the household's preferences. In such a case, $\bar{\lambda}$ is a function of the structural parameters and is given by $\bar{\lambda} = \frac{\kappa}{\theta}$.[8]

Monetary policy is delegated to a central bank. The value for the central bank is given by

$$V_t^{CB} = u^{CB}(\pi_t, y_t) + \beta E_t V_{t+1}^{CB}, \tag{7}$$

where the central bank's contemporaneous utility function, $u^{CB}(\cdot,\cdot)$, is given by

$$u^{CB}(\pi, y) = -\frac{1}{2}\left(\pi^2 + \lambda y^2\right). \tag{8}$$

Note that, while the central bank's objective function resembles the private sector's, the relative weight that it attaches to the stabilization of the output gap, $\lambda \geq 0$, may differ from $\bar{\lambda}$. We assume that the central bank does not have a commitment technology. Each period t, the central bank chooses the inflation rate, the output gap, and the nominal interest rate in order to maximize its

[7] κ is related to the structural parameters of the economy as follows: $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\eta\theta)}\left(\sigma^{-1} + \eta\right)$, where $\alpha \in (0, 1)$ denotes the share of firms that cannot reoptimize their price in a given period, $\eta > 0$ is the inverse of the elasticity of labor supply, and $\theta > 1$ denotes the price elasticity of demand for differentiated goods.

[8] See Woodford (2003).

objective function (7) subject to the behavioral constraints of the private sector (1)-(2) and the ZLB constraint $i_t \geq 0$, with the policy functions at time $t+1$ taken as given.

A *Markov-Perfect equilibrium* is defined as a set of time-invariant value and policy functions $\{V^{CB}(\cdot),\ y(\cdot),\ \pi(\cdot),\ i(\cdot)\}$ that solves the central bank's problem above, together with society's value function $V(\cdot)$, which is consistent with $y(\cdot)$ and $\pi(\cdot)$. As shown in the Appendix, there are two Markov-Perfect equilibria in this economy: One fluctuates around a positive nominal interest rate and zero inflation/output (*the standard Markov-Perfect equilibrium*), and the other fluctuates around a zero nominal interest rate and negative inflation/output (*the deflationary Markov-Perfect equilibrium*). We focus on the standard Markov-Perfect equilibrium in this paper.

The main exercise of the next section will be to examine the effects of λ on society's welfare. We quantify the welfare of an economy by the equivalent perpetual consumption transfer (as a share of its steady state) that would make a household in the hypothetical economy without any fluctuations indifferent to living in the economy,

$$W := (1-\beta)\,\frac{\theta}{\kappa}\left(\sigma^{-1}+\eta\right)E[V], \tag{9}$$

where the mathematical expectation is taken with respect to the unconditional distribution of r_t^n.[9]

3 Results

After providing conditions for the existence of the standard Markov-Perfect equilibrium, this section shows how output and inflation in the two states depend on the central bank's relative weight on output stabilization λ and shows that $\lambda = 0$ is optimal.

Let x_k denote the value of variable x in state k, where $k \in \{H, L\}$. The standard Markov-Perfect equilibrium is then given by a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that satisfies the system of linear equations and inequality constraints described in Appendix B.

Proposition 1: The standard Markov-Perfect equilibrium exists if and only if

$$p_L \leq p_L^*(\Theta_{(-p_L)}),$$
$$p_H \leq p_H^*(\Theta_{(-p_H)}),$$

where i) for any parameter x, $\Theta_{(-x)}$ denotes the set of parameter values excluding x, and ii) the cutoff values $p_L^*(\Theta_{(-p_L)})$ and $p_H^*(\Theta_{(-p_H)})$ are given in Appendix B.

See Appendix B for the proof.[10] For a given parameterization of the model (including the transition probabilities for the two-state shock) the standard Markov-perfect equilibrium exists if and only if

[9] For a derivation of the welfare-equivalent consumption transfer, see Appendix A.

[10] The conditions for the existence of the deflationary Markov-Perfect equilibrium turn out to be identical to those for the existence of the standard Markov-Perfect equilibrium; see Appendix J.

this parameterization satisfies both inequality conditions.[11]

When the conditions for the existence of the equilibrium hold, the signs of the endogenous variables are unambiguously determined.

Proposition 2: For any $\lambda \geq 0$, $\pi_H \leq 0$, $y_H > 0$, $i_H < r^n_H$, $\pi_L < 0$, $y_L < 0$, and $i_L = 0$. With $\lambda = 0$, $\pi_H = 0$.

See Appendix B for the proof. In the low state, the ZLB constraint becomes binding, and output and inflation are below target. In the high state, a positive probability of entering the low state in the next period reduces expected marginal costs of production and leads firms to lower prices in anticipation of future crises events. This raises the expected real rate faced by the representative household and gives it an incentive to postpone consumption. The central bank chooses to lower the nominal interest rate to mitigate these anticipation effects. In equilibrium, inflation and output in the high state are negative and positive, respectively, and the non-crisis policy rate is below the natural real interest rate. These analytical results are consistent with the numerical results in the literature (see Nakov (2008), among others). In particular, negative inflation away from the ZLB has been referred to as *deflationary bias*. This proposition provides the first analytical underpinning for the deflation bias.

Notice that the first part of this proposition ($\pi_H \leq 0$, $y_H > 0$, and $i_H < r^n_H$) can be seen as demonstrating the breakdown of the so-called divine coincidence. If there were no ZLB constraint, then inflation and output gap in both states would be zero. Here, in the model with the ZLB constraint, inflation and the output gap are not fully stabilized even in the high state when the ZLB does not bind. This is because the possibility of future ZLB episodes reduces inflation expectations in the high state, which can be thought of as a negative cost-push shock that shifts down the intercept of the Phillips curve. In this regard, accounting for $p_H > 0$ is essential for the analysis.

We now establish several results on how the degree of conservatism affects endogenous variables in both states. In doing so, we assume that parameter values are such that the conditions for equilibrium hold for a reasonable range of $\lambda > 0$.[12]

Proposition 3: For any $\lambda \geq 0$, $\frac{\partial \pi_H}{\partial \lambda} < 0$, $\frac{\partial \pi_L}{\partial \lambda} < 0$, and $\frac{\partial y_L}{\partial \lambda} < 0$. For any $\lambda \geq 0$, $\frac{\partial y_H}{\partial \lambda} < 0$ if and only if $\beta p_H - (1-\beta)\left(\frac{1-p_L}{\kappa\sigma}(1-\beta p_L + \beta p_H) - p_L\right) < 0$.

See Appendix B for the proof. $\frac{\partial \pi_H}{\partial \lambda} < 0$ means that, as the central bank cares more about inflation, inflation in the high state is higher (i.e., the deflation bias in the high state is smaller). Since a lower rate of deflation in the high state increases output and inflation in the low state via expectations, inflation and output in the low state both increase with the degree of conservatism

[11] Richter and Throckmorton (2014) show numerically for a nonlinear New Keynesian model that the boundary of the region of the parameter space where their solution algorithm converges to a minimum state variable solution imposes a trade-off between the frequency and the persistence of ZLB events.

[12] p^*_H and p^*_L do depend on λ, but for values of λ between 0 and $\bar{\lambda}$ the quantitative effect is negligible.

($\frac{\partial \pi_L}{\partial \lambda} < 0$ and $\frac{\partial y_L}{\partial \lambda} < 0$). The effect of conservatism on output in the high state is ambiguous. On the one hand, a more conservative central bank is willing to tolerate a larger overshooting of output given the same inflation expectations. On the other hand, higher inflation in both states improves the trade-off between inflation and output stabilization implied by the Phillips curve, making it possible to reduce the overshooting of output in the non-crisis state.

Proposition 4: Suppose that p_L and p_H are sufficiently low so that $p_L \leq p_L^*(\Theta_{(-p_L)})$ and $p_H \leq p_H^*(\Theta_{(-p_H)})$ for all λ in $[0, \bar{\lambda}]$. Then, welfare is maximized at $\lambda = 0$.

See Appendix B for the proof. As demonstrated in Proposition 3, deflation in the high state is smaller and inflation and output decline less in the low state with a smaller λ. These forces work to improve society's welfare. If $\beta p_H - (1-\beta)\left(\frac{1-p_L}{\kappa\sigma}(1-\beta p_L + \beta p_H) - p_L\right) > 0$, then output in the high state becomes smaller with a smaller λ and the optimality of zero weight is obvious. If $\beta p_H - (1-\beta)\left(\frac{1-p_L}{\kappa\sigma}(1-\beta p_L + \beta p_H) - p_L\right) < 0$, then a smaller λ increases the already positive output gap and thus has ambiguous effects on welfare. Proposition 4 demonstrates that, even in this case, the beneficial effects of a smaller λ on π_H, π_L, and y_L dominate the adverse effect on y_H. In Appendix D, we provide a numerical illustration of the aforementioned model properties.

4 Other institutional solutions

In this section, we compare the optimal inflation conservative central banker with four other institutional configurations that have been studied extensively in the context of the traditional inflation bias problem of discretionary policy: a linear inflation/output gap contract (IC/OC), and an inflation/output gap target (IT/OT). To do so, we consider a modified version of the central bank's period objective function

$$u^{CB}(\pi_t, y_t) = -\frac{1}{2}\left[(\pi_t - f_{IT})^2 + \bar{\lambda}\,(y_t - f_{OT})^2\right] + f_{IC}\pi_t + f_{OC}y_t, \tag{10}$$

where f_{IT}, f_{OT}, f_{IC} and f_{OC} are parameters, and where the weight on the quadratic output gap term in the central bank's objective function is the same as in society's period utility function. Let $j \in \{IT, OT, IC, OC\}$ denote a monetary policy regime that satisfies $f_j \neq 0$ and $f_k = 0$, $\forall k \in \{IT, OT, IC, OC\}$, $k \neq j$. The next proposition shows that the regimes IT, OT, IC and OC are isomorphic to each other.

Proposition 5: Suppose the policy parameter f_j in regime j equals some value $\hat{f} \neq 0$. Then for each monetary policy regime $m \in \{IT, OT, IC, OC\}$, $m \neq j$, there exists a value for policy parameter f_m such that the equilibrium conditions under regimes j and m are the same.

See Appendix C for the proof. Hence, welfare (9) is the same under the optimized IT, OT, IC and OC regimes, provided that an equilibrium exists. Without loss of generality, we therefore focus on the linear inflation contract for the remainder of this section and set $f_{IT}, f_{OT}, f_{OC} = 0$. In Appendix C, we show that the conditions for equilibrium existence provided in the previous section are sufficient for all $f_{IC} \in [0, \bar{f}_{IC})$, where $\bar{f}_{IC} > -r_L$ is a function of structural parameters. We can then establish the following welfare results.

Proposition 6: (i) There exists a linear inflation contract with $f_{IC} = f_{IC}^0$, where $0 < f_{IC}^0 < \bar{f}_{IC}$, that replicates the discretionary equilibrium under the optimal inflation conservative central banker. (ii) Welfare under the optimal linear inflation contract is strictly larger than welfare under the optimal inflation-conservatism regime, and the optimized contract parameter f_{IC}^* satisfies $f_{IC}^0 < f_{IC}^* < \bar{f}_{IC}$. (iii) The discretionary equilibrium under the optimal linear inflation contract features strictly positive inflation in the high state, $\pi_H > 0$.

See Appendix C for the proof. Proposition 5 and Proposition 6 together imply that also the optimal IT, OT and OC regimes lead to better welfare outcomes than the optimal inflation-conservatism regime. A key feature of the optimal IT, OT, IC and OC regimes is that, unlike under the optimal conservatism regime where inflation in the high state is zero, they stabilize inflation in the high state at a level strictly above zero.[13] Positive inflation in the high state mitigates the decline of output and inflation in the low state via expectations and consequently further improves the stabilization trade-off in the high state.[14]

5 A quantitative model

In this section, we consider a more elaborate continuous-state model that allows us to quantify the welfare effects of inflation conservatism and other institutional configurations in an empirically motivated framework. The model features price and wage rigidities as in Erceg, Henderson, and Levin (2000), and non-reoptimized prices and wages may be partially indexed to past inflation. The economy is buffeted by preference shocks to the household's discount factor and by price mark-up shocks.[15]

[13]See Appendix D for a numerical illustration.

[14]Whether a marginal increase in f_j raises or lowers output in the high state is determined by the same condition that determines whether a marginal increase in λ raises or lowers output in the high state under inflation conservatism.

[15]In Appendix H we consider a model variant with wage mark-up shocks.

5.1 The private sector and the central bank

Aggregate private sector behavior is summarized by the following system of semi-loglinear equations

$$\pi_t - \tau_p \pi_{t-1} = \kappa_p \left(\frac{\gamma}{1-\gamma} y_t + w_t \right) + \beta (E_t \pi_{t+1} - \tau_p \pi_t) + u_t, \tag{11}$$

$$\pi_t^W - \tau_w \pi_{t-1} = \kappa_w \left(\left(\sigma^{-1} + \frac{\eta}{1-\gamma} \right) y_t - w_t \right) + \beta (E_t \pi_{t+1}^W - \tau_w \pi_t), \tag{12}$$

$$\pi_t^W = w_t - w_{t-1} + \pi_t, \tag{13}$$

$$y_t = E_t y_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} - r_t^n \right). \tag{14}$$

Equation (11) captures the price setting behavior of firms, where w_t is the composite real wage rate and u_t is a price mark-up shock. Equation (12) summarizes the nominal wage setting behavior of households, where π_t^W denotes nominal wage inflation between periods $t-1$ and t. Parameters τ_p and τ_w represent the degree of indexation of prices and wages to past inflation. Equation (13) relates nominal wage inflation to the change in the real wage rate and the price inflation rate, and equation (14) is the familiar consumption Euler equation. Parameters satisfy $\kappa_p = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{1-\gamma}{1-\gamma+\gamma\theta}$, and $\kappa_w = \frac{(1-\alpha_W)(1-\beta\alpha_W)}{\alpha_W(1+\eta\theta_W)}$, where $\gamma \in (0,1)$ is the capital elasticity of output, $\alpha_W \in (0,1)$ denotes the share of households that cannot reoptimize their nominal wage in a given period and $\theta_W > 1$ is the wage elasticity of demand for differentiated labor services. The two external disturbances follow stationary autoregressive processes

$$r_t^n = \rho_r r_{t-1}^n + (1-\rho_r)\, r^n + \epsilon_t^r,$$
$$u_t = \rho_u u_{t-1} + \epsilon_t^u,$$

where ϵ_t^x, $x \in \{r, u\}$, are i.i.d. N(0,σ_x^2) innovations, and where the process for the preference shock is written in terms of the natural real rate of interest.

As before, society's welfare at time t is given by the expected discounted sum of future utility flows. In the quantitative model, society's contemporaneous utility function $u(\cdot)$ is given by the following second-order approximation to the household's utility, assuming that deterministic steady state distortions are eliminated by appropriate subsidies[16]

$$u(\pi_t, y_t, \pi_t^W, \pi_{t-1}, \pi_{t-1}^W) = -\frac{1}{2} \left[(\pi_t - \tau_p \pi_{t-1})^2 + \bar{\lambda} y_t^2 + \bar{\lambda}_W (\pi_t^W - \tau_w \pi_{t-1}))^2 \right], \tag{15}$$

where the relative weights are functions of the structural parameters.[17]

As before, the central bank acts under discretion. We consider four monetary policy regimes: the benchmark regime where the central bank has the same objective function as society, inflation conservatism, a linear inflation contract and wage inflation conservatism.[18] A wage inflation con-

[16] See Giannoni and Woodford (2004).

[17] Specifically, $\bar{\lambda} = \kappa_p \left(\sigma^{-1} + \frac{\eta+\gamma}{1-\gamma} \right) \frac{1}{\theta}$ and $\bar{\lambda}_W = \bar{\lambda} \frac{(1-\gamma)\theta_W}{\kappa_w \left(\sigma^{-1} + (\eta+\gamma)/(1-\gamma) \right)}$.

[18] The inflation contract regime is isomorphic to several other regimes, including an output contract, a wage inflation

servative central banker is a natural institutional reform candidate in our quantitative model since the presence of nominal wage rigidities implies that society cares about wage inflation stability. Nesting these four regimes, the central bank's contemporaneous utility function $u(\cdot)^{CB}$ is given by

$$u^{CB}(\pi_t, y_t, \pi_t^W, \pi_{t-1}, \pi_{t-1}^W) = -\frac{1}{2}\left[\lambda_\pi(\pi_t - \tau_p\pi_{t-1})^2 + \lambda y_t^2 + \bar{\lambda}_W(\pi_t^W - \tau_w\pi_{t-1})^2\right] + f\pi_t, \qquad (16)$$

where λ_π, λ and f are policy parameters. For $\lambda_\pi = 1$, $\lambda = \bar{\lambda}$ and $f = 0$, the central bank's objective function collapses to society's objective function. Appendix E describes the optimization problem of a generic discretionary policymaker and lists the first order conditions.

5.2 Calibration and model solution

The model is calibrated to the U.S. economy following the parameterization in Schmidt (2016), and is summarized in Table 1. The period length is one quarter. While the baseline calibration sets the indexation parameters τ_p and τ_w to zero, this assumption is relaxed later on. The implied

Table 1: Baseline calibration of quantitative model

β	0.9938	θ	9	α_W	0.72	τ_w	0	ρ_u	0
σ^{-1}	1.22	θ_W	9	γ	0.3	ρ_r	0.8	σ_u	0.17
η	1.69	α	0.72	τ_p	0	σ_r	0.363		

values for the parameters in society's objective function are $\bar{\lambda} = 0.0103$ and $\bar{\lambda}_W = 2.3361$. We use a projection method with finite elements to solve the model numerically, as described in Appendix F. This method allows for an accurate treatment of expectation terms. The calibrated model matches the observed volatility of inflation, short-term interest rates, and real GDP growth in the United States over the previous two decades quite well. Under the benchmark monetary policy regime where the central bank has the same preferences as society, the unconditional standard deviations of annualized inflation, the annualized short-term nominal interest rate, and annualized quarterly real GDP growth are 0.63, 2.36 and 2.07, respectively. In the data, for the period from 1995-Q1 until 2015-Q4 the standard deviation of annualized quarterly U.S. inflation as measured by the CPI less food and energy is 0.61, the standard deviation of the quarterly short-term nominal interest rate as measured by the effective federal funds rate is 2.37 (annualized), and the standard deviation of the annualized quarterly real GDP growth rate is 2.54.

5.3 Optimal institutional configurations

As before, we quantify the effects of alternative monetary policy regimes on society's welfare by the perpetual consumption transfer (as a share of its steady state) that would make a household in the

contract, an inflation target, an output target and a wage inflation target.

artificial economy without any fluctuations indifferent to living in the economy. In the quantitative model, this welfare-equivalent consumption transfer is given by

$$W := (1-\beta)\,\frac{\theta}{\kappa}\left(\sigma^{-1} + \frac{\eta+\gamma}{1-\gamma}\right) E[V]. \tag{17}$$

Table 2 reports the optimized cental bank objective function parameters, welfare (17), and the frequency of ZLB events for the four alternative regimes.[19]

Table 2: Results for the quantitative model - baseline calibration

Regime	Optimized policy parameter (benchmark)	Welfare (in %)	Welfare gain over benchmark (%)	ZLB frequency (in %)
Benchmark discretion	-	-0.101	0	31
Inflation conservatism	$\lambda = 0.002$ (0.01)	-0.074	27	33
Inflation contract	$f = 0.19$ (0)	-0.062	39	24
Wage inflation conserv.	$\lambda_\pi = 0.35$ (1)	-0.070	31	31

Note: The non-optimized policy parameters have the following values. Benchmark discretion: $\lambda_\pi = 1$, $\lambda = 0.010$, $f = 0$. Inflation conservatism: $\lambda_\pi = 1$, $f = 0$. Inflation contract: $\lambda_\pi = 1$, $\lambda = 0.010$. Wage inflation conservatism: $\lambda = 0.002$, $f = 0$.

Under the benchmark regime, society's welfare loss amounts to a permanent reduction in consumption of 0.1% of its deterministic steady state and the ZLB binds in 31% of the simulated periods. Under optimal inflation conservatism, the weight on output gap stabilization in the central bank's objective function is strictly positive—reflecting the presence of price mark-up shocks—but considerably smaller than the weight that society puts on the output gap term. Society's welfare is 27% higher than under the benchmark regime. These welfare gains from inflation conservatism are closely linked to the ZLB. Without the ZLB, appointing an inflation conservative central banker in this model would not be welfare-improving. The effect of inflation conservatism on the frequency of ZLB events is ambiguous. On the one hand, an inflation conservative central banker responds more elastically to variations in inflation than the benchmark central banker does, thereby increasing the frequency of ZLB events. On the other hand, inflation conservatism improves stabilization outcomes at the ZLB, as discussed in Section 3. This mitigates the deflationary bias and thereby reduces the frequency of ZLB events. For our baseline calibration, the former channel is slightly stronger than the latter, leading to a two percentage points increase in the frequency of ZLB events. The optimal linear inflation contract leads to a slightly higher welfare level than the conservatism regime. This goes hand-in-hand with a 7 percentage points reduction in the frequency of ZLB events compared to the benchmark regime. Finally, to explore the desirability of wage inflation

[19] Results are based on 2000 simulations with a length of 1050 periods each, where the first 50 periods are discarded as burn-in periods. For each regime candidate, we calculate the average of the discounted welfare loss across the simulations. Appendix H presents results from sensitivity analyses. Appendix I compares inflation targeting to price-level targeting.

conservatism, we fix the central bank's relative weight on output gap stabilization at the optimized value and ask whether there are additional gains from assigning a weight on inflation stabilization λ_π that deviates from the weight of unity in society's objective. For the baseline calibration, assigning a lower weight on the price inflation term in the central bank's objective function is associated with some small additional welfare gains. Such a regime can be characterized as *wage inflation conservative* in the sense that it puts a higher weight on wage inflation stability relative to price inflation and output gap stability than society does.[20]

Previous work by Adam and Billi (2007) suggests that the welfare costs of purely discretionary policymaking in the presence of the ZLB can be very large when current inflation is partly driven by past inflation rates. To investigate the effects of this form of endogenous inflation inertia on the optimal institutional configurations and welfare, we next consider a model variant where non-reoptimized prices and wages are partially indexed to past inflation. To economize on the number of state variables, we abstract from the price mark-up shock and drop u_t from equation (11). Table 3 reports the optimized policy delegation parameters, welfare as measured by equation (17), and the frequency of ZLB events for the case where $\tau = \tau_W = 0.2$. Without the ZLB, the discretionary

Table 3: Results for the quantitative model with partial indexation

Regime	Optimized policy parameter (benchmark)	Welfare (in %)	Welfare gain over benchmark (%)	ZLB frequency (in %)
Benchmark discretion	-	-0.096	0	40
Inflation conservatism	$\lambda = 0$ (0.01)	-0.018	81	31
Inflation contract	$f = 0.16$ (0)	-0.018	81	23
Wage inflation conserv.	$\lambda_\pi = 5$ (1)	-0.012	87	31

Note: The non-optimized policy parameters have the following values. Benchmark discretion: $\lambda_\pi = 1$, $\lambda = 0.010$, $f = 0$. Inflation conservatism: $\lambda_\pi = 1$, $f = 0$. Inflation contract: $\lambda_\pi = 1$, $\lambda = 0.010$. Wage inflation conservatism: $\lambda = 0$, $f = 0$.

policymaker would be able to replicate the efficient equilibrium in this model variant where natural real rate shocks are the only source of uncertainty. Hence, in this model welfare losses are directly linked to the presence of the ZLB. In spite of the rather modest degree of price and wage indexation, the costs of discretionary policymaking without delegation are quite high, being tantamount to a permanent reduction in consumption of almost 0.1% of deterministic steady state consumption. Optimal inflation conservatism reduces the welfare costs considerably to about one-fifth of those observed under the benchmark regime. In the absence of mark-up shocks, the optimal relative weight on the output gap term in the central bank's objective function is zero, as in the simple two-state model.[21] Unlike under the baseline calibration, appointing an inflation conservative central banker reduces the frequency of ZLB events. The optimal linear inflation contract performs as

[20] This result is sensitive to the type of mark-up shock that enters the model. In Appendix H, we show that if the price mark-up shock is replaced with a wage mark-up shock, then it becomes optimal to choose a $\lambda_\pi > 1$.

[21] In this model variant, welfare could be increased further by allowing for negative values of λ. For instance, if $\lambda = -0.001$, which is the smallest value of λ for which our computational algorithm converges, then society's welfare

well as inflation conservatism and further reduces the frequency of ZLB events. Finally, unlike in the baseline model with price mark-up shocks, in this model variant welfare under a conservative central bank can be increased further by assigning more, not less, weight on the price inflation stability objective relative to the output and wage inflation stability objectives.[22]

In Appendix G, we show how the choice of the monetary policy regime affects the behavior of the economy in a temporary liquidity trap scenario.

6 Conclusion

We have demonstrated, both analytically and numerically, that an economy that experiences occasional ZLB episodes can improve welfare by appointing a conservative central banker who is more concerned with inflation stabilization relative to output stabilization than society is. In the absence of policy commitment, optimal monetary policy suffers from a deflationary bias. Inflation stays below target even when the policy rate is positive because households and firms anticipate that the ZLB can be binding in the future. Subdued inflation rates away from the ZLB in turn exacerbate the decline in output and inflation when the economy is in a liquidity trap. A conservative central banker counteracts this vicious cycle by mitigating the deflationary bias away from the ZLB, thereby improving stabilization outcomes at and away from the ZLB. The welfare gains from inflation conservatism are particularly large in models where current inflation and nominal wage growth are partly determined by past inflation rates.

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increases to -0.015%. We constrain λ to be nonnegative since we want to focus on institutional configurations for which there exist clear counterparts in practice. Against this background, it seems rather unrealistic to contemplate a central banker who aims for increasing output volatility.

[22]The optimal value for λ_π reported in Table 3 is a corner solution since we put an upper bound of 5 on the grid for λ_π.

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Appendix

A Welfare measure for the simple model of Section 2

We quantify the welfare of an economy by the equivalent perpetual consumption transfer (as a share of its steady state) that would make a household in the hypothetical economy without any fluctuations indifferent to living in the actual economy. The welfare measure can be derived as follows.

A permanent reduction in private consumption C by a share W in the hypothetical economy without fluctuations reduces lifetime utility of the representative agent in this economy by $\sum_{j=0}^{\infty}\beta^j U_C C W = \frac{1}{1-\beta}U_C C W$, where U_C is the marginal utility of consumption evaluated at the deterministic steady state.

The second-order approximation to the unconditional expected lifetime utility in the actual economy equals $U_C C \frac{\theta}{\kappa}(\sigma^{-1}+\eta)E[V]$.[23]

Equalizing the two terms and solving for W, we obtain the following expression for the welfare-equivalent permanent consumption transfer

$$W = (1-\beta)\frac{\theta}{\kappa}(\sigma^{-1}+\eta)E[V].$$

B Proofs related to Section 3

In this section, we will provide details of the proofs for the propositions stated in Section 3 in the main text. Since the proofs are algebraically intensive, we will have to omit some details in this section.

B.1 Proof of Proposition 1

The standard Markov-Perfect equilibrium is given by a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves the following system of linear equations

$$y_H = \big[(1-p_H)y_H + p_H y_L\big] + \sigma\big[(1-p_H)\pi_H + p_H\pi_L - i_H + r^n_H\big], \tag{B.1}$$
$$\pi_H = \kappa y_H + \beta\big[(1-p_H)\pi_H + p_H\pi_L\big], \tag{B.2}$$
$$0 = \lambda y_H + \kappa\pi_H, \tag{B.3}$$
$$y_L = \big[(1-p_L)y_H + p_L y_L\big] + \sigma\big[(1-p_L)\pi_H + p_L\pi_L - i_L + r^n_L\big], \tag{B.4}$$
$$\pi_L = \kappa y_L + \beta\big[(1-p_L)\pi_H + p_L\pi_L\big], \tag{B.5}$$
$$i_L = 0, \tag{B.6}$$

[23] See, for instance, Gali (2008).

and satisfies the following two inequality constraints:

$$i_H > 0, \tag{B.7}$$
$$\phi_L < 0. \tag{B.8}$$

ϕ_L denotes the Lagrangean multiplier on the ZLB constraint in the low state:

$$\phi_L := \lambda y_L + \kappa \pi_L. \tag{B.9}$$

We first prove four preliminary propositions (Propositions 1.A–1.D), then use them to prove the main proposition (Proposition 1) on the necessary and sufficient conditions for the existence of the standard Markov Perfect equilibrium.

Let

$$A(\lambda) := -\beta \lambda p_H, \tag{B.10}$$
$$B(\lambda) := \kappa^2 + \lambda(1 - \beta(1 - p_H)), \tag{B.11}$$
$$C := \frac{(1 - p_L)}{\sigma\kappa}(1 - \beta p_L + \beta p_H) - p_L, \tag{B.12}$$
$$D := -\frac{(1 - p_L)}{\sigma\kappa}(1 - \beta p_L + \beta p_H) - (1 - p_L) = -1 - C, \tag{B.13}$$

and

$$E(\lambda) := A(\lambda)D - B(\lambda)C. \tag{B.14}$$

Assumption 1.A: $E(\lambda) \neq 0$.

Throughout the proof, we will assume that Assumption 1.A holds.

Proposition 1.A: There exists a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves (B.1)–(B.6).

Proof:

Rearranging the system of equations (B.1)–(B.6) and eliminating y_H and y_L, we obtain two unknowns for π_H and π_L in two equations:

$$\begin{aligned} &\begin{bmatrix} A(\lambda) & B(\lambda) \\ C & D \end{bmatrix} \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \begin{bmatrix} 0 \\ r_L^n \end{bmatrix} \\ \Rightarrow &\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \frac{1}{A(\lambda)D - B(\lambda)C} \begin{bmatrix} D & -B(\lambda) \\ -C & A(\lambda) \end{bmatrix} \begin{bmatrix} 0 \\ r_L^n \end{bmatrix}. \end{aligned} \tag{B.15}$$

Thus,

$$\pi_H = \frac{A(\lambda)}{E(\lambda)} r_L^n \tag{B.16}$$

and

$$\pi_L = \frac{-B(\lambda)}{E(\lambda)} r_L^n. \tag{B.17}$$

From the Phillips curves in both states, we obtain

$$y_H = \frac{\beta\kappa p_H}{E(\lambda)} r_L^n \tag{B.18}$$

and

$$y_L = -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{\kappa E(\lambda)} r_L^n. \tag{B.19}$$

Proposition 1.B: Suppose (B.1)–(B.6) are satisfied. Then $\phi_L < 0$ if and only if $E(\lambda) < 0$.

Proof: Notice that

$$\begin{aligned}\phi_L &= -\lambda \frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{\kappa E(\lambda)} r_L^n + \kappa \frac{-B(\lambda)}{E(\lambda)} r_L^n \\ &= -\left[\frac{\lambda}{\kappa}\big[(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda\big] + \kappa B(\lambda)\right] \frac{r_L^n}{E(\lambda)}. \end{aligned} \tag{B.20}$$

Notice also that $r_L^n < 0$, $(1-\beta p_L)\kappa^2 > 0$, $(1-\beta)(1+\beta p_H - \beta p_L)\lambda \geq 0$, and $\kappa B(\lambda) > 0$. Thus, if $\phi_L < 0$, then $E(\lambda) < 0$. Similarly, if $E(\lambda) < 0$, then $\phi_L < 0$.

Corollary 1: $E(\lambda) < 0$ implies $C > 0$ and $D < 0$.

Proof: Substitute equations (B.10), (B.11), and (B.13) into equation (B.14) to obtain

$$E(\lambda) = \beta\lambda p_H - \left(\kappa^2 + \lambda(1-\beta)\right) C. \tag{B.21}$$

Hence, $C > 0$ is a necessary condition for $E(\lambda) < 0$. Finally, from equation (B.13) it becomes clear that $C > 0$ implies $D < 0$.

Proposition 1.C: $E(\lambda) < 0$ if and only if $p_L < p_L^*(\Theta_{-p_L})$.

Proof: It is convenient to view $E(\cdot)$ as a function of p_H and p_L instead of λ for a moment.

$$
\begin{aligned}
E(p_H, p_L) &= \beta\lambda p_H - \Gamma\left[\frac{1-p_L}{\sigma\kappa}(1-\beta p_L+\beta p_H) - p_L\right] \\
&= \beta\lambda p_H - \Gamma\left[\frac{1}{\sigma\kappa}(1-\beta p_L+\beta p_H - p_L + \beta p_L^2 - \beta p_H p_L) - p_L\right] \\
&= -\Gamma\frac{1}{\sigma\kappa}\beta p_L^2 + \Gamma\left[\frac{1}{\sigma\kappa}(1+\beta+\beta p_H)+1\right]p_L + \beta\lambda p_H - \Gamma\frac{1}{\sigma\kappa}(1+\beta p_H) \\
&:= q_2 p_L^2 + q_1 p_L + q_0, \qquad\qquad \text{(B.22)}
\end{aligned}
$$

where $\Gamma := \kappa^2 + \lambda(1-\beta)$ and

$$
\begin{aligned}
q_0 &:= \beta\lambda p_H - \Gamma\frac{1}{\sigma\kappa}(1+\beta p_H), &\text{(B.23)}\\
q_1 &:= \Gamma\left[\frac{1}{\sigma\kappa}(1+\beta+\beta p_H)+1\right] > 0, &\text{(B.24)}\\
q_2 &:= -\Gamma\frac{1}{\sigma\kappa}\beta < 0. &\text{(B.25)}
\end{aligned}
$$

This function, $E(\cdot,\cdot)$, has the following properties.

Property 1: $E(p_H, 1) > 0$ **for any** $0 \leq p_H \leq 1$**.**

Proof:

$$
\begin{aligned}
E(p_H, 1) &= -\Gamma\frac{1}{\sigma\kappa}\beta + \Gamma\left[\frac{1}{\sigma\kappa}(1+\beta+\beta p_H)+1\right] + \beta\lambda p_H - \Gamma\frac{1}{\sigma\kappa}(1+\beta p_H) \\
&= \Gamma + \beta\lambda p_H > 0 \qquad\qquad \text{(B.26)}
\end{aligned}
$$

Property 2: $E(p_H, p_L)$ **is maximized at** $p_L > 1$ **for any** $0 \leq p_H \leq 1$**.**

Proof:

$$
\begin{aligned}
\frac{\partial E(p_H, p_L)}{\partial p_L} &= 2q_2 p_L^* + q_1 = 0 \\
\Leftrightarrow p_L^* &= -\frac{q_1}{2q_2} \\
&= \frac{\Gamma\left[\frac{1}{\sigma\kappa}(1+\beta+\beta p_H)+1\right]}{2\Gamma\frac{1}{\sigma\kappa}\beta} \\
&= \frac{\left[\frac{1}{\sigma\kappa}(2\beta+(1-\beta)+\beta p_H)+1\right]}{2\frac{1}{\sigma\kappa}\beta} > 1. \qquad\qquad \text{(B.27)}
\end{aligned}
$$

These two properties imply i) one root of $E(\cdot, p_L)$ is below 1 and ii) $E(\cdot, p_L) < 0$ below this root. Let's call this root $p_L^*(\Theta_{-p_L})$. $p_L^*(\Theta_{-p_L})$ is given by

$$p_L^*(\Theta_{-p_L}) := \frac{-q_1 + \sqrt{q_1^2 - 4q_2 q_0}}{2q_2}. \tag{B.28}$$

If $E(\lambda) < 0$, then $p_L < p_L^*(\Theta_{-p_L})$. Similarly, if $p_L < p_L^*(\Theta_{-p_L})$, then $E(\lambda) < 0$. This completes the proof of Proposition 1.C. Note that Proposition 1.C holds independently of whether the system of linear equations (B.1)–(B.6) is satisfied or not.

Proposition 1.D: Suppose (B.1)–(B.6) are satisfied and $E(\lambda) < 0$. Then $i_H > 0$ if and only if $p_H < p_H^*(\Theta_{-p_H})$.

Proof:

First, notice that i_H is given by

$$\begin{aligned}
i_H =& r_H^n + \frac{1}{\sigma}\big[-p_H y_H + p_H y_L\big] + \big[(1-p_H)\pi_H + p_H \pi_L\big] \\
=& r_H^n + \frac{1}{\sigma} p_H \frac{-(1-\beta p_L)\kappa - (1-\beta)(1+\beta p_H - \beta p_L)\lambda/\kappa - \beta\kappa p_H}{E(\lambda)} r_L^n \\
&+ (1-p_H)\frac{A(\lambda)}{E(\lambda)} r_L^n + p_H \frac{-B(\lambda)}{E(\lambda)} r_L^n \\
=& -\frac{r_L^n}{E(\lambda)}\frac{\beta\Gamma}{\sigma\kappa} p_H^2 - \frac{r_L^n}{E(\lambda)}\left[\frac{(1-\beta p_L)\Gamma}{\sigma\kappa} + \kappa^2 + \lambda\right] p_H + r_H^n.
\end{aligned} \tag{B.29}$$

Since $E(\lambda) < 0$, $i_H > 0$ requires

$$\begin{aligned}
& r_L^n \frac{\beta\Gamma}{\sigma\kappa} p_H^2 + r_L^n \left(\frac{(1-\beta p_L)\Gamma}{\sigma\kappa} + \kappa^2 + \lambda\right) p_H - r_H^n E(\lambda) > 0 \\
\Leftrightarrow\ & r_L^n \frac{\beta\Gamma}{\sigma\kappa} p_H^2 + \left[r_L^n \left(\frac{(1-\beta p_L)\Gamma}{\sigma\kappa} + \kappa^2 + \lambda\right) - r_H^n \beta\lambda + r_H^n \Gamma\beta \frac{1-p_L}{\sigma\kappa}\right] p_H \\
& + r_H^n \Gamma \left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right) > 0.
\end{aligned} \tag{B.30}$$

Dividing both sides by Γ and by $-r_L^n$, we obtain

$$
\begin{aligned}
&- \frac{\beta}{\sigma\kappa}p_H^2 - \left[\frac{(1-\beta p_L)+(1-p_L)\beta\frac{r_H^n}{r_L^n}}{\sigma\kappa} + \frac{\kappa^2+(1-\beta\frac{r_H^n}{r_L^n})\lambda}{\Gamma}\right]p_H \\
&- \left[\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right]\frac{r_H^n}{r_L^n} > 0.
\end{aligned} \tag{B.31}
$$

Let

$$
P(p_H) := \phi_2 p_H^2 + \phi_1 p_H + \phi_0, \tag{B.32}
$$

where

$$
\phi_0 := -\left[\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right]\frac{r_H^n}{r_L^n}, \tag{B.33}
$$

$$
\phi_1 := -\frac{(1-\beta p_L)+(1-p_L)\beta\frac{r_H^n}{r_L^n}}{\sigma\kappa} - \frac{\kappa^2+(1-\beta\frac{r_H^n}{r_L^n})\lambda}{\Gamma}, \tag{B.34}
$$

$$
\phi_2 := -\frac{\beta}{\sigma\kappa} < 0. \tag{B.35}
$$

Property 1: $\phi_0 > 0$

Proof: Notice that $i_H = r_H^n > 0$ when $p_H = 0$. Since $E(\lambda) < 0$, the sign of i_H is the same as the sign of $\phi_2 p_H^2 + \phi_1 p_H + \phi_0$. Thus, $\phi_0 > 0$. This completes the proof of Property 1.

$\phi_0 > 0$ and $\phi_2 < 0$ imply that one root of (B.32) is non-negative and $i_H > 0$ if and only if p_H is below this non-negative root, given by

$$
p_H^*(\Theta_{-p_H}) := \frac{-\phi_1 - \sqrt{\phi_1^2 - 4\phi_0\phi_2}}{2\phi_2}. \tag{B.36}
$$

This completes the proof of Proposition 1.D.

With these four preliminary propositions (1.A–1.D), we are ready to prove our Proposition 1.

Proposition 1: There exists a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves the system of linear equations (B.1)–(B.6) and satisfies $\phi_L < 0$ and $i_H > 0$ if and only if $p_L < p_L^*(\Theta_{-p_L})$ and $p_H < p_H^*(\Theta_{-p_H})$.

Proof of "if" part: Suppose that $p_L < p_L^*(\Theta_{-p_L})$ and $p_H < p_H^*(\Theta_{-p_H})$. According to Proposition 1.A there exists a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves (B.1)–(B.6). According to Propositions

1.B and 1.C, $E(\lambda) < 0$ and $\phi_L < 0$. According to Proposition 1.D and the fact that $E(\lambda) < 0$, $i_H > 0$. This completes the proof of the "if" part.

Proof of "only if" part: Suppose that $\phi_L < 0$ and $i_H > 0$. According to Proposition 1.A there exists a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves (B.1)–(B.6). According to Propositions 1.B and 1.C, $E(\lambda) < 0$ and $p_L < p_L^*(\Theta_{-p_L})$. According to Proposition 1.D and the fact that $E(\lambda) < 0$, $p_H < p_H^*(\Theta_{-p_H})$. This completes the proof of the "only if" part.

B.2 Proof of Proposition 2

Proposition 2 characterizes the sign of inflation and output in both states. Using i) the restriction on $E(\lambda)$ (i.e. $E(\lambda) < 0$), ii) $r_L^n < 0$, and iii) inequalities on $A(\lambda)$, $B(\lambda)$, C, and D given by equations (B.10)–(B.13), it is straightforward to check that:

$$\pi_H = \frac{A(\lambda)}{E(\lambda)} r_L^n \leq 0, \tag{B.37}$$

$$\pi_L = \frac{-B(\lambda)}{E(\lambda)} r_L^n < 0, \tag{B.38}$$

$$y_H = \frac{\beta\kappa p_H}{E(\lambda)} r_L^n > 0, \tag{B.39}$$

and

$$y_L = -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{\kappa E(\lambda)} r_L^n < 0. \tag{B.40}$$

Inflation and output are negative in the low state. Inflation in the high state is negative, which is what we call deflation bias. Positive output in the high state is consistent with negative inflation in the high state and the optimality condition of the central bank.

B.3 Proof of Proposition 3

Proposition 3 characterizes how λ affects inflation and output in both states. In so doing, we make use of Corollary 1.

$$
\begin{aligned}
\frac{\partial \pi_H}{\partial \lambda} &= \frac{A'(\lambda)E(\lambda) - A(\lambda)E'(\lambda)}{E(\lambda)^2} r_L^n \\
&= \frac{A(\lambda)B'(\lambda) - A'(\lambda)B(\lambda)}{E(\lambda)^2} C r_L^n \\
&= \frac{-\beta p_H \lambda \left(1 - \beta + \beta p_H\right) + \beta p_H \left(\kappa^2 + \left(1 - \beta + \beta p_H\right) \lambda\right)}{E(\lambda)^2} C r_L^n \\
&= \frac{\beta p_H \kappa^2}{E(\lambda)^2} C r_L^n < 0,
\end{aligned} \tag{B.41}
$$

where $A'(\lambda)$ and $B'(\lambda)$ denote the partial derivatives of $A(\cdot)$ and $B(\cdot)$ with respect to λ.

$$
\begin{aligned}
\frac{\partial \pi_L}{\partial \lambda} &= \frac{-B'(\lambda)E(\lambda) + B(\lambda)E'(\lambda)}{E(\lambda)^2} r_L^n \\
&= \frac{A'(\lambda)B(\lambda) - A(\lambda)B'(\lambda)}{E(\lambda)^2} D r_L^n \\
&= -\frac{\beta p_H \kappa^2}{E(\lambda)^2} D r_L^n < 0
\end{aligned} \tag{B.42}
$$

$$
\begin{aligned}
\frac{\partial y_H}{\partial \lambda} &= \frac{-\beta \kappa p_H E'(\lambda)}{E(\lambda)^2} r_L^n \\
&= -\frac{\beta \kappa p_H \left(A'(\lambda)D - B'(\lambda)C\right)}{E(\lambda)^2} r_L^n \\
&= -\frac{\beta \kappa p_H}{E(\lambda)^2} \left[\beta p_H - (1-\beta)C\right] r_L^n \\
&= -\frac{\beta \kappa p_H}{E(\lambda)^2} \left[\beta p_H - (1-\beta)\left(\frac{1-p_L}{\kappa\sigma}\left(1 - \beta p_L + \beta p_H\right) - p_L\right)\right] r_L^n
\end{aligned} \tag{B.43}
$$

$$
\begin{aligned}
\frac{\partial y_L}{\partial \lambda} &= -\frac{(1-\beta)(1-\beta p_L + \beta p_H)E(\lambda) - \left((1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta p_H)\lambda\right) E'(\lambda)}{\kappa E(\lambda)^2} r_L^n \\
&= -\Bigg[\frac{(1-\beta)(1-\beta p_L + \beta p_H)(A(\lambda)D - B(\lambda)C)}{\kappa E(\lambda)^2} \\
&\quad - \frac{\left((1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta p_H)\lambda\right)\left(A'(\lambda)D - B'(\lambda)C\right)}{\kappa E(\lambda)^2}\Bigg] r_L^n \\
&= \frac{\beta \kappa p_H}{E(\lambda)^2} \left[(1-\beta)C + (1-\beta p_L)\right] r_L^n < 0
\end{aligned} \tag{B.44}
$$

B.4 Proof of Proposition 4

Proposition 4 states that welfare is maximized at $\lambda = 0$.

Society's unconditional expected value is given by

$$EV(\lambda) = \frac{1}{1-\beta}\left[\frac{1-p_L}{1-p_L+p_H}u(\pi_H, y_H) + \frac{p_H}{1-p_L+p_H}u(\pi_L, y_L)\right]. \tag{B.45}$$

To show that $E[V(\lambda)]$ is maximized at $\lambda = 0$, we show that

$$\frac{\partial EV(\lambda)}{\partial \lambda} < 0 \tag{B.46}$$

for all $\lambda \geq 0$.

The derivative of the unconditional expected value is given by

$$\frac{\partial EV}{\partial \lambda} = \frac{1}{1-\beta}\left[\frac{1-p_L}{1-p_L+p_H}\frac{\partial u(\pi_H, y_H)}{\partial \lambda} + \frac{p_H}{1-p_L+p_H}\frac{\partial u(\pi_L, y_L)}{\partial \lambda}\right]. \tag{B.47}$$

The partial derivatives of society's utility are given by

$$\begin{aligned}
\frac{\partial u(\pi_H, y_H)}{\partial \lambda} &:= \frac{\partial}{\partial \lambda}\left[-\frac{1}{2}(\bar{\lambda} y_H(\lambda)^2 + \pi_H(\lambda)^2)\right] \\
&= -\left(\bar{\lambda}\frac{\beta\kappa p_H}{E(\lambda)} r_L^n \left(-\frac{\beta\kappa p_H E'(\lambda)}{E(\lambda)^2} r_L^n\right) + \frac{A(\lambda)}{E(\lambda)} r_L^n \frac{A'(\lambda)E(\lambda) - A(\lambda)E'(\lambda)}{E(\lambda)^2} r_L^n\right) \\
&= -\frac{\bar{\lambda}\beta^2\kappa^2 p_H^2 E'(\lambda) + \lambda\beta^2 p_H^2 E(\lambda) - \lambda^2\beta^2 p_H^2 E'(\lambda)}{E(\lambda)^3}(r_L^n)^2 \\
&= \frac{\beta^2\kappa^2 p_H^2}{E(\lambda)^3}\left(\bar{\lambda} E'(\lambda) + \lambda C\right)(r_L^n)^2 \\
&= \frac{\beta^2\kappa^2 p_H^2}{E(\lambda)^3}\left[\bar{\lambda}\left(\beta p_H - (1-\beta)C\right) + \lambda C\right](r_L^n)^2.
\end{aligned} \tag{B.48}$$

Note that we have already shown that the sign of the first term in square brackets, $\beta p_H - (1-\beta)C$, determines the sign of $\frac{\partial y_H}{\partial \lambda}$. If $\beta p_H - (1-\beta)C > 0$, then $\frac{\partial y_H}{\partial \lambda} > 0$ and, applying Corollary 1, we have $\frac{\partial u(\pi_H, y_H)}{\partial \lambda} < 0$. If instead $\beta p_H - (1-\beta)C < 0$, then the sign of $\frac{\partial u(\pi_H, y_H)}{\partial \lambda}$ is ambiguous.

$$\begin{aligned}
\frac{\partial u(\pi_L, y_L)}{\partial \lambda} &:= \frac{\partial}{\partial \lambda}\left[-\frac{1}{2}(\bar{\lambda} y_L(\lambda)^2 + \pi_L(\lambda)^2)\right] \\
&= \frac{\beta p_H}{E(\lambda)^3}\Big(\bar{\lambda}\left[(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda\right]\left[(1-\beta)C + (1-\beta p_L)\right] \\
&\quad + \kappa^2\left[\kappa^2 + \lambda(1-\beta(1-p_H))\right](1+C)\Big)(r_L^n)^2 \\
&:= \frac{\beta p_H}{E(\lambda)^3}\left(\bar{\lambda}\Phi_1(\lambda) + \Phi_2(\lambda)\right)(r_L^n)^2 < 0,
\end{aligned} \tag{B.49}$$

where

$$\Phi_1(\lambda) := \Phi_{1,1} + \Phi_{1,2}\lambda, \tag{B.50}$$
$$\Phi_2(\lambda) := \Phi_{2,1} + \Phi_{2,2}\lambda, \tag{B.51}$$

and

$$\Phi_{1,1} := (1-\beta p_L)\kappa^2\big[(1-\beta)C + (1-\beta p_L)\big] > 0, \tag{B.52}$$
$$\Phi_{1,2} := \big[(1-\beta)C + (1-\beta p_L)\big](1-\beta)(1+\beta p_H - \beta p_L) > 0, \tag{B.53}$$
$$\Phi_{2,1} := \kappa^4(1+C) > 0, \tag{B.54}$$
$$\Phi_{2,2} := \kappa^2(1-\beta(1-p_H))(1+C) > 0. \tag{B.55}$$

Hence,

$$\begin{aligned}\frac{\partial EV}{\partial\lambda} &= \frac{(1-\beta)^{-1}(r_L^n)^2}{1-p_L+p_H}\left((1-p_L)\frac{\beta^2\kappa^2 p_H^2}{E(\lambda)^3}\left[\bar{\lambda}\left(\beta p_H - (1-\beta)C\right) + \lambda C\right] + p_H\frac{\beta p_H}{E(\lambda)^3}\left(\bar{\lambda}\Phi_1(\lambda) + \Phi_2(\lambda)\right)\right) \\ &= \frac{(1-\beta)^{-1}\beta p_H^2(r_L^n)^2}{(1-p_L+p_H)E(\lambda)^3}\left(\beta\kappa^2(1-p_L)\left[\bar{\lambda}\left(\beta p_H - (1-\beta)C\right) + \lambda C\right] + \left(\bar{\lambda}\Phi_1(\lambda) + \Phi_2(\lambda)\right)\right).\end{aligned} \tag{B.56}$$

Let

$$\Omega(\lambda) := \beta\kappa^2(1-p_L)(\beta p_H - (1-\beta)C)\bar{\lambda} + \beta\kappa^2(1-p_L)C\lambda + \bar{\lambda}(\Phi_{1,1} + \Phi_{1,2}\lambda) + \Phi_{2,1} + \Phi_{2,2}\lambda. \tag{B.57}$$

If $\Omega(\lambda) > 0$ for all $\lambda \geq 0$, then $\frac{\partial EV(\lambda)}{\partial\lambda} < 0$ for all $\lambda \geq 0$. Notice that $\Omega'(\lambda)$ is positive since the coefficients on λ are all positive. Thus, we only need to show $\Omega(0) > 0$ to show that $\Omega(\lambda) > 0$ for all $\lambda \geq 0$.

$$\begin{aligned}\Omega(0) &= \beta\kappa^2(1-p_L)\big[\beta p_H - (1-\beta)C\big]\bar{\lambda} + \bar{\lambda}\Phi_{1,1} + \Phi_{2,1} \\ &= \big[\beta\kappa^2(1-p_L)\big[\beta p_H - (1-\beta)C\big] + \Phi_{1,1}\big]\bar{\lambda} + \Phi_{2,1} \\ &= \big[\beta^2\kappa^2(1-p_L)p_H + (1-\beta)^2\kappa^2 C + (1-\beta p_L)^2\kappa^2\big]\bar{\lambda} + \kappa^4(1+C) > 0,\end{aligned} \tag{B.58}$$

given that $C > 0$ for the equilibrium to exist (see Corollary 1) and $\bar{\lambda} > 0$. This completes the proof.

C Proofs related to Section 4

In this section, we will provide details of the proofs for the propositions stated in Section 4 in the main text.

C.1 Proof of Proposition 5

If the central bank's objective function is modified to allow for a constant output gap/inflation target and/or for a constant linear output gap/inflation contract

$$u^{CB}(\pi_t, y_t) = -\frac{1}{2}\left[(\pi_t - f_{IT})^2 + \bar{\lambda}\,(y_t - f_{OT})^2\right] + f_{IC}\pi_t + f_{OC}y_t \tag{C.1}$$

then the system of equilibrium conditions under optimal discretionary policy is given by

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1} \tag{C.2}$$

$$y_t = E_t y_{t+1} - \sigma\,(i_t - E_t\pi_{t+1} - r_t^n) \tag{C.3}$$

$$0 = i_t\left(\bar{\lambda} y_t + \kappa\pi_t - \omega\right) \tag{C.4}$$

$$i_t \geq 0 \tag{C.5}$$

$$0 \geq \bar{\lambda} y_t + \kappa\pi_t - \omega, \tag{C.6}$$

where $\omega := \kappa f_{IT} + \bar{\lambda} f_{OT} + \kappa f_{IC} + f_{OC}$. Let ω be equal to some $\bar{\omega} \geq 0$. It is straightforward to verify that the four regimes $(f_{IT} = \bar{\omega}/\kappa; f_{OT}, f_{IC}, f_{OC} = 0)$, $(f_{OT} = \bar{\omega}/\bar{\lambda}; f_{IT}, f_{IC}, f_{OC} = 0)$, $(f_{IC} = \bar{\omega}/\kappa; f_{IT}, f_{OT}, f_{OC} = 0)$ and $(f_{OC} = \bar{\omega}; f_{IT}, f_{OT}, f_{IC} = 0)$ are isomorphic to each other.

C.2 Existence of standard Markov-Perfect equilibrium under a constant linear inflation contract

If the monetary authority's mandate is modified to include a constant linear inflation contract, the central bank's period objective function is given by

$$u^{CB}(\pi_t, y_t) = -\frac{1}{2}\left(\pi_t^2 + \bar{\lambda} y_t^2\right) + f_{IC}\pi_t \tag{C.7}$$

where $f_{IC} \geq 0$ is a parameter characterizing the linear inflation contract.

The standard Markov-Perfect equilibrium is given by a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves the following system of linear equations

$$y_H = \big[(1-p_H)y_H + p_H y_L\big] + \sigma\big[(1-p_H)\pi_H + p_H\pi_L - i_H + r_H^n\big], \tag{C.8}$$

$$\pi_H = \kappa y_H + \beta\big[(1-p_H)\pi_H + p_H\pi_L\big], \tag{C.9}$$

$$0 = \bar{\lambda} y_H + \kappa\,(\pi_H - f_{IC})\,, \tag{C.10}$$

$$y_L = \big[(1-p_L)y_H + p_L y_L\big] + \sigma\big[(1-p_L)\pi_H + p_L\pi_L - i_L + r_L^n\big], \tag{C.11}$$

$$\pi_L = \kappa y_L + \beta\big[(1-p_L)\pi_H + p_L\pi_L\big], \tag{C.12}$$

$$i_L = 0, \tag{C.13}$$

and satisfies the following two inequality constraints:

$$i_H > 0, \tag{C.14}$$

$$\phi_L < 0. \tag{C.15}$$

ϕ_L denotes the Lagrangean multiplier on the ZLB constraint in the low state:

$$\phi_L := \bar{\lambda} y_L + \kappa \left(\pi_L - f_{IC}\right). \tag{C.16}$$

Proposition: There exists a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves the system of linear equations (C.8)–(C.13) and satisfies $\phi_L < 0$ and $i_H > 0$ for any $f_{IC} \in [0, \bar{f}_{IC})$ if $p_L < p_L^*(\Theta_{-p_L})$ and $p_H < p_H^*(\Theta_{-p_H})$, where $\bar{f}_{IC} = -\left(1 + (1-\beta)\frac{\bar{\lambda}}{\kappa^2}\right) r_L^n > -r_L^n$.

This proposition states that the equilibrium existence conditions provided for the case $f_{IC} = 0$ are sufficient for any $f_{IC} \in [0, \bar{f}_{IC})$.
Proof:

Rearranging the system of equations (C.8)–(C.13) and eliminating y_H and y_L, we obtain two unknowns for π_H and π_L in two equations:

$$\begin{aligned} &\begin{bmatrix} A(\bar{\lambda}) & B(\bar{\lambda}) \\ C & D \end{bmatrix} \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \begin{bmatrix} \kappa^2 f_{IC} \\ r_L^n \end{bmatrix} \\ \Rightarrow &\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \frac{1}{E(\bar{\lambda})} \begin{bmatrix} D & -B(\bar{\lambda}) \\ -C & A(\bar{\lambda}) \end{bmatrix} \begin{bmatrix} \kappa^2 f_{IC} \\ r_L^n \end{bmatrix}, \end{aligned} \tag{C.17}$$

where $A(\cdot), B(\cdot), C, D$ and $E(\cdot)$ are defined in Equations (B.10) - (B.14).

Thus,

$$\pi_H = -\frac{C}{E(\bar{\lambda})}\kappa^2 f_{IC} + \frac{A(\bar{\lambda})}{E(\bar{\lambda})} r_L^n \tag{C.18}$$

and

$$\pi_L = \frac{D}{E(\bar{\lambda})}\kappa^2 f_{IC} - \frac{B(\bar{\lambda})}{E(\bar{\lambda})} r_L^n. \tag{C.19}$$

From the Phillips curves in both states, we obtain

$$y_H = \frac{\beta p_H - (1-\beta)C}{E(\bar{\lambda})}\kappa f_{IC} + \frac{\beta\kappa p_H}{E(\bar{\lambda})} r_L^n \tag{C.20}$$

and

$$y_L = \frac{\beta p_L - 1 - (1-\beta)C}{E(\bar{\lambda})}\kappa f_{IC} - \frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\bar{\lambda}}{\kappa E(\bar{\lambda})} r_L^n. \tag{C.21}$$

Suppose (C.8)–(C.13) hold. Consider ϕ_L :

$$\phi_L = -\frac{\kappa^2 + (1-\beta p_L + \beta p_H)\bar{\lambda}}{E(\bar{\lambda})} f_{IC} - \frac{\left(\kappa^2 + (1-\beta p_L + \beta p_H)\bar{\lambda}\right)\left(\kappa^2 + (1-\beta)\bar{\lambda}\right)}{\kappa^2 E(\bar{\lambda})} r_L^n \tag{C.22}$$

Notice that $r_L^n < 0$ and, from Proposition 1, $E(\bar{\lambda}) < 0$. Thus, $\phi_L < 0$ if and only if $f_{IC} < \bar{f}_{IC}$. Finally, notice that from Proposition 1.C $E(\bar{\lambda}) < 0$ if and only if $p_L^* < (\Theta_{-p_L})$.

Next, consider i_H. We know from Proposition 1.D that given $f_{IC} = 0$ and $E(\bar{\lambda}) < 0$, $i_H > 0$ if and only if $p_H < p_H^*(\Theta_{-p_H})$.

$$i_H = r_H^n - \frac{C + p_H}{E(\bar{\lambda})}\kappa^2 f_{IC} - \frac{\kappa^2 + \bar{\lambda}}{E(\bar{\lambda})} r_L^n - \frac{p_H}{\sigma}\frac{1-\beta p_L + \beta p_H}{E(\bar{\lambda})}\kappa f_{IC} - \frac{p_H}{\sigma}\frac{(\kappa^2 + (1-\beta)\bar{\lambda})(1-\beta p_L + \beta p_H)}{\kappa E(\bar{\lambda})} r_L^n \tag{C.23}$$

Notice that

$$\frac{\partial i_H}{\partial f_{IC}} = -\left(\frac{C + p_H}{E(\bar{\lambda})}\kappa^2 + \frac{p_H}{\sigma}\frac{1-\beta p_L + \beta p_H}{E(\bar{\lambda})}\kappa\right) > 0. \tag{C.24}$$

Hence, $i_H > 0$ if $p_H < p_H^*(\Theta_{-p_H})$ for all $f_{IC} \in [0, \bar{f}_{IC})$.

C.3 Proof of Proposition 6

(i) There exists a linear inflation contract with $f_{IC} = f_{IC}^0$, where $0 < f_{IC}^0 < \bar{f}_{IC}$, that replicates the discretionary equilibrium under the optimal inflation conservative central banker.

Proof:

Let $f_{IC}^0 =: \{f_{IC} | \pi_H = 0\}$. Using Equation (C.18), one obtains $f_{IC}^0 = -\frac{\beta\bar{\lambda}p_H}{C\kappa^2} r_L^n > 0$.

Substituting f_{IC}^0 for f_{IC} into (C.19), (C.20), (C.21), we obtain $\pi_L = \frac{1}{C} r_L^n$, $y_H = -\frac{\beta p_H}{\kappa C} r_L^n$, and $y_L = \frac{1-\beta p_L}{\kappa C} r_L^n$. It is then straightforward to verify that these policy functions are identical to those under the optimal inflation conservative central banker.

Finally, in (ii) we show that $f_{IC}^0 < f_{IC}^* < \bar{f}_{IC}$ and therefore $f_{IC}^0 < \bar{f}_{IC}$.

(ii) Welfare under the optimal linear inflation contract is strictly larger than welfare under the optimal inflation-conservatism regime, and the optimized contract parameter f_{IC}^* satisfies $f_{IC}^0 < f_{IC}^* < \bar{f}_{IC}$.

Proof:

We first show that the welfare measure $E(V)$ improves for a marginal increase in f_{IC} conditional on $f_{IC} = f_{IC}^0$:

$$\frac{\partial E(V)}{\partial f_{IC}}|(f_{IC}=f^0_{IC})$$
$$= \frac{(1-\beta)^{-1}}{1-p_L+p_H}\left(\left(\kappa^2+\bar{\lambda}(1-\beta)^2\right)p_H C + \left(\kappa^2+\beta^2(1-p_L)p_H\bar{\lambda}+(1-\beta p_L)^2\bar{\lambda}\right)p_H\right)\frac{r^n_L}{CE(\bar{\lambda})} > 0. \tag{C.25}$$

The optimal linear inflation contract satisfies

$$\frac{\partial E(V)}{\partial f_I C} = \frac{1}{1-\beta}\left[\frac{1-p_L}{1-p_L+p_H}\frac{\partial u^{CB}_H}{\partial f_{IC}} + \frac{p_H}{1-p_L+p_H}\frac{\partial u^{CB}_L}{\partial f_{IC}}\right] = 0. \tag{C.26}$$

Solving for the optimal contract parameter f^*_{IC}

$$f^*_{IC} = -\frac{f^*_{num}}{f^*_{den}} r^n_L \tag{C.27}$$

where

$$\begin{aligned} f^*_{num} =& \frac{\left(\kappa^2+(1-\beta)^2\bar{\lambda}\right)\left(\kappa^2+(1-\beta p_L+\beta p_H)\bar{\lambda}\right)}{\kappa^2} p_H C \\ &+ \frac{\left(\kappa^2+(1-\beta)\bar{\lambda}\right)\left(\kappa^2+(1-\beta p_L+\beta p_H)(1-\beta p_L)\bar{\lambda}\right)}{\kappa^2} p_H + \beta^2 p^2_H \bar{\lambda}, \end{aligned} \tag{C.28}$$

$$\begin{aligned} f^*_{den} =& \left(\kappa^2+(1-\beta)^2\bar{\lambda}\right)\left(2p_H+(1-p_L+p_H)C\right)C + \left(\kappa^2+(1-\beta p_L+\beta p_H)(1-\beta p_L)\bar{\lambda}\right)p_H \\ &-(1-\beta)\beta p^2_H\bar{\lambda}. \end{aligned} \tag{C.29}$$

In order to show that $f^*_{IC} < \bar{f}_{IC}$, note that

$$1 > f^*_{IC}/\bar{f}_{IC} \Leftrightarrow \left(\kappa^2+(1-\beta)\bar{\lambda}\right)f^*_{den} - \kappa^2 f^*_{num} > 0.$$

After some algebra:

$$\left(\kappa^2+(1-\beta)\bar{\lambda}\right)f^*_{den} - \kappa^2 f^*_{num} = -\left(\kappa^2+(1-\beta)^2\bar{\lambda}\right)\left(p_H+(1-p_L+p_H)C\right)E(\bar{\lambda}) > 0. \tag{C.30}$$

(iii) The discretionary equilibrium under the optimal linear inflation contract features strictly positive inflation in the high state, $\pi_H > 0$.

Proof:

Note that

$$\frac{\partial \pi_H}{\partial f_{IC}} = -\frac{C}{E(\bar{\lambda})}\kappa^2 > 0 \tag{C.31}$$

for all f_{IC}. Since $\pi_H = 0$ for $f_{IC} = f^0_{IC}$, $\pi_H > 0$ for any $f_{IC} > f^0_{IC}$. This completes the proof.

C.4 Comparative statics

We have already shown that $\frac{\partial \pi_H}{\partial f_{IC}} > 0$. Furthermore,

$$\frac{\partial \pi_L}{\partial f_{IC}} = -\frac{C+1}{E(\bar{\lambda})}\kappa^2 > 0, \tag{C.32}$$

$$\frac{\partial y_H}{\partial f_{IC}} = \frac{\beta p_H - (1-\beta)C}{E(\bar{\lambda})}\kappa, \tag{C.33}$$

$$\frac{\partial y_L}{\partial f_{IC}} = \frac{\beta p_L - 1 - (1-\beta)C}{E(\bar{\lambda})}\kappa > 0, \tag{C.34}$$

where $\frac{\partial y_H}{\partial f_{IC}} > 0$ if and only if $\beta p_H - (1-\beta)C < 0$. This is the same condition that is necessary and sufficient to ensure that under inflation conservatism a marginal reduction in the central bank's relative weight on output stabilization λ raises output in the high state.

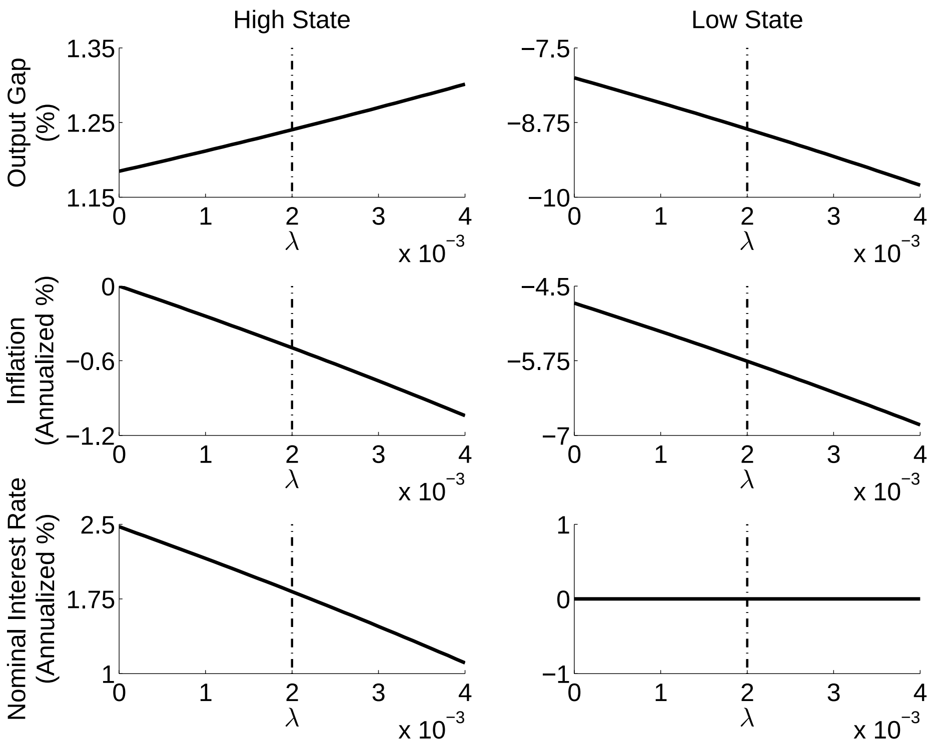
D Numerical illustration for the two-state model

Table 4: Parameterization (Two-state shock model)

Parameter	Value	Economic interpretation
β	0.99	Subjective discount factor
σ	0.5	Intertemporal elasticity of substitution in consumption
η	0.47	Inverse labor supply elasticity
θ	10	Price elasticity of demand
α	0.8106	Share of firms per period keeping prices unchanged
$r^n_H \times 400$	4.04	Natural real rate in the high state
$r^n_L \times 400$	-5.00	Natural real rate in the low state
p_H	0.02	Frequency of contractionary demand shock
p_L	0.875	Persistence of contractionary demand shock

This section provides a numerical illustration of the analytically derived properties of the two state model using specific parameter values. The exercise is not meant to assess the quantitative relevance of the analytical results established in sections 3 and 4 of the main manuscript but merely aims at visualizing them. The structural parameters are calibrated using the parameter values from Eggertsson and Woodford (2003), as listed in Table 4. The frequency of the crisis shock is chosen so that the ZLB episode occurs once every 12 years, on average. In the majority of the papers who have adopted this framework, p_H is assumed to be zero. The persistence of 0.875 means that the expected duration of the crisis is two years.

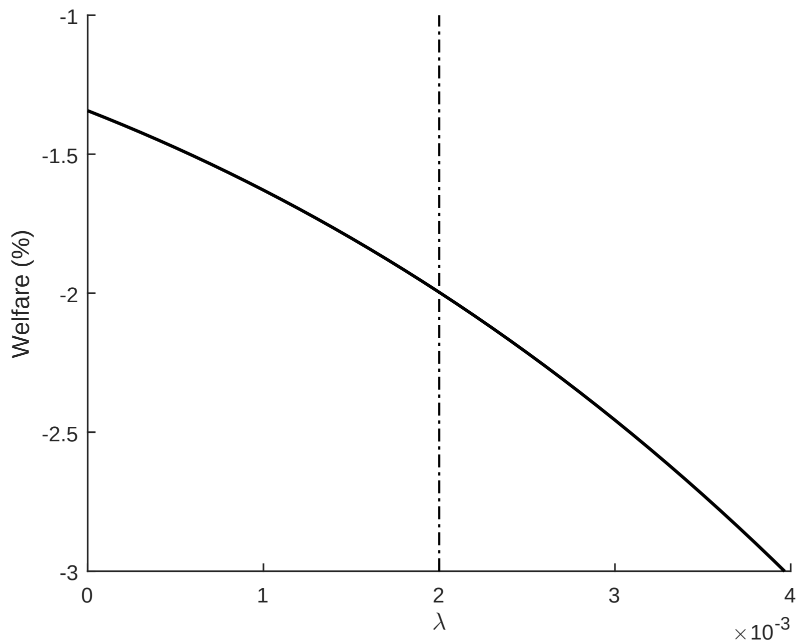
Figure 1: Inflation conservatism - Output gap, inflation, and nominal interest rate



Note: The figure displays how the output gap, the inflation rate, and the nominal interest rate in both states vary with λ. The dash-dotted vertical lines indicate society's weight, $\bar{\lambda}$.

Figure 1 shows how the output gap, inflation, and the nominal interest rate in both states vary with the weight on output stabilization λ when the inflation contract parameter f_{IC} is set equal to zero. The dash-dotted vertical lines show society's weight, $\bar{\lambda}$. Consistent with Proposition 2, output and inflation in the high state are positive and negative, respectively, for any λ. The nominal interest rate is below the natural rate of interest, which is 4 percent. In the low state, output and inflation are negative, and the nominal interest rate is zero. Consistent with Proposition 3, as λ decreases (i.e., as the central bank becomes more conservative), the deflation bias in the high state is reduced. This comes at the cost of a higher positive output gap in the high state, but a smaller deflation bias in the high state mitigates the decline in inflation and output in the low state.

Figure 2: Welfare effects of inflation conservatism

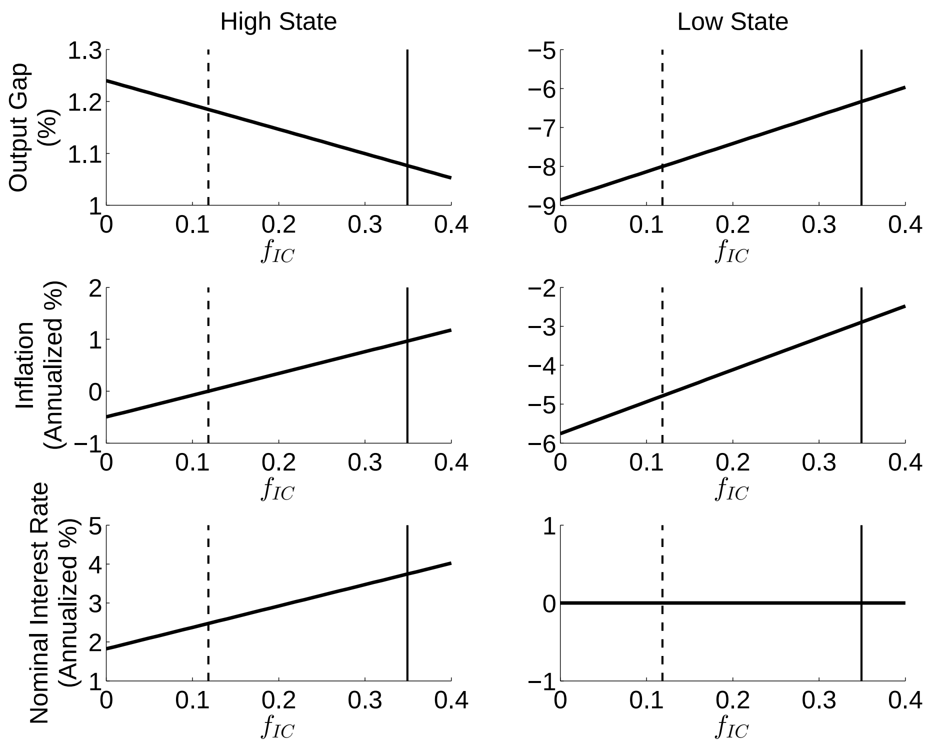


Note: The figure displays how welfare varies with λ in the two-state shock model. The dash-dotted vertical lines indicate society's weight, $\bar{\lambda}$.

The benefits of the smaller rate of deflation in the high state and larger output and inflation in the low state dominate the negative effect of a larger output gap in the high state. Accordingly, welfare increases with the degree of conservatism, as shown in Figure 2. Consistent with Proposition 4, the optimal weight is zero.

Next, we apply the numerical example to the linear inflation contract. Figure 3 shows how the output gap, inflation, and the nominal interest rate in both states vary with the contract parameter f_{IC} when the central bank's weight on output stabilization λ is set equal to society's weight on output stabilization $\bar{\lambda}$. The dashed vertical lines indicate f_{IC}^0, the value of the inflation contract parameter for which the contract replicates the optimal conservative central banker. The solid vertical lines indicate f_{IC}^*, the optimal value for the contract parameter. Consistent with

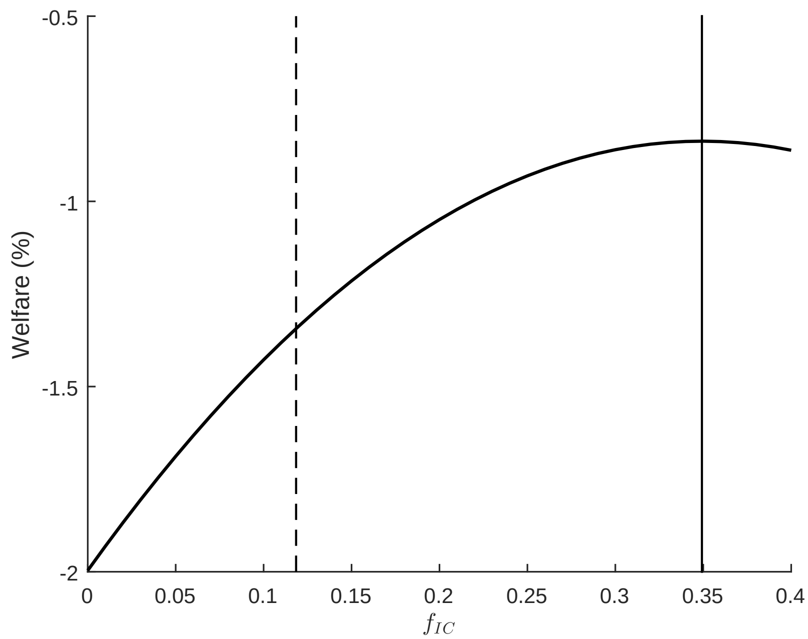
Figure 3: Inflation contract - Output gap, inflation, and nominal interest rate



Note: The figure displays how the output gap, the inflation rate, and the nominal interest rate in both states vary with f_{IC}. The dashed vertical lines indicate the inflation contract parameter f_{IC}^{0} for which the contract replicates the the conservatism regime with $\lambda = 0$. The solid vertical lines indicate the inflation contract parameter f_{IC}^{*} for which society's welfare is maximized.

Proposition 6, inflation in the high state is strictly positive under the optimal linear inflation contract. Consistent with Appendix B, output and inflation in both states are strictly increasing in f_{IC}.[24] Finally, Figure 4 illustrates how society's welfare varies with f_{IC}. The dashed vertical line earmarks the contract parameter value f_{IC}^0 for which the contract replicates the the conservatism regime with $\lambda = 0$, and the solid vertical line earmarks the contract parameter value f_{IC}^* for which society's welfare is maximized. Consistent with Proposition 6, $f_{IC}^* > f_{IC}^0$.

Figure 4: Welfare effects of the inflation contract



Note: The figure displays how welfare varies with f_{IC} in the two-state shock model. The dashed vertical line indicates the inflation contract parameter f_{IC}^0 for which the contract replicates the the conservatism regime with $\lambda = 0$. The solid vertical line indicates the inflation contract parameter f_{IC}^* for which society's welfare is maximized.

E Policy problem in the quantitative model

Each period t, the generic discretionary central banker maximizes his objective function from period t onwards, taking the decision rules of the private sector and of future central bankers as given. We consider stationary Markov-perfect equilibria. In the baseline variant of the model, the vector of state variables s_t consists of the natural real rate shock r_t^n, the price mark-up shock u_t, the composite real wage rate of the previous period w_{t-1} and the inflation rate in the previous period π_{t-1}. The policy problem reads

[24] Note that the parameter calibration satisfies the necessary and sufficient condition which implies that output in the high state is strictly increasing in f_{IC}, $\beta p_H - (1-\beta)\left(\frac{1-p_L}{\kappa\sigma}(1-\beta p_L + \beta p_H) - p_L\right) < 0$.

$$
\begin{aligned}
V^{CB}(s_t) = \max & -\frac{1}{2}\left[\lambda_\pi(\pi_t-\tau_p\pi_{t-1})^2+\lambda y_t^2+\bar{\lambda}_W(\pi_t^W-\tau_w\pi_{t-1})^2\right]+f\pi_t+\beta E_t V^{CB}(s_{t+1}) \\
& + \phi_t^{PCP}\left[\pi_t-\tau_p\pi_{t-1}-\kappa_p\left(\frac{\gamma}{1-\gamma}y_t+w_t\right)-\beta(E_t\pi(s_{t+1})-\tau_p\pi_t)-u_t\right] \\
& + \phi_t^{PCW}\left[\pi_t^W-\tau_w\pi_{t-1}-\kappa_w\left(\left(\sigma^{-1}+\frac{\eta}{1-\gamma}\right)y_t-w_t\right)-\beta(E_t\pi^W(s_{t+1})-\tau_w\pi_t)\right] \\
& + \phi_t^w\left[\pi_t^W-(w_t-w_{t-1})-\pi_t\right] \\
& + \phi_t^{IS}\left[y_t-E_t y(s_{t+1})+\sigma\left(i_t-E_t\pi(s_{t+1})-r_t^n\right)\right] \\
& + \phi_t^{ZLB} i_t,
\end{aligned}
$$

taking into account the laws of motion of the exogenous shocks. The functions $V^{CB}(s_{t+1})$, $\pi(s_{t+1})$, $\pi^W(s_{t+1})$ and $y(s_{t+1})$ are the central banker's continuation value, the rate of price inflation, the rate of wage inflation and the output gap that the central banker expects to be realized in period $t+1$ in equilibrium, contingent on the realizations of the exogenous shocks in period $t+1$.

The consolidated first-order conditions are

$$
\begin{aligned}
& \pi_t-\tau_p\pi_{t-1}-f-\tau_p\beta(E_t\pi(s_{t+1})-\tau_p\pi_t)+\tau_p\beta E_t\phi^{PCP}(s_{t+1})-\tau_w\beta E_t\phi^w(s_{t+1}) \\
& -\left(1+\tau_p\beta-\beta\frac{\partial E_t\pi(s_{t+1})}{\partial\pi_t}\right)\phi_t^{PCP}+\left(\frac{\partial E_t y(s_{t+1})}{\partial\pi_t}+\sigma\frac{\partial E_t\pi(s_{t+1})}{\partial\pi_t}\right)\phi_t^{IS} \\
& -\left(\tau_w\beta-\beta\frac{\partial E_t\pi^W(s_{t+1})}{\partial\pi_t}\right)\left(\bar{\lambda}_W(\pi_t^W-\tau_w\pi_{t-1}^W)-\phi_t^w\right)+\phi_t^w=0, && \text{(E.1)} \\
& \beta E_t\phi^w(s_{t+1})-\left(\kappa_p+\beta\frac{\partial E_t\pi(s_{t+1})}{\partial w_t}\right)\phi_t^{PCP}-\left(\frac{\partial E_t y(s_{t+1})}{\partial w_t}+\sigma\frac{\partial E_t\pi(s_{t+1})}{\partial w_t}\right)\phi_t^{IS} \\
& +\left(\kappa_w-\beta\frac{\partial E_t\pi^W(s_{t+1})}{\partial w_t}\right)\left(\bar{\lambda}_W(\pi_t^W-\tau_w\pi_{t-1}^W)-\phi_t^w\right)-\phi_t^w=0, && \text{(E.2)} \\
& \lambda y_t+\kappa_p\frac{\gamma}{1-\gamma}\phi_t^{PCP}+\kappa_w\left(\sigma^{-1}+\frac{\eta}{1-\gamma}\right)\left(\bar{\lambda}_W(\pi_t^W-\tau_w\pi_{t-1}^W)-\phi_t^w\right)=\phi_t^{IS}, && \text{(E.3)} \\
& \phi_t^{IS} i_t=0, && \text{(E.4)} \\
& i_t\geq 0, && \text{(E.5)} \\
& \phi_t^{IS}\leq 0, && \text{(E.6)}
\end{aligned}
$$

and the behavioral constraints of the private sector.

F Computational algorithm to solve the quantitative model

We approximate the policy functions with a finite elements method using collocation. For the basis functions we use cubic splines. The algorithm proceeds in the following steps:

1. Construct the collocation nodes. The nodes are chosen such that they coincide with the

spline breakpoints. Use a Gaussian quadrature scheme to discretize the normally distributed innovations to the exogenous shocks.

2. Start with a guess for the basis coefficients.

3. Use the current guess for the basis coefficients to approximate the partial derivatives of the expectation terms with respect to the endogenous state variables evaluated at the collocation nodes.

4. Approximate the expectation functions for price inflation, wage inflation, output, and the Lagrange multipliers on the New Keynesian price Phillips curve and the equation relating price and wage inflation, using the same quadrature scheme as in step 3.

5. Solve the system of equilibrium conditions for price inflation, wage inflation, output, the real wage rate, the policy rate and the two Lagrange multipliers at the collocation nodes, assuming that the zero lower bound is not binding. For those nodes where the zero bound constraint is violated solve the system of equilibrium conditions associated with a binding zero bound constraint.

6. Update the guess for the basis coefficients. If the new guess is sufficiently close to the old one, proceed with step 7. Otherwise, go back to step 4.

7. Check whether the new set of partial derivatives based on the updated basis coefficients is sufficiently close to the previous ones. If this is the case, you are done. Otherwise, go back to step 3.

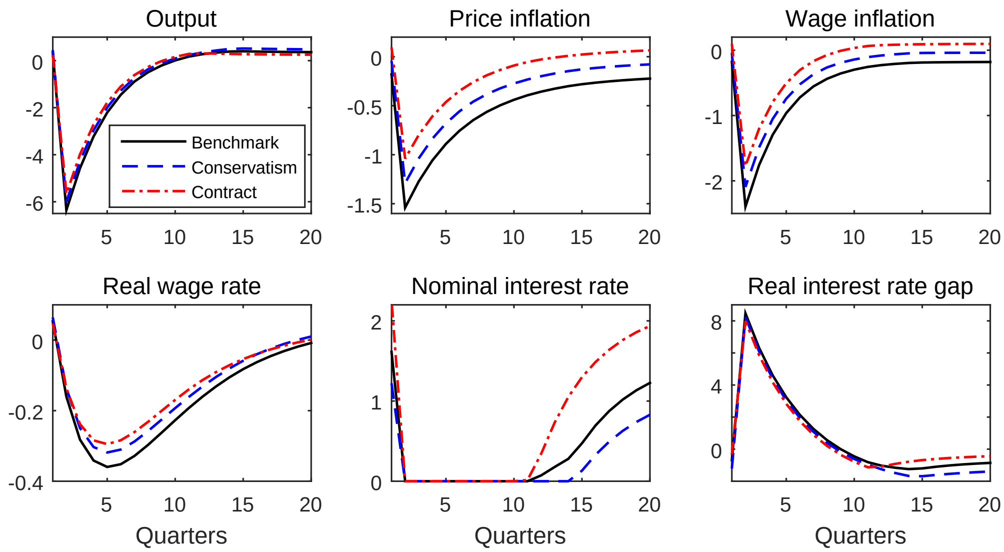
We use MATLAB routines from the CompEcon toolbox of Miranda and Fackler (2002) to obtain the Gaussian quadrature approximations of the innovations and to evaluate the spline functions.

For a more detailed exposition of the computational algorithm see Schmidt (2016), Appendix D.

G A liquidity trap scenario in the quantitative model

This section shows how the choice of the monetary policy regime affects the behavior of the economy in a liquidity trap situation. Figure 5 depicts the equilibrium dynamics in the baseline version of the quantitative model when the economy is buffeted by a large negative natural real rate shock that temporarily drives it away from the risky steady state. We consider the benchmark discretionary regime (solid black lines), the optimal inflation conservatism regime (blue dashed lines) and the optimal inflation contract regime (red dash-dotted lines). Under all regimes the central bank lowers the policy rate to zero where it stays for several quarters. At the same time, there is a pronounced increase in the gap between the real interest rate and the natural real rate. This gap acts as a drag on aggregate demand and pushes down output, price inflation and wage inflation. The decline in output and inflation is most severe under the benchmark discretionary regime. Inflation

Figure 5: Liquidity trap scenario in the baseline quantitative model

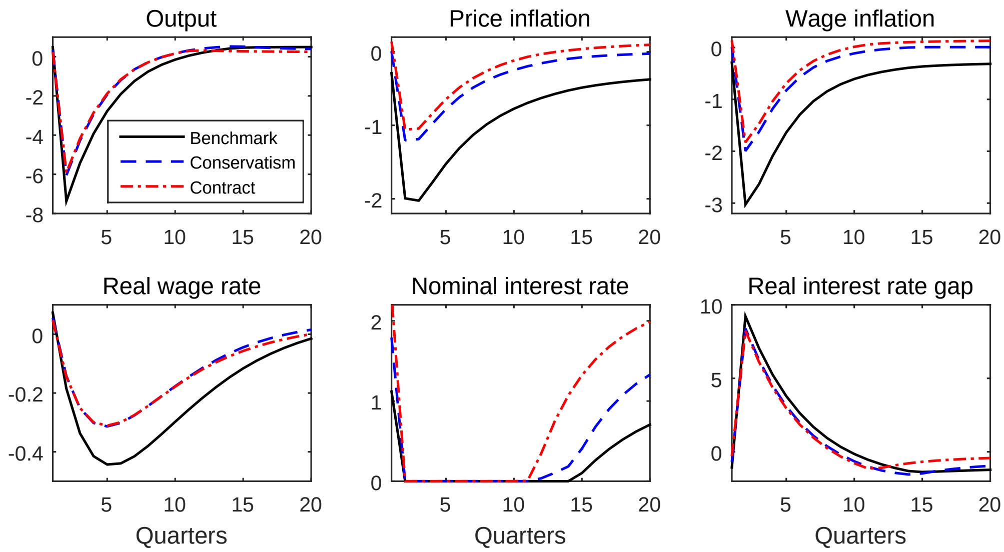


Note: The figure displays impulse responses to a negative natural real rate shock for the baseline quantitative model. Inflation rates and interest rates are expressed in annualized percentage terms. Output and the real wage rate are expressed in percentage deviations from the deterministic steady state. The real interest rate gap is defined as the discrepancy between the real interest rate and the natural real rate of interest.

conservatism improves stabilization outcomes in the liquidity trap situation. Interestingly, the conservative central bank keeps the policy rate at its zero floor for longer than the benchmark central bank does, reflecting the fact that inflation conservatism makes the policy rate more elastic. Importantly, the deflationary bias away from the ZLB is smaller if the central bank is conservative than if the central bank has the same preferences as society. The private sector's anticipation of a more muted deflationary bias under the conservative central bank results in a smaller ex-ante real interest rate gap at the ZLB and, consequently, less severe declines in output and inflation. The drop in output and inflation at the ZLB is even smaller under the optimal inflation contract regime. The latter aims to stabilize inflation at a strictly positive level, which further improves private sector expectations.

Figure 5 shows the equilibrium dynamics for the same liquidity trap scenario in the quantitative model with partial indexation of prices and nominal wages to past inflation. The decline in output, price inflation and nominal wage growth in response to the natural real rate shock is more severe than in the baseline model variant, and the differences in stabilization outcomes between the benchmark discretionary regime and the two delegation regimes becomes more distinct. In contrast to the baseline model, in the model with partial indexation the inflation conservative central bank lifts the policy rate from the lower bound three quarters earlier than the benchmark central bank, that is, the positive general equilibrium effects that lead to a smaller deflationary bias more than offset the effect of the higher policy rate elasticity on the equilibrium policy rate path.

Figure 6: Liquidity trap scenario in the quantitative model with partial indexation



Note: The figure displays impulse responses to a negative natural real rate shock for the model variant with partial price and wage indexation. Inflation rates and interest rates are expressed in annualized percentage terms. Output and the real wage rate are expressed in percentage deviations from the deterministic steady state. The real interest rate gap is defined as the discrepancy between the real interest rate and the natural real rate of interest.

H Robustness analysis for the quantitative model

H.1 Parameters

This section examines the sensitivity of the welfare results obtained for the baseline model with natural real rate and price mark-up shocks to some key parameters: the subjective discount factor β, the interest rate elasticity of consumption σ, and the degree of nominal price and wage rigidities α and α_W. All other parameters are calibrated according to the baseline parameterization of the quantitative model. We focus on the comparison between the benchmark discretionary regime and the optimal conservative regime. Table 5 reports the results.

Table 5: Inflation conservatism vs. benchmark regime for alternative parameter values

	Baseline	Lower β	Higher σ	Lower α, α_W
Parameter value(s)		0.9932	1	0.68
$\bar{\lambda}$	0.0103	0.0103	0.0097	0.0142
Optimized λ	0.002	0.003	0.0015	0.001
Welfare gain (in %)	27	13	43	48

Lowering the discount factor to 0.9932 increases the annualized deterministic steady state real interest rate to 2.74%. The larger steady state buffer to the ZLB implies that the ZLB constraint is ceteris paribus binding less often. Consequently, the welfare gains from inflation conservatism are smaller than for the baseline calibration. A higher interest rate elasticity of output σ increases the contractionary effect of a given positive gap between the actual real interest rate and the natural real rate. Increasing σ from 1/1.22 to 1 thus raises the welfare gains from inflation conservatism and reduces the optimal weight in the central bank's objective function on the output gap term. Finally, more flexible prices and nominal wages ceteris paribus heighten the decline in output and inflation in a liquidity trap (e.g. Werning, 2012) and increase the deflationary bias in those states where the ZLB is not binding. Therefore, the welfare gains from inflation conservatism increase with the degree of price and wage flexibility, and the optimal relative weight on output gap stabilization in the central bank's objective function becomes smaller.

H.2 Model with wage mark-up shocks

This section reports results on optimized delegation parameters, welfare, and the frequency of ZLB events for the quantitative model when the price mark-up shock is replaced with a wage mark-up shock. Specifically, u_t is dropped from the New Keynesian price Phillips curve and the New Keynesian wage Phillips curve is augmented with an *i.i.d.* shock e_t

$$\pi_t^W - \tau_w \pi_{t-1} = \kappa_w \left(\left(\sigma^{-1} + \frac{\eta}{1-\gamma} \right) y_t - w_t \right) + \beta (E_t \pi_{t+1}^W - \tau_w \pi_t) + e_t, \tag{H.1}$$

where the standard deviation of e_t is set to 0.05. All other parameters are calibrated according to the baseline parameterization of the quantitative model. Table 6 reports the results for the alternative monetary policy regimes.

Table 6: Results for the quantitative model with wage mark-up shocks

Regime	Optimized policy parameter (benchmark)	Welfare (in %)	Welfare gain over benchmark (%)	ZLB frequency (in %)
Benchmark discretion	-	-0.073	0	30
Inflation conservatism	$\lambda = 0.003$ (0.01)	-0.050	32	34
Inflation contract	$f = 0.20$ (0)	-0.027	63	25
Wage inflation conserv.	$\lambda_\pi = 5$ (1)	-0.030	59	31

Note: The non-optimized policy parameters have the following values. Benchmark discretion: $\lambda_\pi = 1$, $\lambda = 0.010$, $f = 0$. Inflation conservatism: $\lambda_\pi = 1$, $f = 0$. Inflation contract: $\lambda_\pi = 1$, $\lambda = 0.010$. Wage inflation conservatism: $\lambda = 0.003$, $f = 0$.

The results are qualitatively very similar to those obtained for the baseline model with price mark-up shocks. The only exception is the wage inflation conservatism regime (3rd row of the table). In the baseline model with price mark-up shocks it is desirable to reduce the relative weight on price inflation stability in the central bank's objective function conditional on setting the relative weight on output gap stability equal to its optimized value. Instead, in the model with wage mark-up shocks it is desirable to increase the relative weight on price inflation stability in the central bank's objective.[25] Hence, the desirability of wage inflation conservatism observed for the baseline model is sensitive to the type of shocks buffeting the economy.

I Comparison of inflation conservatism to price-level targeting

In the main manuscript, we have focused our analysis on institutional configurations of the central bank's objective that prevail the (flexible) inflation-targeting framework currently common to all major central banks in practice. This section compares the inflation conservative central banker to a central banker who aims to stabilize the price level $p_t \equiv p_{t-1} + \pi_t$. Eggertsson and Woodford (2003) have shown that a central bank that can commit to a targeting rule that requires to stabilize a weighted sum of the price level and the output gap (subject to the ZLB constraint) achieves a welfare level that is close to the one obtained under the optimal commitment policy. Here, we consider the desirability of price-level targeting (PLT) in the context of optimal discretionary policies. We use the quantitative model, where the parameters are calibrated according to the baseline parameterization of the model summarized in Table 1 of the main manuscript. To economize on the number of state variables, we abstract from price mark-up shocks and drop u_t from the New Keynesian Phillips curve.

The objective function of a discretionary central bank with a price-level target reads as follows

$$V_t^{CB} = -\frac{1}{2}\left[(1-\lambda_p)\left(\pi_t^2 + \bar{\lambda} y_t^2 + \bar{\lambda}_W (\pi_t^W)^2\right) + \lambda_p p_t^2\right] + \beta E_t V_{t+1}^{CB}, \tag{I.1}$$

where $\lambda_p \in [0,1]$. For $\lambda_p = 0$, the central bank's objective is identical to society's objective. For $\lambda_p = 1$, the central bank is only concerned with the stabilization of the expected discounted sum of current and future price levels. Table 7 summarizes the results.

Table 7: Results for inflation conservatism vs. price-level targeting

Regime	Optimized policy parameter	Welfare (in %)	ZLB frequency (in %)
Benchmark discretion	-	-0.052	31
Inflation conservatism	$\lambda = 0$	-0.015	31
Price-level targeting	$\mu = 1$	-0.003	17

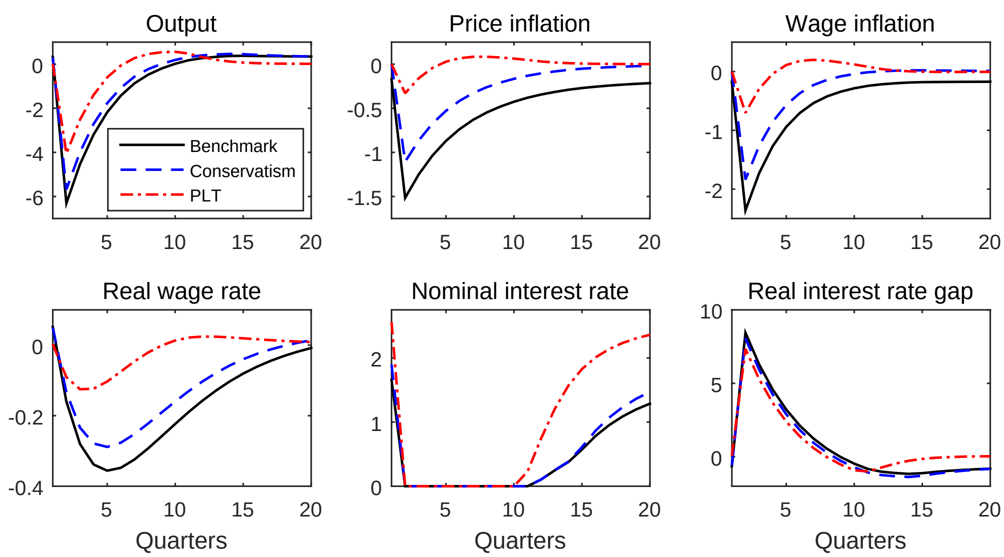
Strict PLT, i.e. $\lambda_p = 1$, turns out to be the optimal configuration. Welfare under the strict PLT regime is higher than under the inflation conservatism regime. In light of the analysis by Eggertsson and Woodford (2003) this result is not too surprising. Augmenting the objective of a discretionary central bank with a price-level target induces a desirable form of history dependence into the policymaking process. In particular, if today's inflation rate is too low and the price level falls below its target then the central bank needs to engineer higher inflation in the future to bring

[25] The optimal value for λ_π reported in Table 6 is a corner solution since we put an upper bound of 5 on the grid for λ_π.

the price level back to its target path. Forward-looking agents thus anticipate that a liquidity trap event with low inflation will be followed by a transitory inflationary boom. Expectations of high inflation in the future lower the ex-ante real interest rate and mitigate the decline in output and inflation at the ZLB.

This is illustrated in Figure 7 which shows impulse responses to a large negative natural real rate shock when the economy is initially at the risky steady state. Under strict PLT (red dash-dotted

Figure 7: Impulse responses to a contractionary natural real rate shock



Note: The figure displays impulse responses to a negative natural real rate shock. Inflation rates and interest rates are expressed in annualized percentage terms. Output and the real wage rate are expressed in percentage deviations from the deterministic steady state. The real interest rate gap is defined as the discrepancy between the real interest rate and the natural real rate of interest.

lines), output, price inflation and wage inflation fall much less than under the benchmark regime (black solid lines) and the inflation conservative regime (blue dashed lines), and the downturn is followed by a temporary boom in output and inflation. Similar to the inflation conservative central banker that focuses only on inflation stabilization, PLT eliminates the deflationary bias away from the ZLB.

J Existence of other Markov-Perfect equilibria in the two-state model

While the paper focuses on the standard Markov-Perfect equilibrium in which the ZLB constraint binds in the low state but not in the high state, there are potentially three other types of Markov-Perfect equilibria: i) one in which the ZLB constraint binds in both states (*the deflationary Markov-Perfect equilibrium*), ii) one in which the ZLB constraint does not bind in both states (*the ZLB-free Markov-Perfect equilibrium*), and iii) one in which the ZLB binds in the high state but not in the low state (*the topsy-turvy Markov-Perfect equilibrium*). In this section, we examine whether and under what conditions any of these other types of Markov-Perfert equilibria exist. Our main results are that i) the conditions for the existence of the deflationary Markov-Perfect equilibrium are the same as those for the existence of the standard Markov-Perfect equilibrium and ii) the other two types do not exist under any parameter configurations.[26]

J.1 Existence of the deflationary Markov-Perfect equilibrium

The deflationary Markov-Perfect equilibrium is given by a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves the following system of linear equations

$$y_H = \big[(1-p_H)y_H + p_H y_L\big] + \sigma\big[(1-p_H)\pi_H + p_H\pi_L - i_H + r^n_H\big], \tag{J.1}$$
$$\pi_H = \kappa y_H + \beta\big[(1-p_H)\pi_H + p_H\pi_L\big], \tag{J.2}$$
$$i_H = 0, \tag{J.3}$$
$$y_L = \big[(1-p_L)y_H + p_L y_L\big] + \sigma\big[(1-p_L)\pi_H + p_L\pi_L - i_L + r^n_L\big], \tag{J.4}$$
$$\pi_L = \kappa y_L + \beta\big[(1-p_L)\pi_H + p_L\pi_L\big], \tag{J.5}$$
$$i_L = 0, \tag{J.6}$$

and satisfies the following two inequality constraints:

$$\phi_H < 0, \tag{J.7}$$
$$\phi_L < 0. \tag{J.8}$$

ϕ_H and ϕ_L denote the Lagrangean multipliers on the ZLB constraint in the high state and in the low state:

$$\phi_H := \lambda y_H + \kappa\pi_H, \tag{J.9}$$
$$\phi_L := \lambda y_L + \kappa\pi_L. \tag{J.10}$$

The following proposition states that the conditions for the existence of the deflationary Markov-Perfect equilibrium are identical to the conditions for the existence of the standard Markov-Perfect equilibrium.

Proposition 7: The deflationary Markov-Perfect equilibrium exists if and only if

$$p_L \leq p^*_L(\Theta_{(-p_L)}),$$
$$p_H \leq p^*_H(\Theta_{(-p_H)}),$$

[26] There is a continuum of sunspot equilibria which may randomly move between the standard and deflationary Markov-Perfect equilibria. Characterizing the conditions for the existence of such sunspot equilibria is outside the scope of the paper.

where the cutoff values $p_L^*(\Theta_{(-p_L)})$ and $p_H^*(\Theta_{(-p_H)})$ are defined by (B.28) and (B.36) in Appendix A. We first prove six preliminary propositions, then use them to prove Proposition 7.

Let

$$\tilde{A} := -\left(\frac{p_H}{\sigma\kappa}(1-\beta p_L+\beta p_H)+p_H\right), \tag{J.11}$$

$$\tilde{B} := -\tilde{A}-1, \tag{J.12}$$

and

$$\begin{aligned}\tilde{E} &:= \tilde{A}D-\tilde{B}C \\ &= -\tilde{A}+C,\end{aligned} \tag{J.13}$$

where C and $D<0$ are defined in (B.12) and (B.13).

Assumption 7.A: $\tilde{E}\neq 0$.

Throughout the proof, we will assume that Assumption 7.A holds.

Proposition 7.A: There exists a vector $\{y_H,\pi_H,i_H,y_L,\pi_L,i_L\}$ that solves (J.1)–(J.6).

Proof:

Rearranging the system of equations (J.1)–(J.6) and eliminating y_H and y_L, we obtain two unknowns for π_H and π_L in two equations:

$$\begin{aligned}&\begin{bmatrix}\tilde{A} & \tilde{B}\\ C & D\end{bmatrix}\begin{bmatrix}\pi_L\\ \pi_H\end{bmatrix}=\begin{bmatrix}r_H^n\\ r_L^n\end{bmatrix}\\ \Rightarrow&\begin{bmatrix}\pi_L\\ \pi_H\end{bmatrix}=\frac{1}{\tilde{E}}\begin{bmatrix}D & -\tilde{B}\\ -C & \tilde{A}\end{bmatrix}\begin{bmatrix}r_H^n\\ r_L^n\end{bmatrix}.\end{aligned} \tag{J.14}$$

Thus,

$$\pi_H := \frac{\tilde{A}}{\tilde{E}}r_L^n-\frac{C}{\tilde{E}}r_H^n \tag{J.15}$$

and

$$\pi_L := \frac{-\tilde{B}}{\tilde{E}}r_L^n+\frac{D}{\tilde{E}}r_H^n. \tag{J.16}$$

From the Phillips curves in both states, we obtain

$$y_H=\frac{(1-\beta)\tilde{A}-\beta p_H}{\kappa\tilde{E}}r_L^n-\frac{(1-\beta)C-\beta p_H}{\kappa\tilde{E}}r_H^n \tag{J.17}$$

and

$$y_L = \frac{(1-\beta)\tilde{A} + (1-\beta p_L)}{\kappa \tilde{E}} r_L^n - \frac{(1-\beta)C + (1-\beta p_L)}{\kappa \tilde{E}} r_H^n. \tag{J.18}$$

Proposition 7.B: Suppose (J.1)–(J.6) are satisfied. Then $\phi_L < 0$ only if $\tilde{E} > 0$.

Proof by contradiction:

First, notice that

$$\phi_L = \frac{1}{\tilde{E}} \left[-(1+C)\kappa r_H^n - (1-\beta)C\frac{\lambda}{\kappa} r_H^n - (1-\beta p_L)\frac{\lambda}{\kappa} r_H^n + (1+\tilde{A})\kappa r_L^n + \frac{\lambda}{\kappa}(1-\beta)\tilde{A} r_L^n + \frac{\lambda}{\kappa}(1-\beta p_L) r_L^n \right]. \tag{J.19}$$

Suppose that $\tilde{E} < 0$. From the equation above we know that, given $\tilde{E} < 0$, $\phi_L < 0$ if and only if

$$-(1+C)\kappa r_H^n - (1-\beta)C\frac{\lambda}{\kappa} r_H^n - (1-\beta p_L)\frac{\lambda}{\kappa} r_H^n + (1+\tilde{A})\kappa r_L^n + \frac{\lambda}{\kappa}(1-\beta)\tilde{A} r_L^n + \frac{\lambda}{\kappa}(1-\beta p_L) r_L^n > 0. \tag{J.20}$$

Collecting terms, this condition can be simplified to

$$\left(\kappa + \frac{\lambda}{\kappa}(1-\beta p_L)\right) [(1+A) r_L^n - (1+C) r_H^n] > 0. \tag{J.21}$$

From (J.13), we know that $\tilde{E} < 0$ if and only if $C < \tilde{A}$, where $\tilde{A} < 0$. Furthermore, from (B.12) we know that $C > -1$. Suppose $C \to -$; then $A > -1$, which proves that (J.21) cannot hold.

Proposition 7.C: Suppose (J.1)-(J.6) are satisfied and $\tilde{E} > 0$. Then $\phi_L < 0$ if $\phi_H < 0$.

Proof: This follows directly from noticing that

$$\phi_L = \phi_H + \frac{\kappa^2 + \lambda}{\kappa \tilde{E}} (r_L^n - r_H^n). \tag{J.22}$$

Proposition 7.D: Suppose (J.1)–(J.6) are satisfied and $\tilde{E} > 0$. Then $\phi_H < 0$ if and only if $p_H < p_H^*(\Theta_{-p_H})$.

Proof:

First, notice that

$$\phi_H = \frac{1}{\tilde{E}}\left([-C\kappa - (1-\beta)\frac{\lambda}{\kappa}C + \beta\frac{\lambda}{\kappa}p_H]r_H^n + [\kappa\tilde{A} + (1-\beta)\frac{\lambda}{\kappa}\tilde{A} - \beta\frac{\lambda}{\kappa}p_H]r_L^n\right). \tag{J.23}$$

Since $\tilde{E} > 0$, $\phi_H < 0$ requires

$$-C\kappa - (1-\beta)\frac{\lambda}{\kappa}C + \beta\frac{\lambda}{\kappa}p_H]r_H^n + [\kappa\tilde{A} + (1-\beta)\frac{\lambda}{\kappa}\tilde{A} - \beta\frac{\lambda}{\kappa}p_H]r_L^n < 0. \tag{J.24}$$

Multiplying both sides by $\frac{\kappa}{\Gamma}\frac{1}{r_L^n}$ and collecting terms, we get

$$\begin{aligned}&-\frac{\beta}{\sigma\kappa}p_H^2 - \left(\frac{(1-\beta p_L) + (1-p_L)\beta\frac{r_H^n}{r_L^n}}{\sigma\kappa} + \frac{\kappa^2 + (1-\beta\frac{r_H^n}{r_L^n})\lambda}{\Gamma}\right)p_H \\ &-\left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right)\frac{r_H^n}{r_L^n} > 0.\end{aligned} \tag{J.25}$$

Let

$$P(p_H) := \phi_2 p_H^2 + \phi_1 p_H + \phi_0 : \tag{J.26}$$

where

$$\begin{aligned}\phi_0 &:= -\left[\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right]\frac{r_H^n}{r_L^n} \\ \phi_1 &:= -\frac{(1-\beta p_L) + (1-p_L)\beta\frac{r_H^n}{r_L^n}}{\sigma\kappa} - \frac{\kappa^2 + (1-\beta\frac{r_H^n}{r_L^n})\lambda}{\Gamma} \\ \phi_2 &:= -\frac{\beta}{\sigma\kappa} < 0,\end{aligned} \tag{J.27}$$

which is similar to the definition in Appendix A. $\phi_0 > 0$ and $\phi_2 < 0$ imply that one root of (J.26) is non-negative and $\phi_H < 0$ if and only if p_H is below this non-negative root, given by

$$p_H^*(\Theta_{-p_H}) := \frac{-\phi_1 - \sqrt{\phi_1^2 - 4\phi_0\phi_2}}{2\phi_2}. \tag{J.28}$$

This completes the proof of Proposition 7.D.

Proposition 7.E: $\tilde{E} > 0$ and $p_H < p_H^*(\Theta_{-p_H})$ only if $E(\lambda) < 0$.

Proof:

Suppose that $\tilde{E} > 0$ and $p_H < p_H^*(\Theta_{-p_H})$. Then $\tilde{E} + P(p_H) > 0$.

$$
\begin{aligned}
\tilde{E} + P(p_H) = &\frac{\beta}{\sigma\kappa}p_H^2 + \left(1 + \frac{(1-\beta p_L) + (1-p_L)\beta}{\sigma\kappa}\right)p_H + \left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right) \\
&- \frac{\beta}{\sigma\kappa}p_H^2 - \left(\frac{(1-\beta p_L) + (1-p_L)\beta\frac{r_H^n}{r_L^n}}{\sigma\kappa} + \frac{\kappa^2 + (1-\beta\frac{r_H^n}{r_L^n})\lambda}{\Gamma}\right)p_H \\
&- \left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right)\frac{r_H^n}{r_L^n} \\
&= \left[\beta\frac{\lambda}{\Gamma}p_H - \beta\frac{1-p_L}{\sigma\kappa}p_H - \left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right)\right]\left(\frac{r_H^n}{r_L^n} - 1\right). \qquad \text{(J.29)}
\end{aligned}
$$

Since $\left(\frac{r_H^n}{r_L^n} - 1\right) < 0$, the following condition has to hold:

$$
\beta\frac{\lambda}{\Gamma}p_H - \beta\frac{1-p_L}{\sigma\kappa}p_H - \left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right) < 0. \qquad \text{(J.30)}
$$

Collecting terms, we get

$$
-\Gamma\frac{1}{\sigma\kappa}\beta p_L^2 + \Gamma\left[\frac{1}{\sigma\kappa}(1+\beta+\beta p_H) + 1\right]p_L + \beta\lambda p_H - \Gamma\frac{1}{\sigma\kappa}(1+\beta p_H) = E(\lambda) < 0. \qquad \text{(J.31)}
$$

This completes the proof of Proposition 7.E. Note that Proposition 7.E holds independently of whether the system of linear equations (J.1)–(J.6) is satisfied or not.

Proposition 7.F: $E(\lambda) < 0$ **only if** $\tilde{E} > 0$**.**

Proof: This follows directly from noticing that

$$
\tilde{E} = -\frac{E(\lambda)}{\Gamma} + \frac{\beta\lambda}{\Gamma}p_H + \frac{1}{\sigma\kappa}\left(\beta p_H + 1 + \beta(1-p_L)\right)p_H. \qquad \text{(J.32)}
$$

Note that Proposition 7.F holds independently of whether the system of linear equations (J.1)–(J.6) is satisfied or not.

With these six preliminary propositions (7.A–7.F), we are ready to prove Proposition 7.

Proposition 7: There exists a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ **that solves the system of linear equations (J.1)–(J.6) and satisfies** $\phi_L < 0$ **and** $\phi_H < 0$ **if and only if** $p_L < p_L^*(\Theta_{-p_L})$ **and** $p_H < p_H^*(\Theta_{-p_H})$**.**

Proof of "if" part: According to Proposition 7.A, there exists a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves (J.1)–(J.6). Suppose that $p_L < p_L^*(\Theta_{-p_L})$ and $p_H < p_H^*(\Theta_{-p_H})$. According to Proposition 1.C (which does not rely on the system of linear equations), $E(\lambda) < 0$. According to Proposition

7.F, then $\tilde{E} > 0$. According to Proposition 7.D, this implies $\phi_H < 0$. Finally, according to Proposition 7.C, this implies $\phi_L < 0$. This completes the proof of "if" part.

Proof of "only if" part: According to Proposition 7.A, there exists a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves (J.1)–(J.6). Suppose that $\phi_L < 0$ and $\phi_H < 0$. According to Proposition 7.B, $\tilde{E} > 0$. According to Proposition 7.D, then $p_H < p_H^*(\Theta_{-p_H})$. According to Proposition 7.E, this implies $E(\lambda) < 0$. According to Proposition 1.C (which does not rely on the system of linear equations), $p_L < p_L^*(\Theta_{-p_L})$. This completes the proof of the "only if" part.

J.2 Nonexistence of the topsy-turvy Markov-Perfect equilibrium

The topsy-turvy Markov-Perfect equilibrium is given by a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves the following system of linear equations

$$y_H = \big[(1-p_H)y_H + p_H y_L\big] + \sigma\big[(1-p_H)\pi_H + p_H\pi_L - i_H + r_H^n\big], \tag{J.33}$$

$$\pi_H = \kappa y_H + \beta\big[(1-p_H)\pi_H + p_H\pi_L\big], \tag{J.34}$$

$$i_H = 0, \tag{J.35}$$

$$y_L = \big[(1-p_L)y_H + p_L y_L\big] + \sigma\big[(1-p_L)\pi_H + p_L\pi_L - i_L + r_L^n\big], \tag{J.36}$$

$$\pi_L = \kappa y_L + \beta\big[(1-p_L)\pi_H + p_L\pi_L\big], \tag{J.37}$$

$$0 = \lambda y_L + \kappa\pi_L, \tag{J.38}$$

and satisfies the following two inequality constraints:

$$\phi_H < 0, \tag{J.39}$$

$$i_L > 0. \tag{J.40}$$

ϕ_H denotes the Lagrangean multiplier on the ZLB constraint in the high state:

$$\phi_H := \lambda y_H + \kappa\pi_H. \tag{J.41}$$

Proposition 8: The topsy-turvy Markov-Perfect equilibrium does not exist.

We first prove three preliminary propositions, then use them to prove Proposition 8.

Let

$$\hat{C}(\lambda) := \kappa^2 + \lambda\left(1 - \beta p_L\right), \tag{J.42}$$

$$\hat{D}(\lambda) := -\beta\lambda\left(1 - p_L\right), \tag{J.43}$$

and

$$\hat{E}(\lambda) := \tilde{A}\hat{D}(\lambda) - \tilde{B}\hat{C}(\lambda), \tag{J.44}$$

where $\tilde{A}$ and $\tilde{B}$ are defined in (J.11) and (J.12).

Assumption 8.A: $\hat{E}(\lambda) \neq 0$.

Throughout the proof, we will assume that Assumption 8.A holds.

Proposition 8.A: There exists a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves (J.33)–(J.38).

Proof:

Rearranging the system of equations (J.33)–(J.38) and eliminating y_H and y_L, we obtain two unknowns for π_H and π_L in two equations:

$$\begin{aligned} &\begin{bmatrix} \tilde{A} & \tilde{B} \\ \hat{C}(\lambda) & \hat{D}(\lambda) \end{bmatrix} \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \begin{bmatrix} r_H^n \\ 0 \end{bmatrix} \\ \Rightarrow &\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \frac{1}{\hat{E}(\lambda)} \begin{bmatrix} \hat{D}(\lambda) & -\tilde{B} \\ -\hat{C}(\lambda) & \tilde{A} \end{bmatrix} \begin{bmatrix} r_H^n \\ 0 \end{bmatrix}. \end{aligned} \tag{J.45}$$

Thus,

$$\pi_H := -\frac{\hat{C}(\lambda)}{\hat{E}(\lambda)} r_H^n \tag{J.46}$$

and

$$\pi_L := \frac{\hat{D}(\lambda)}{\hat{E}(\lambda)} r_H^n. \tag{J.47}$$

From the Phillips Curves in both states, we obtain

$$y_H = -\frac{(1-\beta)\hat{C}(\lambda) + \beta p_H \Gamma}{\kappa \hat{E}(\lambda)} r_H^n \tag{J.48}$$

and

$$y_L = -\frac{(1-\beta p_L)\hat{D}(\lambda) + (1-p_L)\beta\hat{C}(\lambda)}{\kappa \hat{E}(\lambda)} r_H^n. \tag{J.49}$$

Proposition 8.B: Suppose (J.33)–(J.38) are satisfied. Then $\phi_H < 0$ if and only if $\hat{E}(\lambda) > 0$.

Proof:

Notice that

$$\phi_H = -\frac{1}{\kappa \hat{E}(\lambda)} \left((\kappa^2 + (1-\beta))\hat{C}(\lambda) + \beta p_H \Gamma \right) r_H^n, \tag{J.50}$$

where $(\kappa^2 + (1-\beta))\hat{C}(\lambda) + \beta p_H \Gamma > 0$ and $r_H^n > 0$. Hence, $\phi_H < 0$ if and only if $\hat{E}(\lambda) > 0$.

Proposition 8.C: Suppose (J.33)–(J.38) are satisfied. Then $i_L > 0$ only if $\hat{E}(\lambda) < 0$.

Proof:

Notice that

$$\begin{aligned}
i_L = r_L^n - \frac{1}{\kappa \hat{E}(\lambda)} \Bigg[& -\kappa p_L \hat{D}(\lambda) + \kappa(1-p_L)\hat{C}(\lambda) + \frac{1}{\sigma}(1-p_L) \\
& \Big((1-\beta)\hat{C}(\lambda) + \beta p_H \Gamma + (1-\beta p_L)\hat{D}(\lambda) + (1-p_L)\beta \hat{C}(\lambda) \Big) \Bigg] r_H^n, && \text{(J.51)} \\
= r_L^n - \frac{1}{\kappa \hat{E}(\lambda)} \Bigg[& -\kappa p_L \hat{D}(\lambda) + \kappa(1-p_L)\hat{C}(\lambda) + \frac{1}{\sigma}(1-p_L) \\
& \Big((1-\beta p_L)\hat{C}(\lambda) + \beta p_H \Gamma + (1-\beta p_L)\hat{D}(\lambda) \Big) \Bigg], && \text{(J.52)} \\
= r_L^n - \frac{1}{\kappa \hat{E}(\lambda)} \Bigg[& -\kappa p_L \hat{D}(\lambda) + \kappa(1-p_L)\hat{C}(\lambda) + \frac{1}{\sigma}(1-p_L)\left(\beta p_H + (1-\beta p_L)\right)\Gamma \Bigg] r_H^n, && \text{(J.53)}
\end{aligned}$$

where $-\kappa p_L \hat{D}(\lambda) + \kappa(1-p_L)\hat{C}(\lambda) + \frac{1}{\sigma}(1-p_L)\left(\beta p_H + (1-\beta p_L)\right)\Gamma > 0$, $r_H^n > 0$, and $r_L^n < 0$. Hence, $i_L > 0$ only if $\hat{E}(\lambda) < 0$.

With these three preliminary propositions (8.A-8.C), we are ready to prove Proposition 8.

Proposition 8: There exists no vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves the system of linear equations (J.33)–(J.38) and satisfies $i_L > 0$, $\phi_H < 0$.

Proof by contradiction: According to Proposition 8.A, there exists a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves (J.33)–(J.38). Suppose that $\phi_H < 0$ and $i_L > 0$. According to Proposition 8.B, $\phi_H < 0$ implies $\hat{E}(\lambda) > 0$. According to Proposition 8.C, $i_L > 0$ implies $\hat{E}(\lambda) < 0$, which contradicts ($i_L > 0$, $\phi_H < 0$).

J.3 Nonexistence of the ZLB-free Markov-Perfect equilibrium

The ZLB-free Markov-Perfect equilibrium is given by a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves the following system of linear equations

$$y_H = \big[(1-p_H)y_H + p_H y_L\big] + \sigma\big[(1-p_H)\pi_H + p_H\pi_L - i_H + r_H^n\big], \tag{J.54}$$

$$\pi_H = \kappa y_H + \beta\big[(1-p_H)\pi_H + p_H\pi_L\big], \tag{J.55}$$

$$0 = \lambda y_H + \kappa\pi_H, \tag{J.56}$$

$$y_L = \big[(1-p_L)y_H + p_L y_L\big] + \sigma\big[(1-p_L)\pi_H + p_L\pi_L - i_L + r_L^n\big], \tag{J.57}$$

$$\pi_L = \kappa y_L + \beta\big[(1-p_L)\pi_H + p_L\pi_L\big], \tag{J.58}$$

$$0 = \lambda y_L + \kappa\pi_L, \tag{J.59}$$

and satisfies the following two inequality constraints:

$$i_H > 0, \tag{J.60}$$

$$i_L > 0. \tag{J.61}$$

Proposition 9: The ZLB-free Markov-Perfect equilibrium does not exist.

Proof:

Let

$$\hat{E} = \big[1 - \beta(1-p_H) + \frac{\kappa^2}{\lambda}\big](1 - \beta p_L + \frac{\kappa^2}{\lambda}) - \beta^2 p_H(1-p_L). \tag{J.62}$$

Assumption 9.A: $\hat{E} \neq 0$.

Throughout the proof, we will assume that Assumption 9.A holds.

Notice that i_H and i_L only appear in the consumption Euler equations. Thus, we can first find a vector of $\{y_H, \pi_H, y_L, \pi_L\}$ that satisfies the Phillips curves and the government's optimality condition in both states, then use the two consumption Euler equations to find i_H and i_L. Rearranging the system of equations (J.55), (J.56), (J.58), and (J.59) and eliminating y_H and y_L, we obtain two unknowns for π_H and π_L in two equations:

$$\pi_H = -\frac{\kappa^2}{\lambda}\pi_H + \beta\big[(1-p_H)\pi_H + p_H\pi_L\big] \tag{J.63}$$

and

$$\pi_L = -\frac{\kappa^2}{\lambda}\pi_L + \beta\big[(1-p_L)\pi_H + p_L\pi_L\big] \tag{J.64}$$

$$\Rightarrow \begin{bmatrix} 1-\beta(1-p_H)+\frac{\kappa^2}{\lambda} & -\beta p_H \\ -\beta(1-p_L) & 1-\beta p_L+\frac{\kappa^2}{\lambda} \end{bmatrix} \begin{bmatrix} \pi_H \\ \pi_L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{J.65}$$

$$\Rightarrow \begin{bmatrix} \pi_H \\ \pi_L \end{bmatrix} = \frac{1}{\hat{E}} \begin{bmatrix} 1-\beta p_L+\frac{\kappa^2}{\lambda} & \beta(1-p_L) \\ \beta p_H & 1-\beta(1-p_H)+\frac{\kappa^2}{\lambda} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{J.66}$$

From the Phillips curves in both states, we obtain

$$y_H = 0 \tag{J.67}$$

and

$$y_L = 0. \tag{J.68}$$

From the consumption Euler equations in both states, we obtain

$$i_H = r_H^n > 0 \tag{J.69}$$

and

$$i_L = r_L^n < 0. \tag{J.70}$$

These two inequalities hold because we assume that $r_H^n > 0$ and $r_L^n < 0$. Thus, the inequality condition for the policy rate in the low state is violated. Accordingly, there is no vector that solves (J.54)–(J.59) and satsifies both $i_H > 0$ and $i_L > 0$.